Books I and IV of Kūshyār ibn Labbān's *Jāmi*^{c} *Zīj*: An Arabic astronomical handbook by an eleventh-century Iranian scholar

De boeken I en IV van de *Jāmi*^{c} *Zīj* van Kūshyār ibn Labbān: een Arabisch sterrenkundig handboek van een elfde-eeuwse Iraanse geleerde

(met een samenvatting in het Nederlands)

Proefschrift

ter verkrijging van de graad van doctor aan de Universiteit Utrecht op gezag van de rector magnificus, prof. dr. W.H. Gispen, ingevolge het besluit van het college voor promoties in het openbaar te verdedigen op woensdag 20 december 2006 des middags te 12.45 uur

door

Mohammad Bagheri

geboren op 5 januari 1951 te Rasht, Iran

Promotoren: Prof. dr. H.J.M. Bos
Prof. dr. J.P. Hogendijk

Beoordelingscommissie: Dr. B. van Dalen (Frankfurt am Main)
Prof. dr. J.J. Duistermaat (Utrecht)
Prof. dr. T. de Jong (Amsterdam)
Prof. dr. D.A. King (Frankfurt am Main)
Prof. dr. R. Kruk (Leiden)

ISBN-10: 90-393-4388-8
ISBN-13: 978-90-393-4388-3
2000 Mathematics Subject Classification: 01A30, 01A20, 01A35.

Acknowledgements

I would like to express my profound gratitude to Prof. Dr. Henk J. M. Bos for accepting the supervision of my PhD dissertation and for his unfailing encouragement and support throughout the period of my research for the present work. I am also grateful to Prof. Dr. Jan P. Hogendijk without whose scientific, practical and spiritual support, I would never have been able to carry out this job. I also thank Prof. E. S. Kennedy (Princeton University), a leading scholar in the history of Islamic exact sciences for his precious scientific support especially in the terminology of Islamic astronomy. Prof. Dr. Fuat Sezgin (Director of the Institute for the History of Arabic-Islamic Science, Johann Wolfgang Goethe University, Frankfurt) offered me the opportunity to work on my PhD dissertation at his Institute during the period June-August 2005. I greatly appreciate this kind gesture which was decisive for my work and I will always remember the ready assistance I received from colleagues at his Institute. He also helped me in preparing some manuscript copies which were vital for my work. I sincerely thank Dr. Hushang A'lam (Encyclopaedia Islamica Foundation, Tehran) and Dr. Benno van Dalen (Institute for the History of Science, Johann Wolfgang Goethe University, Frankfurt) who supported me with their scholarly comments in preparing the translation and commentary of Kūshyār's *zīj*.

Prof. Gregg De Young (American University of Cairo) kindly helped me with the difficult task of preparing manuscript copies from Egypt and I am grateful to him for his arduous efforts. I owe a lot to the late Mr. Abu'l-Qasem Qurbani, a pioneering figure in the studies of the history of mathematics and astronomy in Iran, who always encouraged me in my work and provided me with a copy of one of the Istanbul manuscripts. The memory of him will always remain alive in our hearts and in all our works. I also wish to thank Prof. Dr. E. Ihsanoğlu (IRCICA, Istanbul), Prof. Michio Yano (Kyoto Sangyo University, Japan), Dr. Gleb Mikhailov (Russia), Mr. M. Mar'ashi Najafi (Qum, Iran), Mr. A. Hadzad (Germany) and Mr. S. Farrokh Seresht (The Netherlands) for their assistance in preparing the manuscript copies.

I thank my dear friends, Mr. Claudio Cecotti (Italy) and Mr. Hüseyin Şen (the Netherlands), who have been very helpful with the text processing and computer work. My Institute, Encyclopaedia Islamica Foundation (Tehran) has also been most supportive and encouraging of my work. I especially wish to thank Mr. Hassan Taromi (Scientific Vice Director) for his full and friendly cooperation and my colleagues Mr. A. Sobut and Dr. S. 'Udi who supported me with their expert knowledge of the Arabic language. Last but not the least, I want to express my heart-felt gratitude to my mother Mahboob (Effat) to whom I owe all my achievements, and to my wife Nepton who has supported me throughout with compassion and admirable patience, and to whom I dedicate this thesis.

Mohammad Bagheri
Wassenaar (The Netherlands), October 2006

Contents

General Introduction

1. Introduction

Between the eighth and fifteenth centuries A.D., the main centers for the study of the exact sciences were in the Islamic world. Astronomy was one of the most important sciences studied in medieval Islamic civilization, not only because of its relevance to Islamic timekeeping and prayers, but also for its own sake and because of its relationship to astrology. By the eighth century, the mathematical and astronomical traditions of pre-Islamic Iran and India had been assimilated. From the ninth century onwards, the Islamic astronomers studied the Greek tradition, and several Arabic translations were produced of the most important astronomical work of Greek antiquity, the *Almagest* of Ptolemy. The Ptolemaic models of planetary motion are physically 'wrong', because the earth is considered as the center of the universe. Important basic features of the Ptolemaic models can be mathematically transformed to heliocentric ones by a simple change of coordinates. This explains to a modern reader why these Ptolemaic models, which were used by Islamic astronomers, can be fitted very well to observed positions of the planets. The resulting predictions are so accurate that the errors can hardly be noticed with naked eye.

From the ninth century onwards, the Islamic astronomers made new observations in order to check and correct some of the parameter values that were used in Ptolemy's astronomical models, such as the value of the solar eccentricity. In the tenth century and later, the Ptolemaic models were themselves modified. For example, it was discovered that the apogee (point of furthest distance) of the sun had a very slow motion with respect to the signs of the zodiac and also with respect to fixed stars. We shall not be concerned with the modifications of the Ptolemaic models made for purely philosophical reasons in the Eastern Islamic world in the twelfth century and later, in order to make these models completely consistent with the Aristotelian dogma of uniform circular motion.

Libraries in and outside the present Islamic world contain thousands of medieval Arabic astronomical manuscripts. Only few of these have been studied to date.[1] An important genre in the Arabic astronomical literature is a group of treatises called *zīj*es (plural of *zīj*). From the ninth century onwards, the Islamic tradition produced constantly astronomical handbooks with instructions and tables for the computation of solar, lunar and planetary positions. These handbooks were often also provided with auxiliary trigonometrical tables, tables for lunar visibility and prayer times, geographical tables, tables of stellar positions, and astrological tables. The total number of pages with tables in a single handbook may be 150-200. These handbooks were called *zīj*, from Old Persian *zīg*, meaning "thread" or "chord". By extension the word came to mean "the set of parallel threads making up the warp of a fabric". Then, since the closely drawn vertical lines of a numerical table are similar to the parallel strings of a textile, the meaning was further extended to include the former. And finally, the word came to denote whole sets of astronomical tables with instructions.

More than 200 *zīj*es are known to have been written, of which more than 100 are extant, sometimes in many different manuscripts. Surveys of the entire *zīj* literature can be found in [Kennedy 1956]. A new survey of Islamic *zīj*es is currently under preparation by Dr. Benno van Dalen. For more information about *zīj*es see [King and Samsó 2001]. Only a few *zīj*es have been published to date. The *zīj* of al-Battānī appeared between 1899 and 1907 in a critical Arabic edition with a Latin translation in three volumes [al-Battānī 1899-1907]. Al-Bīrūnī's *al-Qānūn al-Mas'ūdī* or *Canon Masudicus* in 13 Books can also be considered a *zīj*. This was published in India between 1954 and 1956 in an uncritical edition [al-Bīrūnī 1954-1956]. The text was not analyzed mathematically and astronomically, and consequently many errors in the manuscripts which were used remained uncorrected. (These errors were not corrected in the Russian translation published in 1973 and 1976 either [al-Bīrūnī 1973-1976]). The Persian introductions to the 15th century *zīj* of Ulugh Beg (but only with the chronological tables) appeared in an edition with French translation in [Sédillot 1847 and 1853]. Some medieval Latin reworkings of Arabic *zīj*es have also been published (e.g., [Al-Khwārimī 1962]). Modern researchers have studied many other Arabic *zij*es, and have often published smaller sections thereof, but no edition with

[1] The most up-to-date lists of mathematical and astronomical manuscripts can be found in [Sezgin 1974 and 1976; Rosenfeld & Ihsanoğlu 2003].

translation of whole *zījes* has appeared, and no Arabic *zīj* has ever been translated into English hitherto. The *zījes* contain a mass of material which, once properly evaluated, will lead to a detailed knowledge of the development and transmission of astronomical and mathematical knowledge in Islamic civilization, and thus shed light on an aspect of Islamic culture which is severely underrepresented and underestimated in modern historical accounts and political discussions. My dissertation is a small contribution to the enormous amount of research work that still remains to be done.

The present dissertation contains an edited Arabic version with an English translation of a large part of the *Jāmiʿ Zīj* ("Comprehensive astronomical handbook with tables") by the Iranian mathematician and astronomer Kūshyār ibn Labbān (ca. 1025 A.D.). Kūshyār's *Jāmiʿ Zīj* has drawn the attention of modern historians of science, and several sections of it have been published on the basis of a few Arabic manuscripts. These publications will be discussed in Part 3 below. The whole *Jāmiʿ Zīj* has never been critically edited. The part which I am presenting in this dissertation consists of Books I and IV, containing the astronomical instructions (Book I) and the corresponding "Proofs" (Book IV). I am currently preparing editions and translations of the remaining Books II (numerical tables) and III (cosmology). The *Jāmiʿ Zīj* will be the first one to be translated into English in its entirety.

The reader may wonder why I have chosen Kūshyār's *zīj* from the 100 or so extant unpublished *zījes*. The fact that Kūshyār was a native my homeland Gīlān (north Iran) constitutes an emotional reason. In addition, there are good scientific and historical reasons. Kūshyār's *zīj* dates back to a relatively early period, and unlike most other early *zījes*, Kūshyār presents not only astronomical instructions (in Book I), but also the corresponding "Proofs" (in Book IV). Thus Kūshyār's work, unlike many other similar works, allows us to have a more intimate knowledge of the eleventh-century astronomical view and thought. As we shall see in Part 4 below, Kūshyār was a competent mathematician, who had studied the works of his predecessors critically, and who seems to be the author of some new methods in his *zīj* not to be found elsewhere. Also, Section I.1 of the *Jāmiʿ Zīj* is one of the earliest extant Arabic treatments of calendars, and provides important information especially on the old Persian calendar that was still in use in his time. The earliest documented change of one of the equations for Mars by a Muslim astronomer is that of the equation of centrum by Kūshyār [Van Dalen 2004a, 22] (cf. [Van Brummelen 1988, 268]). Finally, Kūshyār's work is nearly contemporary with al-Bīrūnī's colossal *Canon Masudicus*, which has already been mentioned, and which is probably the major work of Islamic astronomy (comparable to Ptolemy's *Almagest*). I hope that my edition of Kūshyār's work can serve as an incentive for future editions and translations of other *zīj*es, including the *Canon Masudicus* of al-Bīrūnī.

In Part 2 of this preface, I discuss the available information about Kūshyār's life and his other works. In Part 3, I report the studies of Kūshyār's *Jāmi' Zīj* by modern historians and a summary of its contents. In Part 4, I discuss the characteristics of Kūshyār's *zīj*, and some medieval astronomical terminology with which the reader of the *Jāmi' Zīj* should be familiar. Part 5 deals with the extant Arabic manuscripts of Kūshyār's *Jāmi' Zīj*, and the editorial procedure which I have followed in establishing the Arabic text and the translation.

The reader interested in more discussions about different aspects of the Islamic period astronomy may be referred to *Studies in Islamic exact sciences* by E. S. Kennedy, his colleagues and former students [Kennedy 1983], and the different works by B. Van Dalen, J. P. Hogendijk, D. A. King, F. J. Ragep [Al-Ṭūsī 1993] and G. Saliba, mentioned in the bibliography below (Part 6).

Publications will be referred to by author's name and year of publication, followed by page number(s) if necessary, thus: [Kennedy 1956, 125]. The full reference can be found in the bibliography in Part 6. Dates will be given in Christian chronology (A.D.), Islamic lunar chronology (A.H.), and the Persian Yazdigird chronology (A.Y.). The Islamic chronology began on July 15, 622 A.D., and its years are lunar years of 12 months (354 or 355 days). The Yazdigird chronology began on June 16, 632 A.D., and its years are solar years of 365 days.

I shall use the standard sexagesimal notation for all values, in which, e.g., 3,34;0,15 means $3\times60+34+0/60+15/60^2$. Kūshyār uses the technical term *inḥiṭāṭ* ("lowering") for division by 60 which actually leads to a shift of sexagesimal position.

2. Life and work of Kūshyār ibn Labbān

Kūshyār's complete name was *Kīā* Abu'l-Ḥasan Kūshyār ibn Labbān Bāshahrī al-Jīlī. The word *kīā* meant "king" or "ruler" in classical Persian. It was also prefixed to the names of some authorities and scholars in the Caspian province of Gīlān. His *kunya* Abu'l-Ḥasan, literally "Ḥasan's father", shows that he was a Muslim. However, his given name –Kūshyār– is a pure Persian name connected with the Zoroastrian religion. Its original form was *Gūshyār*, consisting of the name *Gūsh* and the suffix *-yār*. In the pre-Islamic Iranian calendar, each month of a year had 30 days, and each day of the month had a special name. The 14th day of each month was called *Gūsh-rūz* (the day of *Gūsh*; see Chapter 2 of the second section in Book I of the *Jāmi' Zīj*), after Gūsh, the guardian angel of useful quadrupeds in the Zoroastrian religion, which still had some followers in Gīlān in Kūshyār's time. *Gūshyār* literally means "a gift of Gūsh" or "aided by Gūsh", generally taken to mean "fortunate". Maybe the Arabic title *Sa'īd* ("fortunate, auspicious") for Kūshyār, found in some manuscripts of his works, was merely a translation of the word *Gūshyār* [Mu'īn 1952, 202-04]. His *nisba* al-Jīlī (arabicized form of *Gīlī*) attributes him to Gīlān. The arabicized form Jīlī is also used by European authors. He is now referred to in Iran as Kūshyār-e Gīlānī (or Gīlī).

We know little about Kūshyār's biography. He was an eminent Iranian mathematician and astronomer who lived in the second half of the 10th and the early 11th century A.D. [Saidan 1973; Qurbani 1996, 414-420; Yano 1997; Jaouiche 1986; Pingree 2002; Bagheri 2006a]. He was from the Gīlān province situated in the northern part of Iran, on the southern coast of the Caspian Sea. Since he finished writing a copy of his *Jāmi' Zīj* in 393 A.Y./1025 A.D., and that, according to al-Nasawī, he was dead in 416 A.Y./1048 A.D. (see below), he must have died between 1025 and 1048 A.D. In Book I of the *Jāmi' Zīj* (Chapter 5, Section 7), Kūshyār presents an example of a nativity in 332 A.Y./963-4 A.D. that may refer to his own date of birth. He then finds the years that had elapsed from that year up to 389 A.Y./1020-21 A.D., which may be taken as the year in which he wrote Book I of the *Jāmi' Zīj*. A detailed account of the social conditions of Gīlān in Kūshyār's time is provided in the introduction to the French translation of his treatise on arithmetic [Mazahéri 1975].

Kūshyār spent part of his life in Rayy[2], as he explicitly mentions it in I.1.3. We know from al-Bīrūnī [1985, 101,139,143] that he met Kūshyār (evidently in Rayy). Kūshyār told al-Bīrūnī that he had abridged al-Khujandī's *qānūn al-hay'a* (lit., "Rule of cosmology"; i.e., the sine theorem in right spherical triangles) and renamed it *al-Mughnī* (lit., "making [one] able to dispense [with Menelaus' Theorem]"). See IV.3.1 and its commentary.

Kūshyār was probably an astronomer at the court of Voshmgīr (d. 357 A.H./967-8 A.D.), the Iranian local ruler in Māzandarān province, on the southern coast of the Caspian Sea, immediately east of Gīlān. In *Tārīkh-i Māzandarān* ("A history of Māzandarān") composed in the 17th century A.D., we read: "One day in the month of Muḥarram 357 A.H., in the city of Jurjān[3], Kūshyār advised the ruler of Māzandarān, Voshmgīr, not to ride horses throughout that day lest he should be killed. All the saddles were taken off the horses, and the ruler did not ride all day long. However, in the evening he heard the grunt of a wild boar, and he could not help riding. He mounted a horse and followed the wild boar; the boar rushed towards the horse, Voshmgīr fell and died" [Gīlānī 1973, 78]. This account is not consistent with the above assumption for Kūshyār's date of birth. However, older sources such as [Ibn Isfandiyār 1941, part 2, 3-4], composed in the early 14th century A.D., which mentions Kūshyār among the astronomers of Ṭabaristān (an older name for Māzandarān) [ibid., part 1, 137], and [Mar'ashī 1954, 131], composed in the late 15th century A.D., give similar accounts of the same event without naming Kūshyār. Therefore it seems that the astronomer in this story was someone else, and Kūshyār was in fact at the court of Voshmgīr's son, Qābūs (reigned 367-403 A.H./977/8-1012/3 A.D.), to whom al-Bīrūnī presented his *Āthār al-bāqiya* (*Chronology of ancient nations*) in 390 A.H/999-1000 A.D. The following account confirms this conjecture.

In the medical treatise *Dhakhīra-yi Khwārazmshāhī* ("Khwārazmshāh's treasure"), written in Persian by Sayyed Ismā'īl Jurjānī in 504 A.H./1110-11 A.D., the author says that Kūshyār was a learned astronomer from Gīlān who lived in Gurgān in the service of Qābūs (Voshmgīr's son). Then Jurjānī narrates his encounter with some descendants of Kūshyār in Qum. They showed him treatises written by Kūshyār in a very neat and nice form. They told Jurjānī that "Kūshyār wrote only when he was calm and relaxed, and his books are written very

[2] An old city of Iran, now adjacent to the south-eastern part of Tehran.

[3] This is the arabicized name of Gurgān, an old city in Māzandarān province. The ruins of Gurgān are near Gunbad-i Qābūs in present-day Iran, about 100 kilometers north-east of modern Gurgān.

neatly in a nice hand; when Kūshyār was told that his writing style required too much time to complete a single book, he replied, 'yes, it takes much time, but once I am gone, people won't be concerned with how long I took to write them, but rather with the quality and contents of the books'" [Jurjānī 1976, 644].

In his article on Kūshyār, the historian Beyhaqī quotes the following dictum from him: "If two persons are interested in a single thing, the one ignoring the defects of that thing is really unfair to himself." [Beyhaqī 1935, 84].

Sa'dī, the famous Iranian poet of the 13th century, in the following poem on humbleness, names Kūshyār as the symbol of a wise scholar [Sa'dī 1879, 245-246]:

"Some one was a little knowledgeable about stars;
But he was drunk with arrogance.
From afar he went to Kūshyār,
With a heart full of devotion [to him and] a head full of conceit.
The wise man (i.e., Kūshyār) utterly ignored him,
[And] did not teach him anything.
Thus frustrated, when he decided to travel [back home],
The learned glorious man said to him:
'You have imagined yourself full of wisdom;
[Well,] how can a brim-full vessel contain more?
You are full of pretensions; therefore, you go empty-handed from me;
Come [to me] empty; in order to be filled with knowledge'.
...."[4]

In March 1988, Kūshyār's millennium was celebrated at Gīlān University during the 19th Annual Iranian Mathematics Conference. Kūshyār's works have attracted the attention of modern scholars since the early 19th century A.D. All his works are written in Arabic, the lingua franca of his time. For a detailed list of his works and their manuscripts see [Sezgin 1974, 343-345; 1978, 246-249; 1979, 182-183; Rosenfeld-Ihsanoğlu 2003, # 308, 118-119]. For a list of my remarks and additional data on the entry on Kūshyār ibn Labbān in the latter see [Bagheri 2006b, 2].

In Part 3 of this General Introduction, I will list the contents of

[4] English translation by Dr. Hushang A'lam; the last two lines are not translated, because they are not related to Kūshyār.

Kūshyār's *Jāmi' Zīj*, and I review the publications by modern authors related to the *Jāmi' Zīj*. In Part 4, I present some basic astronomical concepts which are necessary to understand the *Jāmi' Zīj*, and I also discuss the innovations made by Kūshyār.

Kūshyār's only known mathematical work is entitled *Uṣūl ḥisāb al-Hind* ("Principles of Hindu Reckoning"). It was translated into Hebrew by Shālôm ben Joseph 'Anābī in the 15th century A.D. (see [Cecotti 2004]). An edition of the Arabic text of this treatise was published by Saidan [Saidan 1967]. In recent decades, it has been translated into English, French, Persian, and Russian [Kūshyār 1965; Mazahéri 1975, 73-133; Kūshyār 1988a; Abdullazade 1990, 233-250]. For a comparative survey of the different versions of this work extant in four mss. in Istanbul, Tehran, Bombay and Cairo, see [Bagheri 2004].

Kūshyār's astrological treatise is titled *al-Madkhal fī ṣinā'at aḥkām al-nujūm* ("Introduction to the art of astrology"). An edition of the original Arabic text has been published by Prof. Michio Yano with an English translation and with an edition of the medieval Chinese translation prepared in 1383 A.D. [Kūshyār 1997]. There are also medieval Persian and Turkish translations of this treatise which have not yet been published [Sezgin 1979, 183; Pingree 2002, 408].

Kūshyār's treatise on the astrolabe is extant in several manuscripts. Mr. Taro Mimura has prepared an edition of the Arabic text under the supervision of M. Yano at Kyoto Sangyo University, and plans to publish it with an English translation. There is an old Persian translation of this work in Tashkent (MS 3894/1). Abdullazade has provided a table of the contents of this treatise [Abdullazade 1990, 194-212] and I have published an edition of the old Persian translation with an introduction [Kūshyār 2004].

3. Kūshyār's *Jāmiʿ Zīj* and its contents

Kūshyār's most important astronomical work is the *Jāmiʿ Zīj* (*al-Zīj al-Jāmiʿ* (lit., "Comprehensive astronomical handbook with tables"). In the Iranian literary tradition, Kūshyār's *zīj* was reputed as involving very elaborated and complicated subjects. An Iranian poet of the 13th century A.D., Muḥammad ibn al-Badīʿ al-Nasawī, writes in a poem quoted by [ʿAwfī 1906, 241]:

چو حل شدست مرا زیج گوشیار سخن

کجا به طیره شوم من ز ریش خند و زنخ

"Since [the problems of] Kūshyār's *zīj* of literature/poetry have been solved for me,
"I may not by any means be angered by [people's] derision and idle talk."

Here "Kūshyār's *zīj*" is used as a metaphor for "complicated and abstruse subjects". Kennedy gives a summary account of Kūshyār's *Jāmiʿ Zīj* in [Kennedy 1956, 125, 156-57]. He maintains that the elements of this *zīj* were taken from al-Battānī's *Ṣābī Zīj*, and that it is improbable that new observational data were incorporated into it. The *Jāmiʿ Zīj* was well known and influential in the Islamic astronomical tradition. Although it is influenced by Ptolemy's *Almagest* and al-Battānī's *zīj*, the *Jāmiʿ Zīj* has a special value because Kūshyār systematically presents geometrical proofs of the underlying theorems and algorithms. This feature is found only in a few other extant *zīj*es, e.g., Abu'l-Wafā's *Almagest*, al-Bīrūnī's *Canon Masudicus*, and al-Kāshī's *Khāqānī Zīj*.

The *Jāmiʿ Zīj* consists of four *maqāla*s ("Books"): Book I on elementary trigonometrical and astronomical calculations; Book II contains numerical trigonometrical and astronomical tables; Book III is on cosmology (*hay'a*); Book IV on "proofs" of the computations in Book I. Muḥammad ibn ʿUmar ibn Abī Ṭālib Tabrīzī translated the first book of the *Jāmiʿ Zīj* into Persian in 483 A.H./1090 A.D. [cf. Bagheri 1998]. Versions in Hebrew characters of different parts of the *zīj* are kept in four manuscripts that cover the whole work altogether [Langermann 1996, 151]. ʿAlī ibn Aḥmad al-Nasawī, probably a disciple of Kūshyār, wrote an Arabic commentary on the first book of the *Jāmiʿ Zīj* entitled *al-Lāmiʿ fī amthilat al-Zīj al-jāmiʿ* ("Explanation of the examples in the *Jāmiʿ Zīj*")

(MS Or. 45/7, Columbia University, New York, fols. 49r-75v)[5]. He presented numerical examples for each of the 85 chapters in Book I of the *Jāmi' Zīj* except for six chapters[6] that, according to him, did not need any example and two chapters which he simply skipped[7]. The folios of this ms. are not in their correct order[8] and there is a lacuna from the middle of chapter 6.14 to the middle of chapter 6.20. It is particularly interesting that on folios 50r and 51v al-Nasawī mentions the year 416 of Yazdigird era (1047-8 A.D.) as "the present year". So he flourished around 1050, and since at the beginning of the treatise he names Kūshyār with the prayer "may God have mercy on him!", this confirms that Kūshyār had died at that date.

A manuscript kept in the National Library of Tunis is said to be a commentary on Kūshyār's astronomical treatise (*Sharḥ kitāb Kūshyār ibn Labbān fi'l-falak*) by 'Abd al-Karim Dakālī [*Fihris* 1977-81, vol. 1, 106]. This may be another commentary on the *Jāmi' Zīj* [cf. Pingree 2002].

Partial editions, translations and studies of the *Jāmi' Zīj* have appeared during the last two and half centuries. Muḥammad A'lā al-T^{h}ānawī in his *Kashshāf iṣṭilāḥāt al-funūn* (A dictionary of the technical terms [used in the sciences of the Musulmans]), composed in 1158 A.H./1745-46 A.D., quoted from Kūshyār's *Jāmi' zīj* about the similarities of the Greek and the Syrian calendars, in his entry on chronology (*al-ta'rikh*) [al-T^{h}ānawī 1862, I, 57].

Ludwig Ideler published an edition of some fragments of the chapter on calendars with German translation [Ideler 1825-1826, II, 623-633]. Joachim Lelewel cited some data from the table of geographical coordinates given in the *Jāmi' Zīj*, and compared them with those of al-Bīrūnī and Ibn Yūnus [Lelewel 1852, xlvi-xlix]. E. Wiedemann translated the preface of the *zīj* into German [Wiedemann 1920, 132]. An edition of Chapter II.32, "On the distances and sizes [of celestial bodies]" appeared in India [Kūshyār 1948], and a Persian translation of it was published in Iran [Kūshyār 1988b]. Prof. J. L. Berggren published a translation with a commentary of Section IV.3 of the *Jāmi' Zīj* on

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[5]. The late Prof. A. S. Saidan erroneously attributed this work to Kūshyār and gave wrong manuscript data for it [Saidan 1973, 531, 533]. He seems to have followed Salih Zeki who in his *Āthār al-bāqiya* (lit., "The existing remnants") provides similar information in the entry on Kūshyār [Zeki 1911, 166].

[6] These are Chapters 2.1 (commentary to Chapter 1 of Section 2), 4.1, 6.6, 8.7, 8.9, and 8.10.

[7] These are Chapters 4.7 and 4.8.

[8] A fragment from the middle of 5.21 to the middle of 6.3 is misplaced into the middle of 7.1; one folio from 7.4 is misplaced into the middle of 5.21; and one folio of a Persian treatise on arithmetic is misplaced into the middle of 7.4.

spherical trigonometry. He concluded that, while Kūshyār's account of the trigonometry of his day was not particularly original, it did contain the latest results and showed Kūshyār's taste for systematic exposition based on simple argumentation [Berggren 1987]. Prof. E. S. Kennedy studied Kūshyār's method for the calculation of the equation of time [Kennedy 1988, 2-4]. Khurshid F. Abdullazade extensively discussed the spherical trigonometry, mathematical astronomy and geographical material in the *Jāmiʿ Zīj* [Abdullazade 1990, 61-193, 213-230]. Dr. Benno van Dalen analyzed the table for the equation of time in Book II of the *Jāmiʿ Zīj* and was able to explain its method of computation, by taking into account that the tabular values are influenced by the displacement of the solar mean motion, as explained in Part 4 of this General Introduction. Van Dalen used statistical methods in order to determine the parameter values which Kūshyār used [Van Dalen 1993, 134-41]. He also analyzed a table for the true solar longitudes found in the sequel of the Berlin ms. of the *zīj* and showed that it probably derives from Yaḥyā ibn Abī Manṣūr [Van Dalen 1994b].

Glen Van Brummelen described Kūshyār's ingenious innovative interpolation scheme for composing double argument tables for the planetary equations of anomaly on the basis of the tables in Book II of the *Jāmiʿ Zīj*. The process significantly simplified the determination of a planet's longitude at a given time, although at the cost of some inaccuracy in the result. Van Brummelen took only the tables in Book II into account, but his mathematical reconstructions are confirmed by Kūshyār's text which we are now publishing; compare Sections I.4 and IV.4. Van Brummelen concludes that "Kūshyār was no mere copyist" [Van Brummelen 1998, 279].

Toshiaki Kashino discussed the planetary longitude theory in the *Jāmiʿ Zīj* and provided an edition of the Arabic text and English translation of Chapters I.4.1 to I.4.8, tables II.12 to II.14, II.16 to II.36, II.56, Chapters III.13, III.16 to III.19 and IV.4.1 to IV.4.7 of Kūshyār's *Jāmiʿ Zīj*, in his unpublished thesis [Kashino 1998].

In his *Introduction to astrology*, Kūshyār mentions his other *zīj* entitled *al-Zīj al-Bāligh* ("The extensive astronomical handbook with tables") [Kūshyār 1997, 6/7, 216/217]. No manuscript of the integral text of this work has been reported up to now. However, a short chapter entitled *Fī istiʿmāl adwār al-kawākib ʿalā madhhab al-Hind min Zīj al-Bāligh li-Kūshyār* ("On the application of the cycles of the planets according to the Indian method from Kūshyār's *Zīj al-Bāligh*") kept in Bombay (MS R. I 86, Mulla Firuz collection, Cama Oriental Institute) is reported by F. Sezgin [1974, 248]. I have discussed the content of this chapter in an unpublished paper presented at the 17th Annual Conference for the History of Arabic Science, Suweida (Syria), 1993.

Abu'l-Fażl ʻAllāmī mentions in his *Ā'īn-i Akbarī* [ʻAllāmī 1983, vol. 1, 185], besides the *Jāmiʻ Zīj* and the *Bāligh Zīj*, another work by Kūshyār entitled the *ʻAżudī Zīj*. But the existence of such a work has never been confirmed by another reference in the works of Kūshyār or other authors. We know only one work named *al-Zīj al-ʻAżudī*, composed by Ibn A'lam (ca. 960 A.D.) which has not come down to us [Kennedy 1956, 134].

Now I present very briefly a list of the contents of the *Jāmiʻ Zīj*. Books I and IV each consist of 8 sections. I explain the subjects of Book I and Book IV jointly, because they are directly related to each other. This list will be followed in Part 4 by an explanation of some basic concepts which Kūshyār uses.

In Section I.1, Kūshyār discusses different types of calendars used in ancient times and in his own time. He describes the methods for converting a date between any of the three calendars used in his time (Greek, Arabic and Persian). He also presents a method for finding the weekday corresponding to any date in any of these three calendars, and he lists the feasts in the three calendars.

In Section I.2, he discusses the trigonometric functions sine, cosine, and versed sine. Since he takes the radius of the base circle equal to 60 parts, his trigonometric functions are always 60 times the functions we use. He also discusses the chord function that was used by Ptolemy. He gives the values of the chords of 1/3, 1/4, 1/5, and 1/10 of the circle, and the chords of the sum and the difference of two arcs. He also provides the values of the sine and cosine of 1 degree and their application in setting up a sine table. The definitions of the trigonometric functions are not presented in Book I. We find them in Section IV.1, where Kūshyār also proves the validity of the results for the above-mentioned trigonometric functions which he presents in I.2.

In Section I.3 Kūshyār discusses the trigonometric functions tangent and cotangent and methods to compute them from sine tables. Again his tangent and cotangent are 60 times the functions we use. He also mentions other definitions in which the coefficient of the cotangent is 7 or 12 instead of 60. If a vertical gnomon is divided into 12 units, then the length of the cotangent or, as Kūshyār calls it, the horizontal shadow, is measured in the same units, called digits. If the gnomon is divided into 7 units, the cotangent is measured in units called feet. The validity of the methods of this section is proved in Section IV.2.

In Section I.4, Kūshyār provides his methods for computing the position of the sun, the moon, its nodes, and the five planets. He also discusses the equation of time and the latitudes of the moon and the five planets. He ends the section with a discussion on the retrogradation of the planets. The geometrical background of some of these methods is discussed in Section IV.4. The previous Section IV.3 provides some preliminary theorems in spherical trigonometry.

Section I.5 is devoted to the calculations of different quantities used in mathematical astronomy, such as the first and the second declinations, right and oblique ascensions, ortive amplitudes, the day arc for the sun, the altitude of the sun and the ascendant (We will explain some of these concepts below). Kūshyār also discusses the astrological concept of houses, which involves mathematical computation. Geometrical proofs of the mathematical calculations in this section are given in Section IV.5.

Section I.6 is about lunar and solar eclipses and their magnitudes and durations. Kūshyār also discusses the parallax of the moon and the sun and lunar crescent visibility. The corresponding "proofs" are given in Section IV.6.

Section I.7 is on operations relating to astrology. Here Kūshyār deals with astrological concepts as "projection of rays", prorogation etc. The geometrical background of the method for projection of rays is given in Section IV.7.

Section I.8 is entitled "On operations which are less needed". In this section he provides methods for finding the following quantities, among others: the geographical latitude of a locality from the duration of its longest day, the altitude of the sun or a planet when it is due East or West, the apparent distance between two stars from their ecliptical coordinates, the terrestrial meridian line, the direction to Mecca, etc.
At the end of this section Kūshyār presents the names of the fixed stars, some features for recognizing them, and the lunar mansions. The

"proofs" of the mathematical methods of this section are presented in Section IV.8.

Book II includes 55 tables. The first 7 tables are related to calendrical calculations. The next 4 tables are tables of the sine, versed sine, tangent and cotangent. Then follow 5 tables for finding the true longitude of the sun and 5 tables for finding the true longitude of the moon. For each of the planets Saturn, Jupiter, Mars, Venus and Mercury, 3 tables give the values of mean motion, anomaly and equations. The next 6 tables are for the latitudes of the moon and the planets. Then 6 tables give the first and second declination, right and oblique ascensions, and the equation of day. The next 4 tables are for calculations relating to eclipses. One table is for the astrological function called prorogation. Finally 2 tables provide the names and coordinates of the cities and the fixed stars.

Book III on cosmology (*hay'a*) contains 32 chapters on different astronomical subjects such as the climates, the size of the earth, the ascensions, equinoctial and temporal hours, the orbs of the celestial bodies, retrogradations, sizes and distances of the celestial bodies, lunar phases, and eclipses. Two chapters of the third book entitled *al-Ab'ād wa'l-ajrām* ("<On> the distances and sizes <of the celestial bodies>"), and *Jawāmi' 'ilm al-hay'a* ("A compendium of the science of cosmology" containing definitions of around 130 astronomical terms) were also copied, translated, and circulated as independent treatises.

4. Kūshyār's *Jāmiʿ Zīj* between tradition and innovation

Kūshyār does not inform us what books his reader should have read in order to understand this *zīj*. But it is evident that he assumed a thorough knowledge of Books I through VI of the *Elements* of Euclid (ca. 300 B.C.) on plane geometry of straight lines and line segments, triangles, circles, ratios and proportions. The reader of Kūshyār's Book IV, on "Proofs", should have a good command of Euclid's *Data* as well. This work contains theorems of the type that if certain elements are known in a figure, other elements can also be determined. Euclid's *Elements* and *Data* were available in good Arabic translations. Unlike Euclid, Kūshyār and his contemporaries routinely used numerical approximations of irrational ratios. Thus the reader should also be familiar with, e.g., square root extraction. Since much of Kūshyār's work concerns spherical trigonometry, his reader needs to know some materials on the geometry of the sphere, which is explained, for example, in the *Spherics* of Theodosius (ca. 100 B.C.). This work was also available in Arabic translation.

We now introduce the reader to some basic concepts and terminology of later Greek (Ptolemaic) and medieval Islamic mathematical astronomy, which were traditional concepts and terminology in Kūshyār's time. This introduction may facilitate the reading of the translation of Kūshyār's work. Kūshyār himself explains some of these concepts in Book III of his *zīj*, which is not published here. I do not intend to give a complete exposition of the Ptolemaic system or of medieval astronomy. Of course, the reader may pursue the translation, and refer to this exposition only when necessary.

The most important concept in later Greek and Islamic spherical geometry is that of a great circle, which is the intersection of a sphere by a plane through the center of the sphere. In Kūshyār's work, an "arc" on a sphere almost always means an arc of a great circle. Using arcs of great cicles, Menelaus of Alexandria (ca. 75 A.D.) defined spherical triangles. Each pair of points on a sphere, which are not on the same diameter, can be joined by precisely one great circle arc less than 180 degrees, and thus three points on a sphere, such that no two of them are on the same diameter, determine a spherical triangle. Spherical trigonometry was further developed by Islamic geometers in the tenth century A.D., and Kūshyār makes some contributions to this field in Book IV of the *Jāmiʿ Zīj*.

Another important concept, often used by Kūshyār in Book IV, is the "pole" of a circle on a sphere (not necessarily a great circle). The two poles of any circle on a sphere are the two points of intersection of the sphere with the straight line passing through the center of the sphere perpendicular to the plane of the circle. Every point on a sphere is the pole of precisely one great circle. (The reader may recall the familiar example of the terrestrial equator with its two "poles", namely the North and South poles.) Kūshyār sometimes assumes the following property of poles of great circles: If P and Q are the poles of two different circles p and q, the points of intersection of p and q are the poles of the great circle through P and Q.

Kūshyār provides most of the proofs in Book IV with a figure. In the case of theorems on spherical trigonometry, the figures in the works of Kūshyār and his contemporaries are not perspective drawings of the sphere; the theory of perspective was unknown in medieval Islamic mathematics. Kūshyār's figures are symbolic representations in which one side of the sphere (for example, the part above the horizon) is represented on the paper inside one boundary circle (for example, the representation of the horizon circle). Arcs on the sphere are represented as circular arcs on the paper, in such a way that their relative positions on the sphere are conserved. Arcs on the other side of the sphere may extend outside the boundary circle. There is no evidence that Kūshyār (or any of his contemporaries) used a consistent method of projection in drawing figures for geometrical theorems and proofs. Of course, such systems of projections were known at that time, and used in making metal astrolabes and maps.

Kūshyār used the sine as his basic trigonometric function. In Part 3 of this General Introduction, I have referred to Kūshyār's very detailed explanation of the use and computation of Sines in Section 2 of Books I and IV. Here, I only call attention to the capital initial letter in my translation of Sine, which indicates that Kūshyār's Sine is 60 times the modern sine. Kūshyār defines the Sine in a circle whose radius he divides into 60 "parts", and he expresses the Sine sexagesimally in parts, minutes and seconds. Therefore his Sine of 45 degrees is 42 parts, 35 minutes and 25 seconds ($30\sqrt{2} \approx 42+35/60+25/3600+ \ldots$). The term "total Sine", which often occurs in Kūshyār's work and in my translation, means the maximal Sine (of 90 degrees), that is the radius of the circle.

Now we turn from mathematics to mathematical astronomy. Kūshyār, his contemporaries and predecessors used the celestial sphere for many astronomical computations. This is a very large imaginary sphere, which may coincide with the outermost sphere of the universe. The center of the celestial sphere coincides with that of the earth and the sphere is so large that the radius of the earth can be neglected in all computations. On the celestial sphere, different points and circles are defined. The celestial equator and the celestial North and South poles are the intersections of the celestial sphere with the plane of the terrestrial equator, and the line through the terrestrial North and South poles. Almost all ancient and medieval astronomers, including Kūshyār, assumed that the earth is at rest and that the universe performs a daily rotation around the axis through the celestial North and South poles.

The second fundamental circle on the celestial sphere is the ecliptic, which is defined by the motion of the sun. Ancient and medieval astronomers believed that the sun performs a yearly motion around the earth. This motion (more precisely, the motion of the center of the sun) takes place in a plane passing through the center of the earth, and the ecliptic is the intersection of that plane with the celestial sphere. The ecliptic and the equator intersect at two points, which are called the vernal point (or vernal equinox), and the autumnal point (or autumnal equinox). The moments when the sun is at the vernal and autumnal point, define the beginning of the spring and the fall on the northern hemisphere of the earth; then the day and night have equal length. The two points on the ecliptic at maximal distance of the celestial equator are called the two solstitial points, or solstices. When the sun is at the summer solstice, which is in the northern celestial hemisphere, summer begins on the northern hemisphere on the earth, and the day is longest in the temperate regions north of the equator (which include Iran). Similarly, winter begins in these temperate regions when the sun is at the winter solstice; then the day is shortest. In ancient and medieval astronomy, the ecliptic together with the equinoxes and solstices partake in the daily rotation of the universe around the earth.

The Babylonians were the first to define the ecliptic. They divided it into 12 "signs" of equal length. These signs were also used in Greek, Islamic and European astronomy and astrology. In the order of the yearly motion of the sun, and beginning with the beginning of spring, the Latin names of the signs are as follows: Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricorn, Aquarius and Pisces. These arbitrary names were assigned from stellar constellations. In Greek, Islamic and European astronomy, the spring equinox is the beginning of Aries, the summer solstice the beginning of Cancer, the autumnal equinox the beginning of Libra, and the winter solstice the beginning of

Capricorn. The Babylonians divided each sign of the ecliptic into 30 degrees; so the whole ecliptic is divided into 360 equal degrees. The Greek astronomers adopted from the Babylonians this division into signs, and the Greek astronomer Hipparchus (ca. 150 B.C.) divided all other circles into 360 degrees as well.

Hipparchus was the first to realize that the position of the sun at the beginning of spring (when the day and night are equal in length) changes very slowly with regard to the fixed stars. He and his followers, including Kūshyār, supposed that this phenomenon is caused by a very slow motion of the "fixed stars" with respect to the signs of the ecliptic. This motion was supposed to be a uniform rotation around an axis perpendicular to the plane of the ecliptic. The axis intersects the celestial sphere in the two "poles" of the ecliptic. When Kūshyār refers to the "pole" of the ecliptic, he means the North pole of the ecliptic, which is close to the celestial North pole and always above the horizon in Iran. Kūshyār believed that one complete rotation of the fixed stars with respect to the ecliptic takes place in 24,000 years. In modern astronomy, the phenomenon is called "precession of the equinoxes" and described as a motion of the equinoxes, i.e., the plane of the equator, with respect to the fixed stars, rather than the other way around. The precession is explained as the result of a slow motion of the earth's axis. As a result of precession, the signs slowly move away from the stellar constellations from which the names of the signs were originally derived in the first centuries B.C. Thus most part of the constellation "Pisces" is now in the sign "Aries", and so on.

The three basic coordinate systems on the celestial sphere can now be described. The first system uses ecliptical longitude and latitude, and was used by Kūshyār and his contemporaries in the computation of planetary, lunar and solar positions, and in most astrological applications. This system is hardly used in modern astronomy.

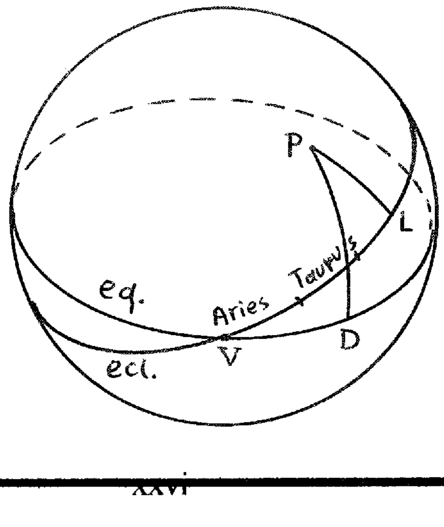

Figure 1

To find the ecliptic coordinates of a point *P*, draw a great circle arc *PL* through it, perpendicular to the ecliptic and less than 90 degrees, meeting the ecliptic in *L*. (Precisely one such arc can be drawn if *P* is not one of the ecliptical poles.) The ecliptical latitude is the length of this arc in degrees, and the latitude is called "northern" if *P* is between the ecliptic and the North pole of the ecliptic, and "southern" if *P* is between the ecliptic and the south pole of the ecliptic. Thus, Kūshyār does not work with negative latitudes. In his words, a point on the ecliptic has "no latitude"; we would say that the point has "zero latitude". The arc between the vernal point *V* and *L*, measured along the direction of the yearly motion of the sun, is the ecliptical "longitude" of the point *L*. The ecliptic longitude is always between 0 and 360 degrees. If *L* is in Aries, the longitude is between 0 and 30 degrees, etc. Figure 1 displays the celestial sphere from the outside. *V* is the vernal point, *P* a point on the northern half of the celestial hemisphere. *L* is approximately in Gemini.

The second coordinate system is defined similarly, but with respect to the celestial equator. We draw a great circle arc *PD* through *P*, perpendicular to the equator (see Figure 1). The arc *VD* on the equator, measured in the direction nearly parallel to the yearly motion of the sun, is called the "right ascension" of point *P*, and the arc *PD* is simply called "distance to the equator".

Modern astronomers use the general concept "declination" instead, but Kūshyār uses the term "declination" (*mayl*) only if *P* is on the ecliptic. If Kūshyār refers to "the declination of (a certain) ecliptical degree", he means the declination of a point on the ecliptic which is the endpoint of an arc beginning at the vernal point and ending at the degree in question. For Kūshyār the declination is "northern" or "southern", but never negative. Another curious term is "total declination", meaning: the declination of one of the two solstitial points. Nowadays this "total declination" is called "obliquity of the ecliptic", and it is equal to the angle between the equator and the ecliptic at the vernal equinox. Kūshyār and most of his Islamic predecessors used the value 23 degrees and 35 minutes.

The third coordinate system which we have to mention is defined with respect to the horizon. The plane tangent to the earth at the locality of the observer intersects the celestial sphere in a circle, which can be considered as a great circle because the radius of the earth is ignored. This great circle is called the "(local) horizon". The line joining the center of the earth to the observer intersects the celestial sphere above the horizon at the zenith (Arabic: *samt al-ra's*) of the locality, and below the horizon at the nadir (Arabic: *naẓīr al-samt*) of the locality. The meridian is the great circle through the zenith and the celestial north and south poles. The meridian intersects the horizon in two points: the North point (closer to the celestial north pole) and the South point. The arcs between the North and South points are bisected by the East and West points. Different localities on earth have different zeniths on the celestial sphere. (In fact, one can map the earth on the celestial sphere by mapping every locality on its zenith. In this way, the celestial sphere was often used for terrestrial computations in medieval Islamic geometry. An example is Kūshyār's determination of the direction of Mecca in IV.8.)

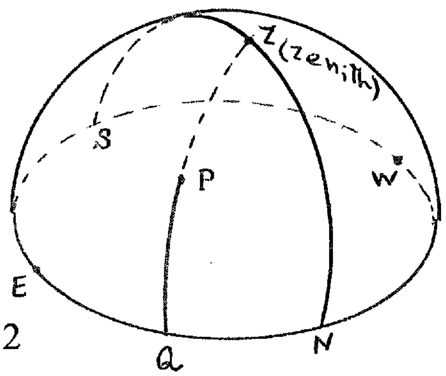

Figure 2

For any point P on the celestial sphere, not the zenith and nadir, we can draw a unique great circle arc PQ less than 90 degrees perpendicular to the horizon, and meeting the horizon at Q. This arc, or its length in degrees, is called the "altitude" (or "depression" if the point is below the horizon). The arc between Q and the East or West point, whichever is closer, is called the azimuth (Arabic: *al-samt*). See Figure 2.

Points on the prime vertical (the great circle through the East and West points and the zenith) are said to have "no azimuth". These conventions are contrary to the modern ones, which prescribe that the zero point of the azimuth is in the North point and that the azimuth ranges between 0 and 180 degrees East and West.

Now we continue with an introduction to some of the traditional concepts and terminology of Ptolemaic astronomy which were used by Kūshyār. This introduction is meant to give the reader some idea about what he may encounter in the translation of Kūshyār's *zīj*. For further details, I refer him to the standard expositions in [Pedersen 1974] and [Neugebauer

1975], and to the translation of *Almagest* by Toomer [Ptolemy 1984]. I often refer to these works in my own commentary. Some of the parameter values of Kūshyār in the following introduction are taken from Books II and III of his *zīj*, which I am currently preparing for publication.

We begin with the motion of the sun. As early as the fifth century B.C., the Babylonians had already observed that the sun does not move uniformly on the ecliptic. In the spring and summer the motion is slightly slower than in the fall and winter. Hipparchus and Ptolemy, who believed in the Aristotelian dogma of uniform motion, explained this "anomaly" by means of the following model (Figure 3).

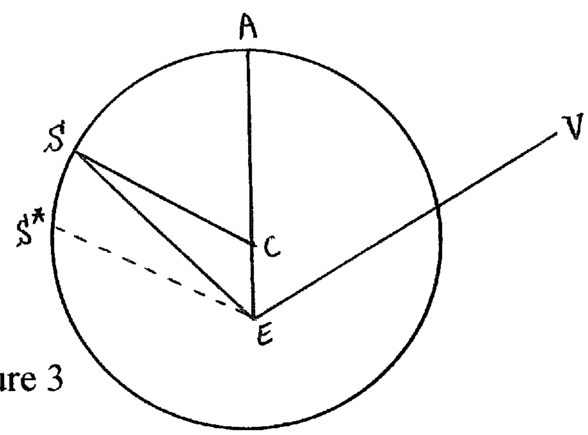

Figure 3

They assumed that the sun S moves uniformly on a circle, the *deferent*, whose center C does not coincide with the center E of the earth. Ptolemy put the radius of the circle equal to 60 "parts". In Book III of the *Almagest* he explains in detail how he (allegedly) derived the parameter values in this model from observations of solar altitude above the horizon. His conclusions are that the solar eccentricity CE is 2;29,30 "parts" (which he often rounded to 2;30 "parts"); that the apogee A is in 5;30 Gemini; and that the sun performs one complete rotation on the deferent in 365+(1/4)–(1/300) days, so the daily motion on the deferent is 0;59,8,17,13,12,31 degrees.[9]

The Ptolemaic model of the solar motion agrees reasonably well with the model based on Newtonian mechanics. That is to say the two models predict approximately the same solar position in the ecliptic. Ancient and

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[9] For a description of the sexagesimal notation which has become standard since the work of Neugebauer, and which I will use, see Part 1 of this General Introduction.

medieval astronomers were unable to measure the variations in the distance between the sun and the earth. According to Newtonian mechanics, the sun is at rest and the earth moves around it in a Keplerian ellipse with the sun at one of the foci (we ignore the gravitational effects of bodies other than the sun and the earth). To an observer on earth, the sun seems to move in a Keplerian ellipse with the earth at one of its foci. Uniform circular motion on a deferent with center C and the earth E close to it, is a reasonable approximation of Keplerian motion on an ellipse with center C and earth E at one of the foci. Thus the concept of "eccentricity" could be transformed from the length of CE (in "parts") in the Ptolemaic solar (and planetary) models to the elliptic models of Kepler, and hence to the geometrical description of the ellipse in general. In modern geometry, the eccentricity of an ellipse means the distance of the center to any focus of the ellipse, divided by half the major axis.

I now explain some additional traditional technical terminology. Figure 3 displays the deferent in the plane of the ecliptic, which is the plane in which the solar motion takes place. V is the (direction of the) vernal point, A is the apogee, and angle VEA is its ecliptical longitude.

Kūshyār calls angle ACS the "mean argument"[10] of the sun and the angular sum $VEA+ACS$ the "mean longitude". These two quantities are linear functions of time. Modern authors often introduce the "mean sun", that is, a point S^* in the ecliptic so that ES^* is parallel to CS. Then the "mean longitude" of the sun S is the ecliptical longitude of S^*.

In order to compute the position of the sun at a given time, we need to know the position of the apogee A, and the position of CS at a conveniently chosen zero point in time. For this zero point, Kūshyār chose noon on the first day of the Yazdigird era at a locality with a terrestrial longitude of 90 degrees east of the Canary Islands (at that time the westernmost part of the known inhabited world). The Yazdigird era (A.Y.) is the most common Iranian calendar of which the first day was June 16, 632 A.D. The Yazdigird era is very convenient for astronomical calculations because every year in this calendar has a constant length of 365 days.[11] From the *zīj* of his predecessor al-Battānī, Kūshyār derived that at this moment of time, the apogee A was in 18;31 degrees Gemini, and the "mean longitude" of the sun was 26;24,36 degrees Gemini (i.e., $VEA+ACS$ = 86;24,36 degrees). The geographical longitude 90 degrees

[10] I have translated the Arabic term *khāṣṣa* by "mean anomaly" for the sake of consistency with the theory of the planets, although this translation may be somewhat misleading in the case of the sun.
[11] Kūshyār discusses the Yazdigird era and other calendars that were used in his time in Section 1 of book I.

was very convenient for Kūshyār because it was assumed to be the longitude of the city of Jurjān where he lived. The reader will notice that Kūshyār's apogee in 18;31 Gemini is different from Ptolemy's apogee in 5;30 Gemini (in particular Ptolemy's solar parameters were quite bad). Kūshyār supposes that the apogee *A* has the same slow motion as the fixed stars, namely 0;0,54 degrees per year.

The extreme precision in the daily motion of the sun is due to the fact that it is based on observations spanning an interval of more than 1000 years. Of course the precision in the position of the sun and also in the position of the apogee is illusory.

The position of the sun as seen from the earth is defined by the angle *VES*, where *V* is the vernal point. To compute this angle at a given moment of time in the Yazdigird era, we first need to find the "adjusted apogee", that is the position of *A* for the given moment, by adding 0;0,54 times the year number to 18;31 Gemini. Then we find the "mean longitude" as the sum of 26;24,36 degrees Gemini plus the number of elapsed days times the daily mean solar motion of 0;59,8,20,46,56,14 degrees. In Book II, Kūshyār provides tables for facilitating this computation. We subtract the "adjusted apogee" (angle *VEA*) from the "mean longitude" (angle *VEA* + angle *ACS*), and thus we obtain the "mean argument", i.e., the angle *ACS*.

From the "mean argument" *ACS*, Ptolemy and Kūshyār compute the correction angle *ESC* by trigonometrical calculations, which will be discussed below. This correction angle *ESC* is called "equation". This term is misleading for a modern reader, because no mathematical equation is involved here. The confusion can be explained by the fact that the word "equation" in the astronomical sense was derived, via the Latin *equatio*, from the Arabic *ta'dīl*. This word has the same root as the Arabic word *mu'ādala*, which means "algebraic equation", and which was also translated into Latin as *equatio*.

Call *c* the "mean argument", angle *ACS*, and $q(c)$ the corresponding equation angle *ESC*. Ptolemy's computation of $q(c)$ is equivalent to the following formula:

$$\sin q(c) = e \sin c \,/\, \sqrt{(d + e\cos c)^2 + (e \sin c)^2}\ ,$$

where $d = CS = 60$, and $e = CE = 2;30$.

Ptolemy and most Islamic astronomers computed the true solar longitude *VES* by adding or subtracting the equation *ESC* to or from the mean longitude *VEA*+*ACS*. In figure 3, the equation has to be subtracted. Apparently Kūshyār wanted the equations to be always additive in order to avoid the possible confusion that quantities from tables had to be added or subtracted depending on sometimes obscure conditions. So he made the following formal change in his computations. I give a general description of his idea below (cf. [van Dalen 2004b, 840-43] and [Van Brummelen 1998]). Choose an integer n greater than the maximum value of $q(c)$ for all c (for the sun, Kūshyār chooses $n = 2$). Kūshyār defines two new quantities: (1) the "displaced" mean argument $c' = c - n$; and (2) the "shifted" equation $q'(c') = n \pm q(c'+n)$. The minus sign is used if $c'+n$ is less than 180 degrees (so the equation has to be subtracted), and the plus sign if $c'+n$ is between 180 and 360 degrees (so the equation has to be added). The terms "displaced" and "shifted" are modern. In Book II, Kūshyār tabulates the shifted equation $q'(c')$ whose values are always positive. Kūshyār then computes the angle VES as $c' + q(c')$.

For some further technical terminology and another innovation of Kūshyār we turn to the motion of the moon. For the sake of simplicity we define in Figure 3 the 'mean sun' as an imaginary body S^* in the ecliptic so that CS is parallel to ES^*. Thus the longitude of S^* is equal to the "mean longitude" of the sun.

We shall now describe Ptolemy's complicated lunar model, which was used by Kūshyār, without further motivation. I realize that the description may be a little bewildering for the reader. For the reason why Ptolemy adopted precisely this model, the reader may refer to [Pedersen 1974, 159-202] and [Neugebauer 1975, 68-99]. The moon moves in a plane which makes a small angle (5 degrees) with the plane of the ecliptic, and the points of intersection with the ecliptic are called the lunar nodes. (The name "ecliptic" is derived from the fact that solar and lunar eclipses take place when the moon is close to the ecliptic.) In the rest of my description, I will ignore the ecliptical latitude of the moon, and identify the moon with its perpendicular projection on the ecliptic. Consequently, Figure 4 displays the ecliptic with earth E, the (direction of the) vernal point V, and the moon M. The body of the moon is represented as a point. Point S^* denotes the (direction of the) mean sun. We assume that the motion of S^* in Figure 4 is counter-clockwise.

The lunar motion is composed of three components: The moon M moves clockwise on an epicycle (a small circle) with center C. Point C moves with a fast counter-clockwise motion on a greater circle (the deferent) with center D, and point D moves clockwise on a small circle (not drawn in the figure) with the earth at its center. We will not discuss the question as to what extent this model is a faithful representation of the lunar motion according to modern theories.

Before describing these motions in detail, it is probably a good idea to introduce the "mean longitude" of the moon, namely angle *VEC*. The mean longitude is a linear function of time. According to Kūshyār, it increases by 13;10,35 degrees per day, and the position of *C* at noon of the first day of the Yazdigird era at Jurjān is 4;10,28 degrees Aries. Angle *CES** is called the "elongation"; it is also a linear function of time, and increases by 13;10,35 – 0;59,8 = 12;11,27 degrees per day.

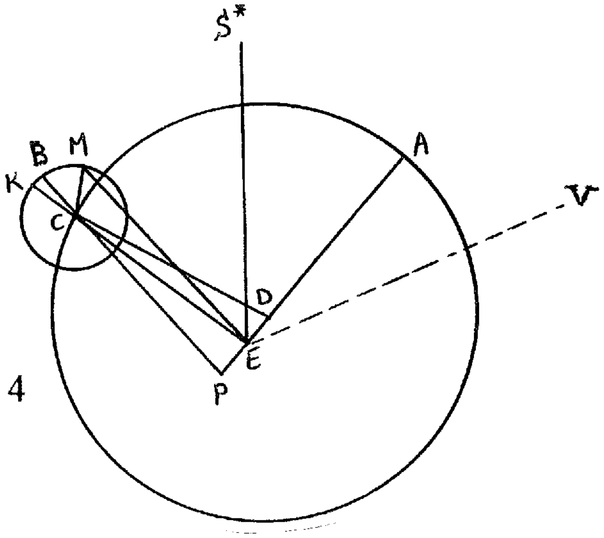

Figure 4

Here are the precise definitions of the three components of the lunar motion. Kūshyār puts the radius *DA* of the deferent equal to 60 "parts".

(1) Point *D* moves uniformly on a circle around the earth *E*, with radius 12;30 parts, in the direction contrary to the solar motion. Line *ED* extended intersects the deferent in its apogee *A*. The daily motion of *D* (and *A*) in the ecliptic is the daily increase of the elongation minus the mean solar daily motion, namely 11;12,19 degrees.

(2) Point *C* moves on the deferent *D* in such a way that the mean lunar longitude *VEC* increases by the above mentioned value 13;10,35 degrees per day. This means that angle *CEA* increases by 24;22,54 degrees per day, that is twice the daily increase in the elongation 12;11,27. Angle *CEA* is accordingly called the "double elongation". The position of *C* is defined by the requirement that line *ES** always bisects angle *AEC*. Thus, if the elongation *AES** is 0 or 180 degrees, point *C* coincides with *A*, and the epicycle center is at maximum distance of 60 "parts". If the elongation *AES** is 90 degrees, *C* is on *EDA* such that *CE* = *CD* – *DE* = 35 "parts".

(3) The moon M moves on this epicycle in the following way: Choose P on DE extended such that $EP = DE$. Draw PC and extend it to meet the epicycle at B. Then the "mean anomaly" angle BCM increases as a linear function of time.[12] The daily increase is according to Kūshyār 13;3,54 degrees, and the epoch value (at noon of the first day of the Yazdigird era at locality 90 degrees East of Canary Islands) is 307;4,26 degrees.

Extend EC to meet the epicycle at K. Then B coincides with K if the elongation is a multiple of 90 degrees, and the absolute value of arc BK is maximal for an elongation equal to 45, 135, 225 or 315 degrees. Ptolemy put $EA = 60$ and found the radius of the epicycle to be equal to 5;15 "parts". Kūshyār does not specify what value he used, but it must have been the product of the scaling factor 1;12,30 times the Ptolemaic value. Later, Kūshyār assumes that one "part" is equal to one earth's radius; his maximum distance 60 "parts" of the epicycle center C to the earth is almost equivalent to the Ptolemaic value of 59 earth radii.

Now the problem is how to compute the true position of the moon, which is the angle VEM, from these data.

First, the mean anomaly (angle BCM) is changed to the "true anomaly" (angle KCM) by adding or subtracting the "first equation" (angle BCK). Kūshyār computes this angle essentially in the Ptolemaic way, but subjected to a cosmetic change to avoid negative values, in a similar way as in the computation of the sun. The details are not to be mentioned here.

Then using the "true anomaly" KCM as an argument, Kūshyār computes the "second equation" MEC. The lunar longitude VEM is computed by adding or subtracting angle MEC to the mean longitude VEC. The computation of the second equation is of interest here, because it involves a change compared to Ptolemy's computation. Kūshyār also applies a cosmetic change in order to avoid subtraction, but we will describe his procedure and compare it to Ptolemy's procedure as if the cosmetic change had not taken place. Call a the true anomaly (angle KCM), and c the double elongation, angle AEC. The distance EC is a function of DE, DA and c and will be denoted as $d(c)$. So $d(c) = EC$.

We have $d(c) = \sqrt{(60 + e\cos c)^2 + (e\sin c)^2}$, with $e = 12;30$.

Call $r = BC$ the radius of the epicycle. Call angle $MEC = q(a,c)$.

We have in modern terms

$$\sin q(a,c) = r\sin a / \sqrt{[d(c) + r\cos a]^2 + (r\sin a)^2}, \text{ with } r = 6;20.$$

[12] The features (2) and (3) imply a contradiction with the principle of uniform circular motion.

A table of this function for all a and c would contain tens of thousands of values. Ptolemy and Kūshyār both use an approximation in order to simplify the computation. Ptolemy computes tables for the two functions $q(a,0)$ and $q(a,180)-q(a,0)$. For a fixed c the maximal value $m(c)$ of $q(a,c)$ can easily be found from $\sin m(c) = r/d(c)$. He then defines an interpolation coefficient $s(c) = 60\frac{m(c)-m(0)}{m(180)-m(0)}$, rounded to integers. The number $s(c)$ is always between 0 and 60 and is called the "sixtieth". His computation of $q(a,c)$ boils down to

$$q(a,c) \approx q(a,0)+[s(c)/60].[q(a,180)-q(a,0)].$$

In the same notation, Kūshyār computes only $q(a,0)$ and the "difference for lesser distance"[13] $m(c)-m(0)$. He also computes a "sixtieth" $S(a) = 60\, q(a,0)/m(0)$, rounded to integers.[14]

He then puts

$$q(a,c) \approx q(a,0)+[S(a)/60].[m(c)-m(0)].$$

This approximation is an interesting variation on the method of Ptolemy. Kūshyār's approximation is somewhat less accurate but saves some computational work.

The reader has now got the flavor of the traditional Ptolemaic astronomical models and computations and their modifications by Kūshyār. The latter makes similar simplifications in the computation of the planetary longitudes, described in [Van Brummelen 1998].

The best description of Ptolemy's theory of planetary latitudes is to be found in [Pedersen 1974, 355-86]. The way in which Kūshyār handles Ptolemy's theory of latitudes has not yet been investigated by modern historians of science. Kūshyār made some modifications in his description of the calculations; see my commentary on the relevant chapters. It seems to me that Kūshyār (and for example his predecessor al-Battānī) understood the geometric rationale of Ptolemy's theory of latitude of planets, but not the fine points of the corresponding computations.

The parameter values in Kūshyār's models (eccentricities and radii of the epicycles) are, apart from scaling factors, almost always the same as in the *Almagest* of Ptolemy or the *zīj* of al-Battānī. In the case of Mercury, Kūshyār says in Book III that he takes the eccentricity $e = 3;10$, while Ptolemy and al-Battānī had taken eccentricity $e = 3;0$ parts. However, according to Kashino [1998, 17] and [Van Brummelen, 268],

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[13] Kashino's translation "difference for the nearest distance" [1998, 26, 45, 98,99] is misleading.

[14] In Kashino's formula (2.34), 2η should be equal to zero [1998, 13].

Kūshyār's tabular values are actually based on taking *e* equal to 3;0 parts, which was used by Ptolemy and al-Battānī. In Section I.4.4, Kūshyār mentions a change he made in the parameter values involving the equation of Mars. He does not specify his new parameter values, and refers only vaguely to observations of meridian altitudes and of conjunctions, without reference to specific observations. According to the mathematical analysis by Van Brummelen [1998, 268] based on Book II of the *Jāmi' Zīj*, Kūshyār changed a parameter value $e = 6$ which Ptolemy and al-Battānī used in the Mars model to a value in the neighbourhood of 6;2,35.

Although Book IV of the *Jāmi' Zīj* is said to contain the "Proofs" on Book I, Book IV does not contain anything like the determination of parameter values of the planetary models from observations, as explained by Ptolemy in the *Almagest*, and by Kūshyār's contemporary al-Bīrūnī in the *Canon Masudicus*. The reader may well ask what the word "proofs" in the title of Book IV really means.

It seems to me that Kūshyār referred to classical Greek geometrical proofs. If a very complicated quantity is computed in Book I, Kūshyār presents in the corresponding section of Book IV a geometrical figure with an abstract proof, in the style of the *Data* of Euclid. In the proof, Kūshyār demonstrates that a certain line segment or arc is "known". The reader is supposed to work out for himself that this line segment or arc in the figure corresponds to the quantity to be computed. Kūshyār often leaves it to his reader to identify in the figure in Book IV the given line segments or arcs, which correspond to the quantities that he supposes to be known in the computation in Book I.

It turns out, not surprisingly, that most innovations in the *Jāmi' Zīj* are mathematical in character. We have already seen the cosmetic changes in order to avoid subtractions, and the simplified interpolation procedure for the "second equation" of the moon and planets.

Kūshyār also presents what may be his own theoretical proofs and procedures in parallax and eclipse computations in Section IV.6. In these calculations, Ptolemy assumes that a number of (small) circular arcs can be approximated by straight line segments. Kūshyār describes Ptolemy's approximate methods (in Sections I.6.13, IV.6.10 and I.6.18, IV.6.13), but he then presents exact methods (in Sections I.6.8, IV.6.13 and I.6.17, IV.6.12), which are probably his own. The difference in the result of computation is often negligible and irrelevant for all practical purposes. Thus Kūshyār belongs to a tradition of Islamic mathematicians who were interested in theoretical proofs and methods, regardless of practical applicability. This tradition included famous mathematicians such as Ibn al-Haytham (ca. 965-1041 A.D.) who wrote a work of over 100 pages on the fact that the moon and planets may not culminate exactly in the meridian if their proper motion is not parallel to the celestial equator[15].

[15] This is the *Kitāb hay'at ḥarakāt al-kawākib al-sab'*, mentioned in [Sezgin 1974, 260, no. 27].

5. Manuscripts and editorial procedures

The manuscripts of the *Jāmiʿ Zīj* which I have used in my work and the abbreviations used for them are as follows (an asterisk * refers to the mss. used as bases for the Arabic edition).

A Alexandria, Baladiya Library, MS 4285 *jim* [Zaydān 1926-1929, I, 216-17]; Books III and IV, copied in 566 A.H./1170-71 A.D. from an autograph dated 393 A.Y./415 A.H./ 1025 A.D., fols. 1v-73v.

B Berlin, Staatsbibliothek, MS Mq. 101 [Ahlwardt 1887-1899, V, 203-206, no. 5751]; Books I and II, copied in 806[1] A.H./1403-04 A.D., pp. 2-221.

C* Cairo, Dār al-kutub, MS Muṣṭafā Fāżil Mīqāt 213/1 [King 1981-1986, I, 414; II, 104]; Book I, copied in 1169 AH/1755-56 A.D., fols. 1v-26r.

F* Istanbul, Fatih, MS 3418/1 [Krause 1936, 472] (Cat., p. 196); Books I-IV, copied in 545 A.H./1150-51 A.D., fols. 1v-175v.

L Leiden Universiteitsbibliotheek, MS Or. 8 [Voorhoeve 1957, 405; De Jong & De Goeje 1865-1866, III, 84-86, no. 1054]; Books I-IV, copied in 634 A.H./1236-37 A.D., fols. 1v-124r.

M Moscow, Russian state Library, MS 154/3 [Matvievskaya & Rosenfeld 1983, II, 217]; Books III and IV, copied in 525 A.H./1130-31 A.D., fols. 36v-111r.

P Leiden Universiteitsbibliotheek, MS Or. 523/1 [De Jong & De Goeje 1865-1866, III, 87-88, no. 1056]; Persian translation of Book I, copied in 689 A.H./1290-91, 31 fols.

V* Istanbul, Vehbi Efendi, MS 893 [Krause 1936, 472]; Book IV, copied in 427 A.H./1035-36 from an autograph, fols. 1r-75r.

Y Istanbul, Yeni Cami, MS 784/3 [Krause 1936, 472], (Cat. Ahmed III, p. 64); Books I-IV, copied in the 6th century A.H./12th century A.D., fols. 230r-362r.

The following mss. extant in Cairo [King 1986, 45] were not accessible to me:
Dār al-kutub Mīqāt: no. 400 (Books I and II, ca. 650 A.H./1250 A.D., [King 1981-1986, I, 62]); no. 691 (Books I and II, ca. 700 A.H./1300 A.D., [King 1981-1986, I, 120]); no. 188/2 (Book II, ca. 1200 A.H./1785 A.D., [King 1981-1986, I, 53]); Ṭalʿat Riyāża 102/3 (Book IV, 1128 A.H./1716 A.D., [King 1981-1986, I, 533]). On these manuscripts see also [King 1981-1986, II, 104-05].

[1]. The date 832 A.H. (=1428-1429 A.D.) is also written on the ms. by a later hand.

A manuscript kept in Hyderabad (Āṣaf. I, 798, no. 305) is mentioned in [Sezgin 1978, 248] as a Persian translation of the *Jāmi' Zīj* (cf. [Rosenfeld & Ihsanoğlu 2003, 118] that mentions it as *Zīj* of Kūshyār al-Jīlī). At my request, my colleagues in the Encyclopaedia Islamica Foundation (Tehran), Mr. Hasan Taromi Rad and Dr. Mohsen Ma'sumi, inspected this manuscript in their trip to Hyderabad in January 2006. This manuscript is not really by Kūshyār. It is a copy of Ulugh Beg's *zīj.*

In order to justify the choice of manuscripts for my edition, it is now necessary to provide some further information on the manuscripts and their relationship.

F is the oldest extant manuscript containing all four books of the *Jāmi' Zīj.* It is written in a clear hand and has very few scribal errors and omissions. F has a lacuna from the middle of the table of contents in the beginning of Book I until the middle of Chapter I.2.2.

The Arabic text of book I is also contained in manuscripts Y, L, B, and C. In B there is a lacuna from the middle of Chapter I.8 (corresponding to I.2.2 in F) to the middle of Chapter I.69 (corresponding to I.6.16 in F).

The Arabic text of Book IV is also contained in manuscripts V, M, A, Y, and L. Manuscript Y has a lacuna from the beginning of Book IV until the end of Chapter IV.9 (corresponding to IV.1.9 in F), and from Chapter IV.33 (corresponding to IV.5.6 in F) until the middle of Chapter IV.43 (corresponding to IV.5.16 in F).

The manuscripts can be divided into three groups according to the way in which Books I and IV are subdivided. (Subdivision of Books II and III are similar in all extant manuscripts).

Group 1: In manuscripts F, C, V and M Books I and IV are subdivided into different sections (*fuṣūl*), and each section is further subdivided into chapters (*abwāb*). Thus, in F, Books I and IV are divided into 8 sections, and Sections 1 through 8 of Book I are divided into 6, 6, 3, 12, 22, 20, 6, and 10 chapters respectively. Thus the total number of the chapters of Book I is 85. Inspection of the manuscript P has shown to me that it was translated from a manuscript of the same group as F.

Group 2: In manuscripts B, and A Books I and IV are directly divided into chapters. Thus, book I is divided into Chapters 1 through 84 in manuscript B. According to [King 1981-1986, II, 104], Book I was also subdivided into 84 chapters in the manuscript Dār al-kutub Mīqāt 400, and Book IV was subdivided into 63 chapters in the manuscript Ṭal'at Riyāża 102/3.

Group 3: Finally, the manuscript L subdivides Book I into consecutive chapters (as in manuscripts B and Y), and Book IV into 8 sections (as in manuscripts F, V and M. See the description in [De Jong and De Goeje 1865-1866, III, 84-86]. We note that Book I is divided into 80 chapters in L. On the other hand, manuscript Y subdivides Book I into sections and Book IV into chapters, which is different from the division of L in both Books I and IV.

The differences between the groups 1 and 2 concern not only the division of the Books, and additions or omissions made to the text (compare IV.3.1 and IV.3.4), but also to some extent the mathematical content. In IV.6.9, for example, the manuscripts F, V, L and M have only one figure for the first four cases of the proof, but manuscripts A and Y have four figures, one for each case. On the whole, the mathematical differences between the two groups are minor.

Since manuscript V (in group 1) and A (in group 2) both contain a statement to the effect that they are a copy of an autograph, I tentatively conclude that Kūshyār compiled more than one version of the *Jāmiʿ Zīj*, and that the groups 1 and 2 descend from different autograph versions. Since the text in A is to some extent mathematically superior to F, it is likely that group 2 represents a later version than group 1. In other words, Kūshyār originally started with the division of Books I and IV into 8 sections, subdivided into chapters, and he later decided to remove the sections and adopt a subsequent numbering of chapters in Books I and IV. He also made some minor mathematical changes to Book IV in the process. The many similarities between manuscripts F, V and C support to my mind the assumption that they descend from the first version of the *Jāmiʿ Zīj* which Kūshyār compiled. Manuscript L and Y in group 3 represent mixed versions.

Manuscript F was written in a classical Arabic language with few deviations from classical grammatical rules. Only very occasionally, the text violates the rules about agreement between genders (masculine and feminine) and between number (single or plural). Such violations may well be due to Kūshyār. At the end of manuscript A the scribe wrote a note to the effect that he found some grammatical flaws, such as confusion between genders and between singular and plural, in the autograph of Kūshyār's text from which A was copied. The scribe adds that he copied the text as it was without any change. One can try to interpret such deviations from the classical norms in Kūshyār's writings as traces of Middle Arabic. However, the deviations may also be due to the fact that Kūshyār's native language was not Arabic but Persian. The *Jāmiʿ Zīj* was a very technical text, written for a specialized audience with a long and thorough training in mathematics and astronomy. In such a text, one expects language of a formal nature. I have recorded all variant readings in manuscripts C and V in my apparatus, and very few of these variants may be traces of Middle Arabic. For example, on p. 25 note 34 of my edition, I have noted the variant readings ست عشر درجة in the manuscript C, which may be a Middle Arabic form for the classical form ست عشرة درجة in F (compare [Blau 1966-1967, I, 239]. I tentatively interpret the variant reading اي درجة (Arabic text p. 77, footnote 27) in V as a Middle Arabic form of اية درجة in F (and in Kūshyār's original), but the deviant form may have been in Kūshyār's autograph. It is impossible to decide such matters.

After much deliberation, I have opted for the following procedure for establishing the Arabic text.

I have chosen manuscript F as the base for my Arabic edition. As the main alternative manuscript for restoring illegible or missing words in F, I have used C for the edition of Book I and V for the edition of Book IV. In reconstructing the original text, I have used the other manuscripts in cases where V and C also have ambiguities or lacunae. I have not corrected minor grammatical flaws in the text of F, because such flaws may be due to the fact that Arabic was not Kūshyār's native language. I have included all variant readings of F, C and V in my critical apparatus. I have only recorded variants in other manuscripts A, B, L, M and Y if they seemed to me in relation to the meaning of the text.[2]

Since it is my aim to reconstruct the first original version of Kūshyār's *Jāmi' Zīj*, and due to the formal scientific character of the text, I have not embarked on a systematic investigation of other manuscripts from a linguistic point of view. For the same reason, I have not hesitated to adapt the orthography to modern standards in some cases, although in most cases I have followed the spelling of F. I have attempted to make the text accessible to modern scientists and historians of science in the middle East and elsewhere, i.e., the modern equivalent of the audience for which Kūshyār wrote his *Jāmi' Zīj*.

I have also used a copy of the only known manuscript of the Arabic treatise *al-Lāmi' fī amthilat al-Zīj al-Jāmi'* ("Explanation of the examples of the *Jāmi' Zīj* ") by Abu'l-Ḥasan 'Alī b. Aḥmad Nasawī (MS Or. 45/7,

[2] Only in exceptional cases, such as Section IV.6.9, I have adopted some of Kūshyār's adaptations in the later version of the *Jāmi' Zīj* in my text and translation, since they contribute to the clarity of the contents. See the commentary to the relevant passages.

Columbia University, New York, fols. 49r-75v), for providing some worked examples.

I have followed F for the spellings of the words. Whenever the Arabic letters are used to denote *Abjad* (sexagesimal alphabetic) numbers, I have printed them in boldface in my Arabic edition. I have not added punctuation marks. The chapters and sections are written continuously in the manuscript F, but I have started each chapter from a new line and each section on a new page. Whenever applicable, I have also divided each chapter into paragraphs for sake of clarity.

I have used the following abbreviations in the apparatus:

om. for omitted word or phrase
add. for added word or phrase

In the body of the Arabic I have used angular brackets < > for restoring the omissions of the text. I have put superfluous words in rectangular brackets [ ]. Significant marginal notes from all other manuscripts are also mentioned in the critical apparatus.

In the English translation, I have tried to maintain the structure of the sentences as much as possible. When this was not possible, then I have added a word or an expression in angular brackets < > to make the translation understandable. My explanatory additions to the translation are provided in parentheses ( ). Kūshyār usually writes the numbers in words, but I have used numbers in digits. For the technical terminology, I have tried to use the most recognized equivalents for Arabic astronomical terms. However, since there are usually different variant for the English equivalents of the Arabic astronomical terms, I have essentially followed Prof. E. S. Kennedy in his different publications. For some concepts that were not translated into English in previous works, I have used the equivalents that Prof. Kennedy wrote in a personal communication to me.
For the sexagesimal numbers, I have used the standard notation in which 21,33,8;24,17 stands for $21\times60^2+33\times60+8+24/60+17/60^2$.
The diacritical marks used for the pronunciation of Arabic terms or proper names are as follows:

ā for the long vowel *alef* ا
ū for the long vowel wāw و

ī for the long vowel *yā'* ي

ʿ = ع ʾ = ء (*hamza*)

th=ث dh=ذ gh=غ q=ق

ḥ=ح ṣ=ص ḍ=ض ṭ=ط ẓ=ظ

I have used roman numbers for the four books of the *Jāmiʿ Zīj*, and Hindu-Arabic numbers for the sections and chapters. In referring to the sections and chapters, I have used the abbreviated form such as I.5 for Section 5 of Book I, and IV.6.8 for Chapter 8 of Section 6 in Book IV.

The commentaries to the chapters of each section are provided at the end of the relevant section, using the above abbreviations to denote the chapters. Since most of the astronomical theories in Kūshyār's *Jāmiʿ Zīj* are essentially Ptolemaic, I have not gone into complete details. I have referred the reader to the standard works such as Toomer's translation of the *Almagest* [Ptolemy 1984], and Pedersen's *Survey of the Almagest* [Pedersen 1974]. On the other hand, Kūshyār is influenced by al-Battāni's *al-Zīj al-Ṣābī.* So I have referred to the corresponding discussion in al-Battānī's work (Nallino's publication of the *Zīj* [al-Battānī 1899-1907]), whenever applicable. For the calculation methods provided by Kūshyār, I have given the formula in modern notation. In some cases, I have provided a worked example to make the method more comprehensible. Prof. J. L. Berggren has published a translation of IV.3 [Berggren 1987] and Mr. T. Kashino has provided an edition and English translation of the chapters and tables relating to planetary longitudes in all four Books of the *Jāmiʿ Zīj*, as mentioned in Part 3 of this preface [Kashino 1998]. I have noticed significant differences between these publications and my edition, in my commentaries to the chapters and in my critical apparatus to the Arabic text. References are given in brackets and include author name, year of publication and page number(s).

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In the name of God the merciful, the compassionate, and we ask for your assistance, o Generous One!

Kūshyār ibn Labbān ibn Bāshahrī al-Jīlī says: When I examined the *zīj*es composed in the art of astronomy and reflected on them, <I found that> there was incorrectness in some of them that needed rectification; some had long-windedness and difficulty that needed simplification; and some had omissions that needed completion. <Even> the *Almagest* is not free of them (i.e., the defects). All of them (i.e., the *zīj*es) <contain> careless calculations, devoid of clear exposition and unsupported by adequate demonstration. <Therefore,> I made up my mind to work out a *zīj* combining theory and practice, in which I <would> rectify the incorrectness, bring closer what was far-fetched, fill up for deficiencies, elucidate every <technical> term with a comment, and provide proofs for every calculation in it. Therefore, any difference found in anything between this <*zīj*> and the others, is <caused by my> rectification of the incorrectness or <my> bringing closer the far-fetched or <my> filling of gaps. I have discussed practice before theory in order to facilitate the beginner's access to it and to quicken his benefiting by it. I have composed this <work> in four Books: the first on elementary calculations, the second on their (i.e., of the calculations) tables, the third on commentary and astronomy, and the fourth on the demonstration of the accuracy of the elementary calculations.

When I resolved to do this and reaffirmed my intention about it, I begged God for success and guidance.

<List of the chapters of> Book I: On elementary calculations, <in> 8 sections and 85 chapters

Section 1: On eras, <in> 6 chapters
1. On the beginnings of ancient eras and <the difference> between any two of them in years and days.
2. On the three eras used in our time.
3. On converting the years of these eras into days, and the days into <corresponding> years by calculation and <by using> the table<s>.
4. On extracting <dates in> these eras from each other.
5. On the weekday of <any date of> these eras.
6. On the feasts and <other> events in these eras.

Section 2: On Sines and Chords, <in> 6 chapters
1. Introduction to the knowledge of the Sine <function>.
2. On interpolation between two lines of the Sine <table> and other tables.
3. On <finding> the Sine of a <given> arc and the arc of a <given> Sine from the table.
4. On <finding> the Sagitta of a <given> arc and the arc of a <given> Sagitta from the <special> table and <from> the Sine table.
5. On <finding> the Chord of a <given> arc and the arc of a <given> Chord from the Sine table.
6. On correcting the Sine whenever we have doubt about any <value> of it.

Section 3: On Tangents and Cotangents, <in> 3 chapters
1. On calculating the Tangent and Cotangent, their two Hypotenuses (i.e., Secant and Cosecant) and their two arcs.
2. On <finding> the Tangent of a <given> arc and the arc of a <given> Tangent from the table.
3. On converting Tangents and Cotangents to different gnomons.

Section 4: On <finding> the true longitudes of the planets, and their situations, <in> 12 chapters
1. On the epoch values and the preliminaries for <finding> the mean longitudes of the planets.
2. On deriving the mean longitudes from their tables.
3. On converting the mean longitudes from <localities having> one <geographical> longitude to another.
4. On the positions of the apogees and the nodes, and <on> their motions.
5. On the equation of time.

6. On the true longitude of the sun.
7. On the true longitude of the moon and its node<s>.
8. On the true longitudes of the five planets.
9. On the latitude of the moon.
10. On the latitudes of the five planets.
11. On the retrogradation of the planets, their direct motion, and their first and last visibility.
12. On the ascension and descension of the planets in their spheres.

Section 5: On the operations relating to the ascendants during the day and the night, <in> 22 chapters
1. On the first declination.
2. On the right ascensions of the <zodiacal> signs.
3. On the second declination.
4. On the distance of the stars from the celestial equator.
5. On the latitude of <any> locality.
6. On the ortive amplitudes of the sun and <any> star.
7. On the equation of daylight of the sun and <any> star.
8. On the ascensions for a locality (i.e., oblique ascensions).
9. On the maximum altitude of the sun and <any> star.
10. On half the day arc of the sun and <any> star.
11. On the <equinoctial> day hours of the sun and <the> stars and the degrees of their <seasonal> hours.
12. On the <ecliptical> degree of the transit of a star through the meridian.
13. On the <ecliptical> degree of the rising and setting of a star.
14. On <finding> the arc of revolution of the celestial equator since the rising of the sun or the star<s> from the altitude of the <sun or the star at a given> time.
15. On <finding> the <elapsed> hours from the arc of revolution.
16. On <finding> the ascendant from the arc of revolution.
17. On <finding> the arc of revolution from the ascendant.
18. On <finding> the altitude of the <sun at a given> time from the arc of revolution.
19. On <finding> the arc of revolution since sunset from the ascendant.
20. On <finding> the ascendant from the arc of revolution since sunset.
21. On a base <value> applying to most operations concerning day and night.
22. On the equalization of houses.

Section 6: On eclipses and what pertains to them, <in> 20 chapters
1. On the motion of the two luminaries in <one> day and <one> hour.
2. On the magnitude of the <apparent> diameter of the two luminaries and the diameter of the shadow <of the earth>.

3. On the <ecliptical> degree of a conjunction and opposition, their hours and ascendants.
4. On the absolute and adjusted magnitudes of a lunar eclipse in digits.
5. On the absolute and adjusted timing of a lunar eclipse.
6. On drawing the figure of a lunar eclipse.
7. On finding the distance of the moon from the earth.
8. On the altitude of the pole of the ecliptic which is called 'the latitude of the clime of visibility'.
9. On the altitude of any desired degree of the ecliptic.
10. On the <equatorial> distance between the meridian and the <right> ascension of a known point of the ecliptic.
11. On the parallax of the two luminaries in the altitude circle.
12. On the six angles which are needed in <the calculation of> solar eclipses.
13. On <finding> the longitudinal and latitudinal parallax of the moon from these angles.
14. On the absolute and adjusted magnitudes of a solar eclipse in digits.
15. On the absolute and adjusted times of a solar eclipse.
16. On drawing the figure of a solar eclipse.
17. On <finding> the altitude of the moon taking account of its latitude.
18. On <finding> the longitudinal and latitudinal parallax of the moon by a method which can be proved.
19. On extracting the longitudes of localities.
20. On <determining> the visibility of the <lunar> crescent and the planets from <certain> arcs defined for them.

Section 7: On the operations relating to astrology, <in> 6 chapters
1. On <finding> the distance between the <ecliptical> degree of a planet and the cardines in <terms of> hours.
2. On <finding> the projection of the ray by means of equal (i.e. ecliptical) degrees.
3. On <finding> the projection of the ray by means of ascension (i.e. equatorial) degrees.
4. On <finding> the prorogations (i.e., astrological progressions).
5. On <finding> the transfers of the years and their ascendants.
6. On converting the ascendant of the world year from one locality to another.

Section 8: On the operations which are less needed, <in> 10 chapters
1. On <finding> the latitude of a locality from the hours (i.e., the duration) of <its> longest day.
2. On the altitude with no (i.e., zero) azimuth.
3. On <finding> the azimuth for any altitude which we assume.

4. On <finding> the altitude from the azimuth.
5. On the distance between two stars of which <only> one has a <non-zero> latitude.
6. On the distance between two stars both having <non-zero> latitudes.
7. On the extraction of the meridian line.
8. On the deviation of <the directions of> localities with known longitudes and latitudes from the meridian of our locality.
9. On of the fixed stars, and the features of some of them in order to recognize them by seeing.
10. On the names of the lunar mansions, and their rising days.

These are the <titles of the> chapters of this Book. I have presented them in order of their importance, and I have devoted most attention to the ones that are most necessary. God is the One who makes <us> successful in what is correct, and to Him we shall return.

Section 1: On eras, <in> 6 chapters
Chapter 1: On the beginnings of the ancient eras and the <numbers of> years and days between any two of them.

The famous eras preserved by the ancients (i.e., those who lived up to the author's time) are: the era of the Deluge, the era of Nabonassar, the era of Philippus, the era of <Alexander> the Two-Horned, the era of Augustus, the era of Diocletianus, the era of the Hejira, and the era of Yazdigird.
The Deluge: The era of the Deluge is used by the authors of the ancient *zīj*es such as the *Sindhind zīj* and *Shāh zīj*. Its beginning was the Friday close to the occurrence of the Flood in the time of Noah – peace be upon him! On that day, at sunrise, the sun was in Aries and the moon was in conjunction with it in the beginning of Aries, and the other planets were around the beginning of Aries. Subsequent eras are related to it (i.e., the Deluge).
Nabonassar: He was Nabonassar I, among the kings of Babylon.* The first day of his era was a Wednesday. Ptolemy rendered the mean motions of the planets in the *Almagest* for this era,* and he rendered the positions of the fixed stars for the beginning of the year 886 of it, which was the first day of the reign of Antoninus. Between Friday, the first day of <the era of> the Deluge, and Wednesday, the first day of this era, there are 860,172 days, which are equal to 2,356 Persian-Egyptian years of 365 days, and 232 completed days.
Philippus: He was Philippus, known as the Mason,* father of the Two-Horned*. He was one of the kings of Athens. He <reigned> after the death of Alexander of Macedonia (Alexander III). Theon of Alexandria based his *zīj*, called the *Canon*, on this era. The first day of his era was a Sunday, between which and the era of the Deluge there were 1,014,834 days or 2,780 years and 134 days.
The Two-Horned: He was Alexander II, known as the Two-Horned.* The first day of his era was a Monday, which was the first day of the seventh year of his reign, when he left the land of Macedonia, traveled over the <whole> Earth, and reached <very remote places of> the inhabited world. Between the Monday <which was the beginning> of this era and the epoch of the Deluge there were 1,019,273 days or 2,792 <completed> years and 193 completed days.
Augustus: He was one of the Roman kings. Christ was born in some year of his <reign>. The first day of this era was a Thursday, between which and the epoch of the Deluge there were 1,122,316 days or 3,074 years and 306 days.

Diocletianus: He was one of the kings of Christendom. The first day of his era was a Wednesday, between which and the epoch of the Deluge there were 1,236,639 days or 3,388 <completed> years and 19 completed days.
The Hejira was the emigration of the Prophet—God bless him and grant him salvation!—from Mecca to Medina. He entered it (i.e., Medina) on Monday, the eighth of the month Rabīʿ al-awwal, and the era is reckoned from the beginning of that year, which was a Thursday, the first day of Muḥarram. Thus between it and that <day of emigration> there are 67 days. The year <of the Hejira calendar> is 354 days plus 1/5 plus 1/6 <of a day>. When <the accumulation of> these fractions exceeds half a day, one day is added to the days of Dhu'l-ḥijjah, so <the number of> its days becomes 30, and <the number of> the days of this year becomes 355. This happens 11 times in the computation of every 30 years, because 11 is 1/5 plus 1/6 of 30. Between this epoch and the epoch of the Deluge there are 1,359,973 days or 3,725 years and 348 days. The determination of the intercalation is such that you should cast out thirties from the <elapsed> years including the desired year, and you should multiply the remainder by 11 and cast out thirties <from the product>. If the remainder is greater than 15, then the <given> year is a leap year, and if it is less, then it is not.
Yazdigird: He was Yazdigird, son of Shahriyār, son of Kisrā, the last of the Persian kings. The first day of the year in which he acceded to the throne was a Tuesday, between which and the epoch of the Deluge there were 1,363,597 days or 3,735 years and 322 days.
If we want to know <the number of the days or years> between any two epochs, we subtract the <number of> years or days closer to the epoch of the Deluge from the <number of> years or days farther from it, and the remainder is the <number of> years or days between them.

Chapter 2: On the three calendars used in our time.

The calendars used among us and in our time are: (a) The calendar of the Two-Horned, which is the Greek and the Syrian <calendar> because there is no difference between them except in the names of the months. The first Greek month is *Kānūn al-thānī* (i.e., *Kānūn* II) with <its> Greek name, and the following <months are based> on its arrangement (i.e., the arrangement of the Syrian months regarding the number of the days in each month); (b) the calendar of the Hejira, that is the Arabian calendar; and (c) the calendar of Yazdigird, that is the Persian calendar.
As to the Syrian <calendar>, its beginning was a Monday as has been mentioned before. The Syrian names of the months and the numbers of their

days, added up and separately, are as I say: *Tishrīn* I, 31 days, **31**; *Tishrīn* II, 30 days, **61**; *Kānūn* I, 31 days, **92**; *Kānūn* II, 31 days, **123**; *Shubāṭ*, 28 days and a quarter of a day, **151**; *Ādhār*, 31 days, **182**; *Nīsān*, 30 days, **212**; *Ayyār*, 31 days, **243**; *Ḥazīrān*, 30 days, **273**; *Tammūz*, 31 days, **304**; *Āb*, 31 days, **335**; *Aylūl*, 30 days, **365**. So a year has 365 days and a quarter of a day. Whenever <the accumulation of> the quarter is greater than half a day, the number of days of *Shubāṭ* is increased by one, so <the number of> its days becomes 29. The <number of> days of this year becomes 366, and it is a leap year. To know it (i.e., the leap year), you cast out fours from the number of years including the desired year. If the remainder is 3, then this is a leap year, and if the remainder is less, it is not.

As to the Arabic <era>, its beginning was a Thursday, the first day of the year in which the Prophet <Muḥammad>—God bless him and grant him salvation!—emigrated <to Medina>. It is the 15th of *Tammūz* of the year 933 of <the era of> the Two-Horned. The names of its months and the numbers of their days, added up and separately, are as I say: *Muḥarram*, 30 <days>; *Ṣafar*, 29 <days>, **59**; *Rabī' I*, 30 <days>, **89**; *Rabī' II*, 29 <days>, **118**; *Jumādā I*, 30 <days>, **148**; *Jumādā II*, 29 <days>, **177**; *Rajab*, 30 <days>, **207**; *Sha'bān*, 29 <days>, **236**; *Ramaẓān*, 30 <days>, **266**; *Shawwāl*, 29 <days>, **295**; *Dhu'l-qa'da*, 30 <days>, **325**; *Dhu'l-ḥijja*, 29 <days> plus a fifth and a sixth of a day, **354; <22>**. Thus a year <has> 354 days plus a fifth and a sixth of a day. Whenever <the accumulation of> these fractions exceeds half a day, its calculation is as has already been mentioned. The <numbers of> the days of these months are found in this way: You subtract the mean daily motion of the sun from the mean daily motion of the moon, and a complete revolution (i.e., 360°) is divided by the remainder. The result is 29;31,50 days approximately. Thus the months were established <as having> 30 days and 29 days alternately, and we add the extra fractions, i.e. the excesses over half a day, at the end of the year; this adds up to a fifth and a sixth of a day.

As to the Persian <calendar>, its beginning was a Tuesday, the first day of the year in which Yazdigird, son of Shahriyār, acceded to the throne. It is the 22nd of Rabī' I of the year 11 of Hejira, and the 16th of Ḥazīrān of the year 943 of the <era of the> Two-Horned. The names of its months and the numbers of their days, separately and added up, are as I say: *Farwardīn-māh*, 30 <days>, **30**; *Ardībahisht-māh*, 30 <days>, **60**; *Khurdād-māh*, 30 <days>, **90**; *Tīr-māh*, 30 <days>, **120**; *Murdād-māh*, 30 <days>, **150**; *Shahrīr-māh*, 30 <days>, **180**; *Mihr-māh*, 30 <days>, **210**; *Abān-māh*, 35 <days>, **245**; *Ādhar-māh*, 30 <days>, **275**; *Day-māh*, 30 <days>, **205**; *Bahman-māh*, 30 <days>, **235**; *Isfandārmadh-māh*, 30 <days>, **265**. Thus a year <has> 365 days. The five days added at the end of Abān-māh are called the *mustaraqa* ("stolen") <days>. Since the Persian year is approximately a quarter of a day less than a solar year, this becomes one day in every four years and one month in every 120 years. During the period of their domination, the Persians observed one intercalary month every 120 years. Thus this year had 13 months. They counted the first month of this year twice: once at the beginning of the year and once more at the end of the year. They put the extra five <days> in the intercalary month (i.e., at the end of the year). <Thus,> the first month of the year was the one in which the sun entered Aries. So, the five <days> and the beginning of the year were moved from one month to the next every 120 years. In the time of Kisrā, son of Qubād, Anūshervān, the sun entered Aries in Ādhar-māh, and the five <days> were placed at the end of Abān-māh. When 120 years had passed, it was the end of the reign of the Persians, the disruption of their government, and <the beginning of> the domination of the Arabs over them. So, this tradition was neglected, and the five <days> remained at the end of Abān-māh until the year 375 Yazdigird, when the sun entered Aries on the first day of Farwardīn-māh. We have been informed that in <the province> Fārs and those areas <near it>, the five <days> were moved to the end of Isfandārmadh-māh according to the ancient tradition. But in our areas, which are Rayy, Jurjān and Ṭabaristān, they are <still observed> at the end of Abān-māh. People think that it is <something related to> the Zoroastrian religion and tradition, and should not be replaced and changed. Each day of the <Persian> months has a special name by which it is called, viz.: *Hurmazd, Bahman, Ardībahisht, Shahrīr, Isfandārmadh, Khurdād, Murdād, Day-ba-ādhar, Ādhar, Abān, Khūr, Māh, Tīr, Kūsh, Day-ba-mihr, Mihr, Surūsh, Rashan, Farwardīn, Bahrām, Rām, Bād, Day-ba-Dīn, Dīn, Ard, Ashtād, Asmān, Zāmyād, Mārasfand, Anīrān*, and the five 'stolen' days <are> *Ahunavad, Ushtavad, Isfandmad, Vahukhshatra*, <and> *Vahishtavasht.*

Chapter 3: On converting the years of these calendars into days, and the days into <the corresponding> years by calculation and by <using> table<s>.

Calculation for the Syrian <calendar>: You multiply the <number of> completed Syrian years by 21,915, you divide the product by 60, and thus the <number of> days in those years will be obtained. If the division has a remainder greater than 30, we restore it to one day. You multiply the given <number of> days by 60 and you divide the product by 21,915: The <number of> years <contained> in those days will be obtained. We divide

the remainder of the division by 60: The <number of > days of the incomplete year will be obtained.
<Calculation for> the Arabian <calendar>: You multiply the <number of> completed Arabian years by 21,262 and you divide the product by 60: The <number of> days in those years will be obtained. You multiply the given <number of> days by 60 and you divide the product by 21,262: The <number of> years <contained> in those days will be obtained. We divide the remainder of the division by 60: The <number of> days of the incomplete year will be obtained.
<Calculation for> the Persian <calendar>: You multiply the <number of> completed Persian years by 365: The <number of> days in those completed years will result. You divide the given <number of> days by 365: The <number of> completed years will be obtained. The remainder is the <number of> days in the incomplete year.
<Conversion by means of> the table: If we compile tables, we record in them the multiple or single years, and months, and opposite them, the numbers of days in them in sexagesimals. Then the first <digit> of them (i.e., these numbers) is the absolute <number of> days. The second of them is a multiple of 60, i.e., once divided by 60. The third one is a multiple of 60×60, i.e., twice divided by 60. The fourth one is a multiple of $60\times60\times60$.
If we want <to find> the <number of> days of given years and months, we enter with the completed years in the table of the multiple years. We take the <number of> days corresponding to the nearest number below it, and write it down (B adds: "on the <dust> board"). Then we enter with the remainder <of the years> in the table for the single years, take the <number of> days corresponding to it, and add it to what we wrote down before, any <sexagesimal> digit to its corresponding <sexagesimal> digit. Then we take the <number of> days corresponding to the completed months and add it to the sum already obtained. Then the <number of> days in the given years and months will be obtained.
If we want <to find> the <numbers of> years and months <corresponding to a certain number> of days, we enter with the days in <the column for> the multiples of days, take the <number of> years corresponding to the nearest lesser number, and write it down. Then we subtract the <number of> days found in the table from the given <number of> days, each digit from its corresponding digit. Then we enter with the remainder of the days in <the column for> the single days and take the <number of> years corresponding to the nearest lesser number. Then we add it to the <number of> years that we wrote down before. We subtract the <number of> days found in the table of single <days> from the <remaining> days that we have, any digit from its

corresponding digit. We take the <number of> months corresponding to the nearest number below the <number of> remaining days. What remains from the <number of> days is the <number of> days of the incomplete month.

Chapter 4: On extracting <dates in> these calendars from each other.

If <a date in> one of these three calendars is known and we want to know <the corresponding date in> another calendar, we convert the known date into days until the present day, and keep it in mind. Then if <the era of> the known <date> precedes the <era in which the date is> unknown, we subtract the <number of> days between the two eras from the <number of> days that we kept in mind. If the <epoch of which the date is> unknown precedes the <epoch of which the date is> known, we add the <number of> days between the two eras to the <number of> days that we kept in mind. Then the remainder or the sum is the unknown <date of the desired> calendar in days. Then we convert it into years as already described. The <beginning of the> Syrian era precedes the <beginning of the> Arabian era by 340,700 days, and precedes the <beginning of the> Persian era by 344,324 days; the <beginning of the> Arabian era precedes the <beginning of the> Persian era by 3624 days. In order to check <the correctness of> the result of <converting> the calendar, we determine the weekday of the given date in the known calendar, and the weekday of the unknown date <in the desired calendar>. If they agree, then it is correct, and if they differ one or two days, we adjust the unknown <date> according to the known <date>.

Chapter 5: On the weekday of <any date of> these calendars.

The Syrian <calendar>: We convert its date into the <number of> days up to the desired day, plus this day. Then we cast out sevens and count the remainder from Monday. The <week->day at which <the number> finishes, will be the weekday <corresponding to> the given day. If we want to, we <may> cast out twenty-eights from the <number of> years including the desired year. We enter with the remainder in the weekday table, and take the weekday of <the beginning of> the desired month.
The Arabian <calendar>: We convert its date into the <number of> days as has already been discussed for the Syrian <calendar>.Then we cast out sevens and we count the remainder from Thursday. The <week->day at which the number finishes will be the weekday of the <given> day. If we want to, we <may> cast out multiples of twohundred-ten from the <number of> years including the given year. We enter with the remainder in the

weekday table and we take <the number corresponding to> the weekday <of the beginning> of the desired year. Then we add to it <the number corresponding to> the weekday of the desired month.
The Persian <calendar>: We cast out sevens from the <number of> years including the given year and we count the remainder from Tuesday. The <week->day at which <the number> finishes will be the weekday of <the beginning of> that year. For each month after Farwardīn we add two days, but we do not add anything for the weekday of Ādhar-māh because the weekday of <the first of> Abān-māh and that of Ādhar-māh are the same on account of the <five> "stolen" <days>.

Chapter 6: On the feasts and <other> events in these calendars.

Syrian <feasts>:
Mā'althā (for the literal meaning of the names of the feasts and their equivalents, see the commentary): If the 29th of Tishrīn I (October) is a Sunday, it is *Mā'althā*, otherwise, <it is> the Sunday which follows it. *Subbār*: If the 28th of Tishrīn II (November) is a Sunday, it is *Subbār*, otherwise, <the Sunday> that follows it.
Mīlād: the night which is followed by the morning of the 25th of Kānūn I (December).
Dinḥ: the 6th of Kānūn II (January).
Ṣaum al-'adhārā: It is the feast of *Ghayṭās*, the Monday which follows *Dinḥ*.
Ṣaum Naynawī: <It consists of> three days beginning on a Monday 22 days before *al-Ṣaum al-kabīr*.
'Īd al-haykal: the 2nd of Shubāṭ (February).
Al-Ṣaum al-kabīr: <For its> calculation we take the years of the Two-Horned <era> with the year we desire (i.e., the current year), and we add five to it. We cast out nineteens and we multiply the remainder by nineteen. If the product is greater than 250, we always subtract one from it; if it is less, we do not subtract anything. We cast out thirties from the result. Then we observe the remainder. If it is equal to <the number of days of> Shubāṭ <in that year> or less than that, then the <beginning of the> fast is on that day of Shubāṭ, if it is a Monday. Otherwise, the Monday after it <is the beginning of the fast>. If it (i.e., the remainder) is greater than the <number of> days of Shubāṭ <in that year>, we subtract the <number of> days of Shubāṭ from it. The remainder, <taken> as <number of the day> of Ādhār, is the beginning of the fast if it is a Monday. Otherwise, the Monday after it <is the beginning of the fast>.

We have compiled a table for it. For working with it, we take the years of <the era of> the Two-Horned with the year we desire (i.e., the current year), and we write it down in two positions. We divide one of the <numbers written in the> two positions by twenty eight and we divide the <number in the> other position by nineteen, after adding five to it. We enter along the length of the table with the remainder of the division by twenty eight, and along the width of the table with the remainder of the division by nineteen. The crossing position of the <column and the row of the> two numbers is the beginning of the fast. If it is <written> in black, it is in Shubāṭ, and if it is <written> in red, then it is in Ādhār.

Another method: It (i.e., the beginning of the fast) is on the nearest Monday to the conjunction which occurs between the 2nd of Shubāṭ (February) and the 8th of Ādhār (March). If we are in doubt about the nearest Monday, then it is <the Monday> which lies between Sha'ānīn and the *Fiṭr* that follows it.

Sha'ānīn: the Sunday, the 42nd of the days of the fast.

Fiṭr: the Sunday next to *Sha'ānīn*.

Al-Sha'ānīn al-ṣaghīra: the Friday following *Fiṭr*.

Sullāq: the Thursday 40 days after *Fiṭr*.

Finṭīqusṭī: the Sunday 10 days after *Sullāq*.

Ṣaum al-Salīḥīn: the Monday after *Finṭīqusṭī*.

Ṣaum Mārt Maryam: the first day of Āb (August).

Ẓuhūr al-Masīḥ: 6th of Āb (August).

Fiṭr Maryam: 15th of Āb (August).

'Īd al-ṣalīb: 14th of Īlūl (September); 13th of Īlūl (September) according to the Nestorians; 15th of Īlūl (September) according to the Romans and the Jacobites.

Suqūṭ al-jimār: the 7th, 14th, and 21st of Shubāṭ (February).

Ayyām al-'ajūz: Seven days starting on the 26th of Shubāṭ (February).

Nayrūz al-Mu'taḍid: 11th of Ḥazīrān (June).

Ayyām al-bāḥūr: Eight days starting on the 19th of Tammūz (July). The variation of the weather on these days indicates that during (the first to the eighth month of) the next year.

Arabian <feasts>:

'Āshūrā: It is the date of the murder of Ḥusayn b. 'Alī—May God honor him and be pleased with him!—<which occurred on> the 10th of Muḥarram.

Maulid al-Nabī - may the exalted God bless him and grant him salvation!: 12th of Rabī' I.

Yaum al-jamal: 15th of Jumādā I.

Mab'ath al-Nabī - may God bless him and grant him salvation!: 26th of Rajab.
Mi'rāj: the night of the 27th of Rajab.
Laylat al-ṣakk: the night of the 15th of Sha'bān.
Ṣaum: the days of Ramażān.
Fatḥ Makka: 20th of Ramażān.
'Īd al-Fiṭr: 1st of Shawwāl.
Al-Tarwīya: 8th of Dhu'l-ḥijja.
'Arafa: 9th of Dhu'l-ḥijja.
'Īd al-ażḥā: 10th of Dhu'l-ḥijja.
Ghadīr Khumm: 18th of Dhu'l-ḥijja.

Persian <feasts>:
Nayrūz: 1st of Farwardīn-māh (i.e., the month of Farwardīn).
Nayrūz al-khāṣṣa: 6th of Farwardīn-māh.
Mihrajān: 16th of Mihr-māh.
Mihrajān al-khāṣṣat al-ṣaghīr: 21st of Mihr-māh.
Gāgīl: 15th of Day-māh.
Bahmanjana: 2nd of Bahman-māh.
Sadaq: the night of the 10th of Bahman-māh.
Wādhīra: 22nd of Bahman-māh.
Katb al-ruqā': 5th of Isfandārmadh-māh, <based on placing> the "stolen" days at the end of Abān-māh.
The six *Jāhanbārs*: first, 26th of Ardībahisht-māh; second, 26th of Tīr-māh; third, 16th of Shahrīr-māh; fourth, 15th of Mihr-māh; fifth, 11th of Day-māh; sixth, the five "stolen" <days> of Isfandārmadh-māh.

Commentary

I.1.1 Historians from the Islamic period have confused Nabonassar, the king of Assyria whose reign began in 747 B.C. and whose era was later used in Ptolemy's *Almagest*, with Nabuchadnezzar (Nabokolassar), king of Babylonia, who reigned in the period 604-562 B.C., and who conquered Jerusalem. So, they have referred to the former by the arabicized form of the latter's name, i.e., *Bukhtanaṣṣar*.

Ptolemy lived in the time of Antoninus Pius (fl. 137 C.E.) and used the era of Nabonassar because, as he says in *Almagest* III.7, this was the era beginning from which ancient observations were preserved down to his time.

The Philippus after whom the epoch 324 B.C. is named, is a son of Alexander III (the Great) and a halfbrother of Alexander IV. His reign started in the same year as that of Alexander IV (323 B.C.), namely with the death of Alexander the Great. The title Mason (*al-bannāʾ*) is mentioned in all mss. except L. It does not occur in other sources that I have seen, save the *Muṣṭalaḥ Zīj* (MS BN arabe 2513), whose chapter on chronology seems to depend, to some extent, on Kūshyār.

In fact, it was Ptolemy's *Handy Tables* (Theon did not write a *zīj*), in which the Philippus era was adopted. This era also occurs in the *Almagest* as 'the death of Alexander' [Ptolemy 1984, 10, fn. 16].

It is generally accepted both by Muslim commentators and occidental scholars that the 'Two-Horned' (*Dhu'l-qarnayn*) mentioned in the Holy Koran, and used by Arab authors, Muslims, and Christians is to be identified with Alexander the Great (356-323 B.C.). He was Alexander III (not Alexander II, as Kūshyār calls him) of Macedonia. The era erroneously named after Alexander is actually the Seleucid era, which started with the death of Alexander IV and the accession of Seleucus, the founder of the Seleucid dynasty, to power [Ginzel 1906-1914, I, 136; Taqizadeh 1939, part 2, 124-27].

Al-Bīrūnī also mentions Diocletianus as "one of the kings of Christendom" [1879, 105], and says elsewhere that "He was the last of the pagan Emperors of Rome; after him they became Christians" [1934, 173]. In the Byzantine tradition, Diocletianus is primarily remembered as a prosecutor, for his edict of prosecution against the Christians that started in 303 C.E.

In early *zīj*es, if the remainder of a division for the determination of the intercalation of the Arabian years was 15, the resulting half of a day was usually truncated, which led to an ordinary 15th year and an intercalary 16th

year in every 30-years cycle. However, in table 2 of Book III of the *Jāmiʿ Zīj* for the number of days in multiples of Arabian years, Kūshyār gives the number of days in 15 Arabian years equal to 5316 =15×(354+11/30)+0.5 days. This means that, as was more common in later Persian *zīj*es, he rounded upwards the half of a day resulting from the accumulation of the fractions which led to an intercalary 15th year [cf. van Dalen 2000, 267].

Following is a summary of the numerical data given in this section:

Era	**Weekday**	**Days after the Deluge**	**Years+days**
Nabonassar (Assyrian, 26 Feb. 747 B.C.)	Wednesday	860172	2356y+232d
Philippus (Greek, 12 Nov. 324 B.C.)	Sunday	1014834	2780y+134d
Alexander (Seleucid, 1 Oct. 312 B.C.)	Monday	1019273	2792y+193d
Augustus (Roman, 30 Aug. 30 B.C.)	Thursday	1122316	3074y+306d
Diocletianus (Roman, 29 Aug. 284 C.E.)	Wednesday	1236639	3388y+19d
Hejira (Arabian, 15 July 622 C.E.)	Thursday	1359973	3725y+348d
Yazdigird (Persian, 16 June 632 C.E.)	Tuesday	1363597	3735y+22d

In this table, we see the number of days that had passed since the Deluge, at the beginning of each of the seven eras. Each number of days is also converted by Kūshyār into Persian years plus remaining days. Kūshyār's data imply that the epoch of the Deluge was taken to be Friday, 18 Feb. 3102 B.C., which was commonly used and is also implied in Kūshyār's astrological treatise [Kūshyār 1997, 140/141].

The above numbers of days for the Nabonassar, Alexander, Hejira and Yazdigird epochs are the most common ones [cf. van Dalen 2000, 266, table 2]. The correct number of days since the Deluge for the Philippus epoch is 1014932. The above number given by Kūshyār (1014834, found in the mss. C, Y, B and P) is probably an error by Kūshyār or the scribes. In the ms. L this number is given as 1014934, which is still wrong but closer to the correct number. Presumably the original digit 9 was miswritten as 8 (a possible error in the Arabic script), and the digit 2 was then changed to 4, in order to accord with the correct weekday (Sunday). For the Augustus era, the number given by Kūshyār (1122316, corresponding to 13 Nov. 30 B.C.) is one of two that are found in various other sources. It is based on the assumption that New Year in the ancient Egyptian and the Coptic calendar coincided in the time of Philippus instead of Augustus [cf. van Dalen 2000, 266]. Also the implied date for the Diocletian era, 12 Nov. 284 C.E., is one of two that were used in various early sources [cf. van Dalen 2000, 266].

I.1.2 In Arabic texts from the Islamic period, the adjective *Rūmī* (Roman) means either 'Roman' or 'Greek'. Here it refers to the Greek era. The modern names (and the numbers of days) of the 'Greek' months are for instance given by al-Bīrūnī in *al-Tafhīm* and his *Chronology*: Yanwārīūs (31), Febrārīūs (28), Mārṭīūs (31), Afrīlīūs (30), Māīūs (31), Yūnīūs (30), Yūlīūs (31), Aghusṭūs (31), Sebṭembrīūs (30), Aqṭubrīūs (31), Nuāmbrīūs (30), and Duqambrīūs (31). Kūshyār has observed the rule for determining the Syrian leap years in table 1 of Book II of the *Jāmi' Zīj* for the number of days in multiples of Syrian years.

The "conventional" Arabian lunar months have alternately 30 and 29 days. In the lunar months based on the visibility of the lunar crescent, generally used in modern time, the first day of any lunar month is the day following the first observation of the lunar crescent. In this system it is possible to have two consecutive 30-day months, or two consecutive 29-day months.

The Iranian calendar at the time of the advent of Islam was based on a vague solar year of 365 days consisting of 12 months of 30 days plus five extra days that were added at the end of the eighth month Abān. This year was originally taken from the Egyptian calendar. Some modern scholars have tried to determine the date of introduction of the Egyptian year in Iran on the basis of Kūshyār's description of the five epagomenai being at the end of Abān in the year 375 of the Yazdigird era (1006-7 C.E.), found in this chapter. For instance, Taqizadeh [1938, 12] believes that the introduction happened in the second decade of the fifth century B.C. However, none of the results have been fully satisfactory [Taqizadeh 1938, 5]. According to Kūshyār, as well as al-Bīrūnī and some other authors, Iranians intercalated one full month in each 120 years to compensate for the difference between the Egyptian year and the tropical year (about one-fourth of a day) and to keep the beginning of their year close to the vernal equinox [see e.g., Ginzel

1906-1914, 290-91]. Taqizadeh thinks that this sort of year was by no means a wholly fictitious year, as some seem to believe [1938, 57]. Recently François de Blois [1996] has tried to show that such an intercalation process was a mere "legend". However, in particular his "negative" argumentation has not convinced me.

De Blois starts his discussion with the assertion that no reference to an Iranian intercalary month is found in ancient sources and no event is reported to have happened in such a month. But from a mathematical point of view, the probability of a random event happening in an intercalary month following a 120 years period as mentioned above is $1/(120\times12+1)=1/1440$, which is less than 0.07%. He then casts doubt on the reliability of the accounts provided by Kūshyār and al-Bīrūnī for the intercalation in the Iranian calendar. Here his argument that Kūshyār prepared a manuscript of his *Jāmiʿ Zīj* in 393 A.H./1002-3 C.E. and hence could not have mentioned a calendar reform in 375 A.Y./1006-7 C.E. turns out to be invalid. Inspection of the Alexandria manuscript of the *zīj* shows that the date of Kūshyār's autograph was 'Sunday the 2nd of Bahman-māh of the year 393' [A.Y./8 Dhu'l-qaʿda 415A.H./10 January 1025 C.E.], so Kūshyār's reference to the reform can be correct. Moreover, in the second chapter of the text presented in this article, Kūshyār says that the transfer of the five epagomenai had not yet been accepted by the inhabitants of Rayy, Jurjān and Ṭabaristān, but in the Persian translation, ms. P, prepared in 483 A.H., Rayy is omitted from the names of the cities. This indicates that Kūshyār and the translator were giving a realistic and up-to-date account of what was going on around them.

In my opinion, de Blois's arguments regarding the problem of having two anniversaries for Zoroastre's death being 8 months apart, mentioned in *Zādspram* (chapter 25), the other passage that he quotes from *Zādspram* (chapter 34), and finally, the reference he makes to *Dinkard* [de Blois 1996, 43] are consistent with Kūshyār's clear description that after each intercalation the first month of the year shifted to the next one, so that the months drifted slowly through the seasons but the epagomenai always kept trace of the vernal equinox (e.g., before 375 A.Y. the year began with Ādhar-māh, but the vernal equinox was at the beginning of Farwardīn-māh). Kūshyār's description of the arrangement of the *Jāhanbārs* also confirms that a calendar reform took place in 375 A.Y. that followed the intercalation system of the pre-Islamic Iranian calendar (see Chapter 6 and its commentary). For a recent discussion of the subject that confirms the intercalation system mentioned by al-Bīrūnī and Kūshyār, see [Ghasemlou 2003, 825-26].

Even after the advent of Islam the Persian solar calendar was used in Iran beside the Hejira lunar calendar until the 5th/11th century. In the year 471 A. H./1079 C.E., the Jalālī or Malikī calendar was constituted. In this calendar the years began with the vernal equinox based on astronomical observation or calculation.

The modern version of the Persian names of the months as mentioned by Kūshyār in this chapter has been used in the formal Iranian calendar since 1925. In this calendar, the year begins with Farvardīn; the first six months have 31 days, the next five months have 30, and the last month, Esfand has 29 days in normal years and 30 days in leap years. The leap years usually occur every four years, but sometimes they are five years apart. This is determined by the exact moment of the vernal equinox being before or after local solar noon on the 29th of Esfand. The 1st of Farvardīn is the first day whose noon is after the exact time of the vernal equinox.

I.1.3 The lengths of Syrian and Arabian years are $21915{:}60=365\frac{1}{4}$ and $21262{:}60=354\frac{11}{30}$ days, respectively.

By "completed" years and months, Kūshyār means those which have passed. An "incomplete" year or month refers to a year or month which has not yet been completed. So, when we are in the month m of the year y of any calendar, $m-1$ completed months and $y-1$ completed years have passed from the beginning of the era. The month m and the year y themselves are incomplete.

The results of Section I.1.3 are used in Section I.1.4.

I.1.4 The Syrian date is based on the Seleucid era. The following chapter gives the method of determining the weekday for any date in each of the calendars. These methods can be used for checking the correctness of a date conversion from one calendar to another.

I.1.5 The second method for finding the weekday of a date in the Syrian calendar is based on the fact that 28 times 365.25 (days) is a multiple of 7. In table 4 of Book II of the *Jāmiʿ Zīj*, the weekdays of the first day of any Syrian month for the years 1 to 28 are given directly. Then it will be easy to find the weekday of any date in a given month. The weekdays are shown in the table in the conventional *abjad* numbers from 0 to 6, corresponding to Saturday, Sunday,..., Friday, respectively. This allows us to convert the final remainder into weekdays directly, because the Arabic names for Sunday up to Thursday are derived from the Arabic words for 'one' to 'five', respectively.

The second method for finding the weekday of a date in the Arabian calendar works because 210 times $354\frac{11}{30}$ is a multiple of 7. Table 5 of Book II of the *Jāmi' Zīj* is in two parts: In one part, the weekdays of the first day of the years 1 to 210 are listed. The other part displays the weekdays of the first day of the 12 Arabian months (assuming 0 for the first month, because its beginning is the same as the beginning of the year).

The method for the Persian years is valid because 365 is a multiple of 7, plus 1. For any month we add 2 days, because $30 = 4\times7+2$. We do not add anything for Ādhar-māh, because with the five epagomenae Abān-māh has 35 days, which is a multiple of 7. Table 6 of Book II gives the number (0 to 6) corresponding to the weekday of the beginning of each Persian month for each remainder r (1 to 7) of the number of years y of the Yazdigird era, if $y = 7k + r$ for an integer k.

Examples:

The weekday of the first day of Tishrīn I of the Syrian year 1359 is found as follows:

1358 (completed years)$\times21{,}915\div60 \approx 496{,}009$

$496{,}009+1= 496010=7\times70858+4$

The fourth day counting from the epoch Monday is Thursday. So the desired weekday is Thursday.

If we want to use table 4 of Book II, we proceed as follows:

$1359 = 28\times48+15$

The table entry for 15 (remainder of the Syrian year) is 5, which corresponds to Thursday.

The weekday of the first day of *Ramażān* of the year 439 of the Hejira era is found as follows:

438 (entire years)$\times21{,}262\div60\approx155{,}213$

The number of the months from the beginning of the year to the first of *Ramażān* is $4\times30+4\times29=236$, and we add one for inclusion of the desired day itself:

$155{,}213+236+1=155{,}450=22207\times7+1$

The first day counting from the epoch Thursday is Thursday itself. So, the desired weekday is Thursday.

If we want to use table 5 of Book II, we proceed as follows:

439=210×2+19

The table entry for 19 (remainder of the Arabian year) is 0, and the table entry for *Ramażān* is 5. Since 5+0=5, the corresponding weekday is a Thursday.

The weekday of the first day of *Mihr-māh* of the year 416 of the Yazdigird era is found as follows:

416=59×7+3

The third day counting from the epoch Tuesday is Thursday. So, the weekday of the beginning of the year is a Thursday. Now, since *Mihr-māh* is the 7th month of the Persian year, we add 12 for the six preceding months:

3+12=15=2×7+1

The first day counting from the epoch Tuesday is Tuesday itself. So, the weekday of the beginning of *Mihr-māh* is Tuesday. In table 6 of Book II, the entry corresponding to r = 3 and *Mihr-māh* is 3, which corresponds to Tuesday.

I have taken these examples from the treatise *al-Lāmi' fī amthilat al-Zīj al-jāmi'* ("Explanation of the examples of the *Jāmi' Zīj*") by Abu'l-Ḥassan 'Alī b. Aḥmad al-Nasawī mentioned in the introduction of this dissertation. Al-Nasawī's calculation (fols. 51r-52r) shows some insignificant differences with what I have provided above because he made a mistake in finding the weekday of the beginning of *Tishrīn I* of the year 1359 of the Syrian era by calculation. Note that all three examples are for the years 1047-8 C.E., the time of composition of al-Nasawī's commentary.

I.1.6 The modern equivalents and the meanings of these feasts are as follows:

NAME	EQUIVALENT	MEANING
Syrian:		
Mā'althā	Presentation of Christ	
Subbār	Annunciation	
Mīlād	Christmas	Birth of Christ
Dinḥ	Epiphany	
Ṣaum al-'adhārā (Ghayṭās)		The Fast of the Virgins
Ṣaum Naynawī		The Fast of Nineveh
'Īd al-haykal	Wax Feast	The Feast of the Temple
Al-Ṣaum al-kabīr	Lent	The great Fast
Sha'ānīn	Palm Sunday	

Al-Sha'ānīn al-ṣaghīra		The lesser Sha'ānīn
Fiṭr	Easter	Fast-breaking
Sullāq	Ascension day	
Finṭīqusṭī	Pentecost, Whitsunday	
Ṣaum al-Salīḥīn		Fast of the Apostles
Ṣaum Mārt Maryam		Fasting for the illness of Mary
Ẓuhūr al-Masīḥ		Advent of Christ
Fiṭr Maryam		Fast-breaking in commemoration of Mary's death
'Īd al-ṣalīb		Feast of the Cross
Suqūṭ al-jimār		Falling of pebbles
Ayyām al-'ajūz		Days of the old woman
Nayrūz al-Mu'tażid		Mu'tażid's New Day
Ayyām al-bāḥūr	Dog days	

Arabic:

'Āshūrā'	The 10th day of Muḥarram
Maulid al-Nabī	Birth of the Prophet
Yaum al-jamal	The day of the Camel Battle
Mab'ath al-Nabī	Appointment day of the Prophet
Mi'rāj	Ascension day of the Prophet
Laylat al-ṣakk	The great Liberation night
Ṣaum	Fasting
Fatḥ Makka	Conquest of Mecca
'Īd al-Fiṭr	Feast of fast-breaking
Al-Tarwīya	Watering
'Arafa	Recognition
'Īd al-ażḥā	Feast of Immolation

Persian:

Ghadīr Khumm		Khumm pool
Nayrūz	Pers. *Nowrūz*	New Day
Al-Nayrūz al-khāṣṣa		*Nayrūz* of the nobility
Al-Mihrajān al-khāṣṣat al-ṣaghīra		The lesser specific Mihrjān
Katb al-ruqā'		Charms against scorpions
Jāhanbārs	Pers. *Gāhanbār-hā*	Seasonal feasts

The calculation of Lent by means of Kūshyār's tables is explained in [Saliba 1970, 197-98]. The explanation for *Ayyām al-bāḥūr* in parentheses in the translation is taken from al-Bīrūnī, whose account is clearer [1934, 184]. All of the feasts and fasts mentioned by Kūshyār are also described by al-Bīrūnī [1879, 199-334; 1934, 174-186; 1954-1956, I, 238-270] whose account is more complete and gives a more extensive explanation for each case. Since al-Bīrūni dedicated his *Chronology* to Qābūs in 390 A.H/999-

1000 C.E., it is highly probable that Kūshyār made use of it. In fact, he repeats the mistakes made by al-Bīrūnī (see below). In only a few cases he gives different data.

Thus Kūshyār says that first of Āb is called *Ṣaum Mārt Maryam*. But according to al-Bīrūnī [1879, 296; 1954-1956, I, 242] this is the *Ṣaum maraż Maryam* ("Fasting on account of the illness of Mary"), and he puts *Ṣaum Mārt Maryam* on the Monday that follows *Subbār* [1879, 310; 1954-1956, 245]. Kūshyār says that the *Ayyām al-bāḥūr* are eight days beginning on the 19th of Tammūz. Al-Bīrūnī's account in *al-Tafhīm* [1934, 184] is the same as Kūshyār's, but in [1879; 268; 1954-1956, I, 270] al-Bīrūnī says that they are seven days beginning on the 18th of Tammūz.

Al-Bīrūnī [1879, 329; 1954-1956, 256] puts *Yaum al-jamal* on the 3rd of Jumādā I. Only in ms. C of the *Jāmi' Zīj* it is mentioned to be on the 15th of Jumādā I. Other mss. do not mention it at all. According to Kūshyār (as found in all mss. that contain Book I), *Fatḥ Makka* ("the Conquest of Mecca") was on the 20th of Ramażān, but al-Bīrūnī [1879, 330; 1954-1956, 256] puts it on the 19th of Ramażān.

Al-Bīrūnī [1879, 214] calls the feast on the 22nd of Bahman *Bād-rūz* instead of Kūshyār's *Wādhīra*. Also instead of *Gāgīl*, we read *Kākthl* and *Kāvkīl* in al-Bīrūnī [1879, 212; 1954-1956, 260].

Each *Jāhanbār* (Persian *Gāhānbār*, lit. "The feasts of the [six] times [of creation]") consists of five days and Kūshyār defines their beginnings. Al-Bīrūnī's account of the beginnings of the six *Jāhanbārs* [1879, 204, 205, 207, 210, 212, 217; 1954-1956, 259-60] is different from Kūshyār's. The dates according to al-Bīrūnī are as follows: I) 11th of Day-māh, II) 11th of Isfandārmadh-māh, III) 26th of Ardībahisht-māh, IV) 26th of Tīr-māh, V) 16th of Shahrīwar-māh, VI) the five 'stolen days' at the end of Abān-māh. There is a shift of two in the numbers of the *Jāhanbārs* between Kūshyār and al-Bīrūnī. Kūshyār puts the 6th *Jāhanbār* at the end of Isfandārmadh-māh and al-Bīrūnī puts it at the end of Abān-māh. Zoroastrian sources are not consistent in this regard [Taqizadeh 1938, 11] and there were different accounts of the beginnings of the *Jāhanbārs*. Kūshyār's account matches with an old Pahlavi text *Āfaringān Gāhanbār* and with the calendar reform of 375 A.Y., and his system is now used by the Zoroastrians [Taqizadeh 1937, footnotes of pp. 18-10].

Most of the feasts listed by Kūshyār (and al-Bīrūnī) are still celebrated, but not always on the same dates. In the present lithurgical calendar of the Syrian Orthodox Church *Mā'althā* is celebrated on February 2nd as the presentation of Christ at the Temple of Jerusalem. Kūshyār's description for *Mā'althā* is valid for the present feast Sanctification of the Church, which corresponds to *'Id al-haykal*. The latter falls on a Sunday in late October or early November. Kūshyār confused these two feasts with each other. The first Sunday of the Advent now falls on the 28th of November if it is a Sunday; otherwise it is the next Sunday. Kūshyār mentions this as *Subbār*. However, at present *Subbār* is celebrated on March 25th. *Ṣaum Maryam* now begins on the 10th of August, and ends at the date given by Kūshyār (the 15th of August). The fast of the Apostles is now celebrated on June 26th-29th, while the corresponding fast in Kūshyār's account, *Ṣaum al-Salīḥīn*, was on the Monday after Pentecost, so depended on Easter.

Nayrūz al-Mu'taẓid was actually a Persian feast, but it was adjusted with the Syrian date 11th of *Hazīrān* (June) [cf. al-Bīrūnī 1934, 185-86]. *Ayyām al-'ajūz* and *Soqūṭ al-jimār* are Arabian occasions but defined by the solar (Syrian) dates. Al-Bīrūnī says that, according to the Greeks, *Ayyām al-bāḥūr* (Dog days) are connected with the (heliacal) rising of the Dog-star of Orion, i.e., Sirius [see al-Bīrūnī 1934, 183].

The Arabian feasts have mostly been preserved up to now, because they are actually connected to Islamic occasions and rituals. However, their importance (manifested in being a formal holiday or not) is not the same in different Islamic countries and among different sects. Also their exact dates are not always agreed unanimously. *Ramaẓān* (the month of fasting) and *'Id al-Fiṭr* (the feast of fast breaking), as well as the occasions connected with the Prophet, i.e., *Maulad al-Nabī* (his birth), and *Mab'ath al-Nabī* (his appointment), and those connected with *Ḥajj* (pilgrimage to Mecca), i.e., *'Arafa* (recognition) and *'Id al-aẓḥā* (immolation), are evenly important in all the Islamic world. *'Ashūrā* and *Ghadīr Khumm* are of particular importance in Shi'ism.

In present Iran, *Nowrūz* (in Arabic *Nayrūz*) is celebrated as the most important formal national feast on 1-4 Farvardīn (usually 21-24 March). *Mihrgān* (in Arabic *Mihrajān*) now falls on the 10th (and not 16th) of *Mihr* because each of the first six Iranian months now have 31 days (not 30 days). *Sadeh* (in Arabic *Sadaq*) still falls on the 10th of *Bahman*. Its name is derived from the Persian word *sad* or *ṣad* which means "hundred", because on this day 50 days plus 50 nights remain until *Nowrūz* [Cf. Bīrūnī 1934, 182; 1954-1956, 260]. The latter two feasts are still remembered and celebrated on a limited level, but not as formal holidays. *Gāhanbār-hā* (in Arabic *Jāhanbārāt*) as well as *Mihrgān* and *Sadeh*, are regarded as important national and religious feasts among the Zoroastrians who also celebrate other old Iranian feasts.

Section 2: On Sines and Chords, <in> 6 chapters
Chapter 1: On introduction to the knowledge of the Sine <function>.

The Sine is a rule (i.e., function) referred to in finding the magnitudes of all arcs. The greatest Sine, i.e. half the diameter of a circle, can be supposed <to be divided into> any <number of > parts, but the easiest and most comprehensive <method> for calculation is <supposing it> to consist of 60 <parts>. The Cosine of an arc is the Sine of its complement to 90 degrees, as in the case of the Cosine of 36 <degrees> by which is meant the Sine of 54 <degrees>, and the Cosine of 54 <degrees> by which is meant the Sine of 36 <degrees>. We shall content ourselves with <calculating> the Sines of the degrees of a quadrant, because, for any <arc> exceeding the quadrant, the Sine is the same as <the Sine of> the degrees of the quadrant <counted> backward from 90 to 1. So, the Sine of 91 <degrees> is equal to that of 89 <degrees>, the Sine of 92 <degrees> is <equal to> the Sine of 88 <degrees>, and so on until the Sine vanishes at 180 <degrees>. Then, we begin a second time, according to the first description up to 360 <degrees>.

The Sagitta of an arc reaches 120 degrees, and this is the diameter of the circle. In calculations, whenever we say that a certain <number> is multiplied by another <number>, lowered, or a certain <number> is divided by another <number>, lowered, we mean that we lower this <second> number by one <sexagesimal> place; so, if it is in degrees, we take it as minutes, and if it is in minutes, we take it as seconds, and so on.

After this introduction, <I say that> the exact value of the Sine of 1 degree cannot be found, but its approximate value has been investigated, so that there is no difference between it and the exact value in <arithmetical> operations. I have derived it by <detailed> investigation to be <equal to> 1;2,49,38,31. We shall explain how to calculate it in the chapter on <its> proof.

The calculation of the Sine of <the arcs> greater than <one> degree is easy, and it should be preceded by an introduction on how to know the Cosine of the arcs whose Sines are known. To calculate it, you subtract the square of the known Sine from the square of the greatest Sine, and take the square root of the remainder. The result is the Cosine of the arc with known Sine. By this calculation, the Cosine of 1 degree, which equals the Sine of 89 <degrees>, is found to be 59;59,27,6,12,39.

If we want <to find> the Sines of other degrees, we multiply the Sine of the preceding degree by the Cosine of 1 degree, lowered, multiply the Sine of 1 degree by the Cosine of the preceding degree, lowered, and add the products to find the desired Sine of the degree. Example: We want <to find> the Sine of 24 <degrees>. We multiply the Sine of 23 <degrees> by the Cosine of 1 degree lowered. Then we multiply the Sine of 1 degree by the Cosine of 23 <degrees> lowered, and add the two products to find the Sine of the desired degree, i.e. the Sine of 24 <degrees>. The Sine of 24 <degrees> is obtained not <only> through 1 and 23 <degrees>, but through any pair of numbers whose sum is 24, and the calculation is the same as for 1 and 23, like 10 and 14, 12 and 12, 18 and 6, and so on.

Chapter 2: On interpolation between two lines of the Sine <table> and other tables.

In all tables, the ratio of the difference between two <successive> values of the argument to the difference between the two <corresponding> entries of the table equals the ratio of <any> part of the difference between the two values of the argument to the <corresponding> part of the difference between the two entries of the table. Thus there are four proportional numbers: the difference A between two <successive> values of the argument; the difference B between two <corresponding> entries of the table; <any> part C of the difference between the two <successive> values of the argument; <the corresponding> part D of the difference between the two entries of the table. The desired unknown E is either the part of the difference between the two entries of the table, or the part of the difference between the two values of the argument. If the part of the difference between the two entries of the table is desired, we multiply the known part c of the difference between the two values of the argument by the difference b between the two entries of the table, and divide it by the difference a between the two values of the argument. If the part of the difference between the two values of the argument is desired, we multiply the known part d of the difference between the entries of the table by the difference a between the two values of the argument and divide it by the difference b between the two entries of the table. Thus the desired unknown will be obtained. Then, if it should be added to the entry in the table or value of the argument, we add it; and if it should be subtracted, we subtract it.

Difference A between arguments in two <successive> rows	Difference B between entries in two <successive rows>
Partial diff. C betw. arguments in two <successive> rows	Partial diff. D betw. entries in two <successive> rows

<figure>

Chapter 3: On <finding> the Sine of a <given> arc and the arc of a <given> Sine from the table.

If we want <to find> the Sine of a given arc, we enter the arc in the row of the arcs, which is the row of the arguments, and take the Sine corresponding to it, <and we correct it, if necessary> according to what has been already said about interpolation. If we want <to find> the arc of a given Sine, we enter the Sine in the table and take the arc corresponding to it, <and we correct it, if necessary> according to what has been said on interpolation.

Chapter 4: On <finding> the Sagitta of a <given> arc and the arc of a <given> Sagitta from the <special> table and from the Sine table.

For the Sagitta a table has been presented from which the Sagitta of a <given> arc and the arc of a <given> Sagitta can be taken in the same way that the Sine of a <given> arc and the arc of a <given> Sine are taken from the Sine table. If we want <to find> the Sagitta of an arc from the Sine table, we look at it: If the arc is less than 90 <degrees>, we subtract it from 90 <degrees>, take the Sine of the remainder, and subtract it (i.e., the Sine) from 60. If the arc is greater than 90 <degrees>, we subtract 90 <degrees> from it, take the Sine of the remainder, and add 60 to it (i.e. the Sine). If we want to <find> the arc of a <given> Sagitta, we look at it: If the Sagitta is less than 60, we subtract it from 60, take the arc of the remainder from the Sine table, and subtract it (i.e., the arc) from 90 <degrees>. If the Sagitta is greater than 60, we subtract 60 from it, take the arc of the remainder, and add it (i.e., the arc) to 90 <degrees>.

Chapter 5: On <finding> the Chord of a <given> arc and the arc of a <given> Chord from the Sine table.

We do not need any of these Chords in this book; we mention them <just> to complete the <list of> operations. If we want <to find> the Chord of a <given> arc, we halve the arc, take its Sine, and double it. If we want <to find> the arc of a <given> Chord, we halve the Chord, convert it to an arc <by means of the Sine table>, and double it.

Chapter 6: On correcting the Sine whenever we have some doubt about any <value> of it.

The calculation of the Sine table, and the checking of its correctness have <already been> finished; so we do not need to repeat anything about it and its calculation. However, if we doubt <the exactness of> the Sine of any degree, we look at it: If half the <number of the> degrees is an integer, we take its (i.e. this) half, multiply its Sine by its Cosine lowered, and double the result. This result will be the Sine of the degree about which there was some doubt. Example: <Let us suppose> we doubt the correctness of the Sine of 24 <degrees>. We multiply the Sine of 12 <degrees> by the Sine of its complement, 78 <degrees>, lowered, and double the result. <Thus> the Sine of 24 <degrees> will be obtained.
If there is no integer half for <the number of> these degrees, we take two arcs the sum of which equals <the number of> these degrees. Then we multiply the Sine of the smaller arc by the Cosine of the greater arc lowered, multiply the Sine of the greater arc by the Cosine of the smaller arc lowered, and add the results. It will be the Sine of the degree about which there was some doubt. Example: <Let us suppose> we doubted <the correctness of> the Sine of 25 <degrees>. Of the many pairs of arcs whose sum equals 25 <degrees>, let us take 10 <degrees> and 15 <degrees>. Then we multiply the Sine of 10 <degrees> by the Sine of 75 <degrees> lowered, multiply the Sine of 15 <degrees> by the Sine of 80 <degrees> lowered, and add the results. Then the Sine of 25 <degrees> will be obtained. If we find the Sine of 24 <degrees> also according to this computation and this example, it will be correct, but that <other> method for even numbers is easier.

Commentary

I.2.1 Kūshyār follows Ptolemy (*Almagest* I.10) in taking the radius of the circle to be equal to 60 units. Thus Kūshyār's Sine and Cosine functions are 60 times the modern sine and cosine functions, respectively. This has the consequence that Kūshyār often has to divide the product of two Sines by 60. Since he is working in the sexagesimal system, the division by 60 is easy. For dividing by 60, he uses a special term "lowered" (*munḥaṭṭan*), because division by 60 corresponds to change of one sexagesimal position. See e.g. the commentary to I.2.6.

Based on the value of the Sine of 1 degree as mentioned by Kūshyār, I have computed the Cosine of 1 degree as 59; 59,27,6,12,38.72. In the text, the sixth sexagesimal digit is rounded to the nearest integer (39).

The fifth sexagesimal digit is given as 57 (instead of 12) in the manuscripts C, L, and B (the other manuscripts do not contain this fragment). A scribe must have misread the *abjad* numeral يب (12) as نز (57).

Kūshyār gives the Sine of 1° to five sexagesimal digits, while he computes the Cosine of 1 degree up to the sixth sexagesimal digit. We can assume that Kūshyār computed Cos 1° according to $Cos1^\circ = \sqrt{(R^2 - Sin^2 1^\circ)}$ with $R = 60$. Then the error in Cos 1° can be shown to be approximately 1/60 of the error in Sin 1°. So, Kūshyār's method can be justified. However, L and B give the sixth sexagesimal digit of the Sine of 1 degree as 0, but a sixth sexagesimal digit is not found in C.

The modern value of the sine of 1 degree is 0.017452406..., which corresponds to the Sine of 1 degree equal to 1;2,49,43,11,... Thus Kūshyār's value is correct to 3 sexagesimal digits. His value corresponds to sin1°= 0.017452046..., which represents the correct value up to the 5th decimal digit. Starting from this smaller Sine, he actually finds a greater Cosine. His result, 59;59,27,6,12,39, corresponds to 0.999847701, whereas the correct values of Cos 1°and cos1° are 59;59,27,6,7,45,... and 0.999847695..., respectively. The calculation method is presented in IV.1.11, where Kūshyār describes how to calculate the Chord of 1 degree. The same method is applicable for deriving the Sine of 1 degree.

At the end of this chapter, he uses the formulae

$Sin^2 (x) + Cos^2 (x) = 60^2$, and

Sin(x+1)=Sin(x)Cos(1)/60+Sin(1)Cos(x)/60,

to find the Sines of arcs greater than 1 degree.

Kūshyār's Sine table in II.8 is quoted in the manuscript of Yaḥyā b. Abī Manṣūr's *Zīj al-Ma'mūnī al-mumtaḥan* [1986, 101] by a scribe who lived after Kūshyār.

I.2.2 Here Kūshyār discusses two problems: 1) If $n<x<n+1$, find Sin(x); 2) If $\mathrm{Sin}(n)<y<\mathrm{Sin}(n+1)$, find x so that Sin(x)=y. For these two purposes, he uses linear interpolation. Thus he assumes that in very small intervals, the increase in Sin(x) is proportional to the increase in x. The Arabic term for argument (independent variable) in the text is the same term for "number": *ʿadad*. For the entry (function or dependent variable) he simply uses the term *jadwal* ("table"). For the "value" of an argument or entry, he uses the word *saṭr* ("line"). Kūshyār uses the Arabic expression *taʿdīl bayn al-saṭrayn* (adjustment between two <consecutive> values) for the process of interpolation.

I.2.3 The word *saṭr* literally means "row", but Kūshyār apparently uses the term *saṭr* in the more general sense of "line". In the Sine tables, arcs are usually written in a column, as is the case in F, Y, and B (and quoted in Yaḥyā's *zīj* mentioned in the commentary of I.2.1). However, the arcs may also be given in a row at the top of the table, as in the detailed Sine tables of L and B (see also the figure in Chapter I.2.2).

I.2.4 Here the Sagitta of any given arc is defined as Sag(x)=60-Cos(x), and the arc of any given Sagitta is discussed. Table II.9 in F is for the Sagittae. We also find it in Y, L, and B. Al-Bīrūnī [1934, 5] and Ḥabash Ḥāsib [in his *zīj*, MS Berlin Ahlwardt 5750 (WE. 90), fol. 80v] call this function as *al-jayb al-maʿkūs* ("Versed Sine"). Al-Bīrūnī [1934, 4] uses the term *sahm* ("Sagitta") as "the line between the middle of a chord and the middle of the corresponding arc", and mentions that the Versed Sine of an arc is equal to the Sagitta of its double.

I.2.5 This chapter is about the Chords of arcs, which are related to Sines and modern sines by Chord(x)=2Sin(x/2)=120sin(x/2). There is no table for Chords in the *Jāmiʿ Zīj*, whereas the table of Chords is the only plane trigonometrical table given by Ptolemy (*Almagest* I.11).

I.2.6 Here Kūshyār provides a method to check the correctness of any entry in the Sine table, using the formulae

$$\mathrm{Sin}(2x)=2\mathrm{Sin}(x)\mathrm{Cos}(x)/60,$$

and

$$\mathrm{Sin}(x+y)=[\mathrm{Sin}(x)\mathrm{Cos}(y)+\mathrm{Cos}(x)\mathrm{Sin}(y)]/60.$$

At the end of *Almagest* I.10, Ptolemy describes similar ways of checking the correctness of the value of a suspected Chord with different methods.

Section 3: On Tangents and Cotangents, <in> 3 chapters
Chapter 1: On calculating the Tangent and Cotangent, their two Hypotenuses (i.e., Secant and Cosecant) and their two arcs.

The Tangent (lit. "the First Shadow") is obtained from gnomons parallel to the horizon plane; it is called the Reversed Shadow. This is the one that we have included in the table for the elementary calculations. The Cotangent (lit. "the Second Shadow") is taken from gnomons perpendicular to the horizon plane; it is called the Horizontal Shadow. This is the one which we have included it in the table for knowing <the length of shadows in> digits and feet at mid-day (see the commentary of the next chapter). It is <also> provided in the calendars.
The gnomon can be supposed <to be divided by> any <number of> parts, however, the easiest <method> in the elementary calculations is <supposing it> to consist of 60 parts. Therefore, we have taken <the values of> the Tangent based on the gnomon being <divided into> 60 parts, and the Cotangent based on the gnomon being <divided into> 12 digits or 7 feet. If the gnomons have identical divisions, the Tangent of any arc is <equal to> the Cotangent of the complement of that arc (i)[1]. If any number is multiplied by the Tangent of any arc, <lowered>, or divided by the Tangent of the complement of the arc, <lowered>, the results> are equal; the product and the quotient are the same (ii).
The Secant (lit. "the Hypotenuse of the Shadow") is the line connecting the tip of the gnomon and the end of the shadow. The arc of a Tangent is the arc of the altitude <angle>, which increases and decreases <depending> on the shadow of the gnomons.
After this introduction, <I say that> if we want the Tangent of an arc, we divide the Sine of the arc by the Cosine of the arc, lowered. The result is the Tangent, based on a gnomon <of> 60 parts (iii). If we want its Secant, we divide the Tangent by the Sine of the arc, lowered: The result is the Secant (iv). If we want <to use another method>, we add the square of the Tangent to the square of the gnomon, and we take its square root (v). If we want the arc of the Tangent <and we have no Tangent table>, we divide the Tangent by its Secant, lowered: The result is the Sine of the arc (vi). If we want the Cotangent of an arc, we divide the Cosine of the arc by the Sine of the arc, lowered: The result is the Cotangent, based on a gnomon <of> 60 parts (vii). If we want its Cosecant, we divide the Cotangent by the Cosine of the arc, lowered: The result is the Cosecant (viii). If we want <to use another method>, we add the square of the Cotangent to the square of the gnomon and we take its square root (ix). If

[1] See the commentary on this chapter, where the corresponding modern formulas are presented referring to the Roman numbers in this translation.

we want the arc of the Cotangent, we divide the Cotangent by the Cosecant, lowered: The result is the Cosine of the arc (x).

Chapter 2: On <finding> the Tangent of a <given> arc and the arc of a <given> Tangent from the table.

If we want the Tangent of an arc, we take <the entry> opposite to the arc, from the Tangent tables, as already mentioned in <the chapter on> the Sine. If we want the arc of a Tangent, we take the arc opposite to the Tangent <from the tables>.

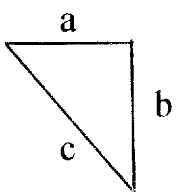
Section: We render the arc of the Tangent in the table up to 45 degrees, because, if it exceeds 45 <degrees>, the difference between <the values in> two <consecutive> lines will be great and the operation will not be correct, except potentially (i.e., theoretically). If we want to multiply any number by the Tangent of an arc, <lowered>, the arc being greater than 45 <degrees>, we divide the number by the Tangent of the complement of the arc <,lowered>. If we want to divide a number by the Tangent of an arc <,lowered>, the arc being greater than 45 <degrees>, we multiply the number by the Tangent of the complement of the arc <, lowered>. The number here is either a Sine or the Tangent of an arc less than 45 <degrees>. However, multiplication of the Tangent of an arc by the Tangent of another arc, both greater than 45 <degrees>, or division of the Tangent of an arc greater than 45 <degrees> by a number, <can> not <be carried out by this method>. In this case, <the operation> will be limited to <using> the Sine and what derives from it, without using the Tangent.

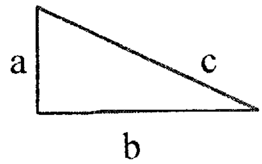
Chapter 3: On converting Tangents to different gnomons.

The ratio of the <number of the> parts <into which> a gnomon <is divided> to the parts of <another> gnomon is like the ratio of a Tangent to <another> Tangent. These are four proportional numbers. We take the <parts of the> gnomon of the known Tangent as the first <number> *A*; <the parts of> the gnomon of the unknown Tangent as the second <number> *B*; the known Tangent as the third <number> *C*, and the unknown Tangent as the fourth <number> *D*. We multiply the second <number> by the third, and divide it by the first <number>: the fourth <number> results. In case of digits and feet, if we multiply the digits by 35 minutes, they turn into feet, based on <taking> the gnomon <divided into> 7 parts (i.e., feet). If we divide the feet by 35 minutes, they turn into digits, based on <taking> the gnomon <divided into> 12 parts (i.e., digits).

Commentary

I.3.1 In modern terminology, the Tangent (Arabic *ẓill*, lit. "shadow") of an arc h is R tg h. It was also called the First or Reversed (Arabic *ma'kūs*) Shadow. The Cotangent of an arc h is R cotg h. It was also called the Second or Horizontal (Arabic *mustawī*) Shadow. For the Tangents, Kūshyār takes $R = 60$, and for the Cotangents, he takes $R = 12$ or $R = 7$. For R/cos h and R/sin h, Kūshyār uses the terms *quṭr al-ẓill al-awwal* ("The Hypotenuse of the First Shadow") and *quṭr al-ẓill al-thānī* ("The Hypotenuse of the Second Shadow"). I have used the modern terms Secant and Cosecant for them.





a = gnomon
b = Tangent (Tg)
(First Shadow, Reversed Shadow)
c = Secant (Sec)
(Hypotenuse of the First Shadow)

a = gnomon
b = Cotangent (Cotg)
(Second Shadow, Horizontal Shadow)
c = Cosecant (Cosec)
(Hypotenuse of the Second Shadow)

Kūshyār uses the term *miqyās* for gnomon. According to al-Bīrūnī [1976, I, 64], the term *miqyās* (lit. "scale, measure") is used for gnomon (*shakhṣ*), especially in calculations.

In this chapter, Kūshyār describes the relations corresponding to the following modern formulas:

(i) $\text{Tg}\, h = \text{Cotg}\,(90^\circ - h)$
(ii) $a\, \text{Tg}\, h = a / \text{Tg}\,(90^\circ - h)$
(iii) $\text{Tg}\, h = \text{Sin}\, h / (\text{Cos}\, h / R)$
(iv) $\text{Sec}\, h = \text{Tg}\, h / (\text{Sin}\, h / R)$
(v) $\text{Sec}\, h = (\text{Tg}^2\, h + R^2)^{\frac{1}{2}}$
(vi) $\text{Sin}\, h = \text{Tg}\, h / (\text{Sec}\, h / R)$
(vii) $\text{Cotg}\, h = \text{Cos}\, h / (\text{Sin}\, h / R)$
(viii) $\text{Cosec}\, h = \text{Cotg}\, h / (\text{Cos}\, h / R)$
(ix) $\text{Cosec}\, h = (\text{Cotg}^2\, h + R^2)^{\frac{1}{2}}$
(x) $\text{Cos}\, h = \text{Cotg}\, h / (\text{Cosec}\, h / R)$

Kūshyār mentions all these rules for $R = 60$. In the mss. C and Y there are additional notes for calculation of the Cotangents when R is equal to

12 digits or 7 feet: "If we want the Cotangent of an arc based on <dividing> the gnomon into 12 digits or 7 feet, we multiply the Cosine of the arc by the <number of the> parts of the gnomon and divide it by the Sine of the arc: the Cotangent will be obtained. If we want its Cosecant, we multiply the Cotangent by the <number of the> parts of the gnomon and divide it by the Cosine of the arc; or we add the square of the <number of the parts of the> gnomon to the square of the shadow, and we take its square root. If we want its arc, we multiply <the number of> the parts of the gnomon by the Cotangent and divide it by the Cosecant; the result will be the Cosine of the arc." These rules correspond to the rules for $R = 60$.

In his treatise on *Shadows*, al-Bīrūnī frequently refers to the contents of this chapter of Kūshyār's *Jāmiʿ Zīj*. For example, he says [1976, 93]: "What Kūshyār proposes for dividing the Cosine of the altitude by the Sine of the altitude, lowered, is exactly what he (al-Nayrīzī) does .... And Abu al-Wafā' proceeded like him, except that he did not lower it, for he had assumed the gnomon to be one".

I.3.2 Kūshyār provides the values of 60 tg h in table II.10 for each degree of h from 1 to 45, with the differences between any two consecutive entries. In table II.11, entitled "Cotangents or Horizontal Shadows for knowing the mid-day Shadows", he gives the values of 12 cotg h and 7cotg h for each degree of h from 1 to 90 degrees.

I don't know why in the "Section" appended to this chapter, Kūshyār requires that "the number here is a Sine or the Tangent of an arc less than 45 degrees".

Al-Bīrūnī [1976,104] says that Kūshyār gave the values of the Tangent for the arcs up to one eighth of a revolution (i.e., 360°), because the tabular differences of the Tangent values for the arguments beyond 45 degrees are so great that the Tangent calculated (by interpolation) can hardly be correct.

I.3.3 Al-Bīrūnī says [1976, 82] that Kūshyār, in his *Jāmiʿ Zīj*, converts a sexagesimal Tangent into other units, through multiplication by the number of the parts into which the gnomon is divided, and division by 60. This is what we read at the beginning of this chapter in a general form. Kūshyār's rules for converting digits and feet into each other are valid because 7/12=35/60=0; 35.

Section 4: On <finding> the true longitudes of the planets, and their situations, <in> 12 chapters

Chapter 1: On mentioning the epoch values and preliminaries for <finding> the mean longitudes of the planets.

We scrutinized the ancient <astronomical> observations as well as the new ones made in al-Ma'mūn's time and later, we examined and checked them through the conjunctions and the meridional altitudes <of the celestial bodies>, and studied each of them exhaustively, after having abandoned our passion, and avoided <personal> inclination to one side, and having abandoned racial and national partisanship, for <many> years. We found that the observations <made> by Muḥammad b. Jābir al-Ḥarrānī, better known as al-Battānī, were the most correct of all, with the least defect and inconsistency, and the closest to our time. <We also found that> its author <had developed the> most precise <point of> view, and <had> examined most thoroughly the observations which he had made. In many things that can be found by observation, he relied on Ptolemy's observations. This is <an evidence that> he (i.e., al-Battānī) <is> most inclined to veracity and fondest of truth. For these reasons, his observations are the most reliable ones, although observations are not (i.e., never) free from inconsistencies. He made observations in <various> localities in Syria, relying however on his observations in Raqqa. He composed a *zīj* in which he provided the true longitudes of the planets for the Syrian and Arabian eras. Using these two eras together with the Persian era is difficult, because they have leap <years> and fractions <for the number of days of the year>, and <there is> difference in <the number of> the days of the months. We have converted the epoch values of the mean longitudes to the Persian era. <By doing this,> we made easier the operation of finding the true longitudes, and we corrected the defect which we found in the composition and presentation of some equations <in the tables>. The description <of our procedure> will be provided in the Book on proofs. If any difference is found between the true longitude of a planet based on this *zīj* and based on the *zīj* of al-Battānī, it is due to the improvement in its equation. This mostly <occurs> in <the case of> Mars, and the difference reaches <a few> degrees. As to the other planets, it is negligible, and in <the case of> the sun and the moon, it does not occur. We subtracted from the epoch values of the mean longitudes <found> for Raqqa the motion <of the respective celestial body> in 1 hour and 7 minutes, so that they (i.e., the tabular mean longitudes) can be based on a <geographical> longitude 90 <degrees> from the Canary Islands, and are <thereby> clearer in their layout, and easier in accessibility. The mean longitudes <corresponding to> the <geographical> longitude difference are always additive <if the

other locality is> between the West (i.e., the Atlantic coast) and the longitude of 90 <degrees>. The Canary Islands are situated off the Atlantic coast. Ptolemy says that they were inhabited in ancient times. <The distance> between them and the coast is 10 degrees of the revolution of the sphere, i.e., two thirds of an hour.

Chapter 2: On deriving the mean longitudes from the tables.

If we want this, we take the Yazdigird (i.e., Persian) years including the year, month and day <for> which we want <to find the mean longitude>. Then we enter with <the number of> the years in the table of the multiple years, and take the mean longitude which corresponds to the closest number which is less than it and write it down on the board. We then take <the mean longitude> corresponding to the remainder of the years in the table of the single years. Then we take what corresponds to the month and the day. We add all that: <the result> will be the mean longitude for the noon of this day at the <geographical> longitude of 90 <degrees>. Then we adjust it by the equation of longitude, as we shall mention later. If there are entire hours left after noon, we take the corresponding <value> in the table of hours. If there are fractions with the hours, and the fractions are in minutes, we take the corresponding <value> in the table of hours, lowered once. If the fractions are in seconds, we take the corresponding <value>, lowered twice, and so on.

Chapter 3: On converting the mean longitudes from <localities having> one <geographical> longitude to another.

We have already said that these mean longitudes have been calculated for the localities whose <geographical> longitude is 90 <degrees east> of the Canary Islands in the Atlantic. We should convert it (i.e., the mean longitude) to the <geographical> longitude of the locality where we are, so that the true longitude <of the planet> comes out correctly. If we want <to do> this, we take the difference between the <geographical> longitude of our locality and the <geographical> longitude of 90 <degrees>. We take one hour for each 15 degrees of difference, and 4 minutes of an hour for each degree. The result is the hour difference between the two <geographical> longitudes. If the <geographical> longitude of our locality is less than 90 <degrees>, we add the hour difference between the two <geographical> longitudes to the given time. If the <geographical> longitude of our locality is greater than 90 <degrees>, we subtract the hour difference between the two <geographical> longitudes from the given time. The sum or the remainder is the time adjusted for the <geographical> longitude difference. On this basis, we derive the mean longitudes for our locality. We find the <geographical> longitudes of the localities, by taking them from the table compiled for them, or by deriving them by calculation, as we shall mention in Chapter 19 of Section 6.

Chapter 4: On the positions of the apogees and the nodes, and <on> their motions.

The positions of the apogees for the beginning of the Yazdigird era are: the sun: Gemini 18; 31°; Saturn: Sagittarius 0;45°; Jupiter: Virgo 10;45°; Mars: Leo 3;15°; Venus: Gemini 18;31°; Mercury: Libra 17;44°. Their motion is one entire cycle in 24,000 solar years, that is 54 seconds in each year. If we want to adjust <the apogee values>, we take the Yazdigird years elapsed after the (i.e., any) known adjusted apogee, and subtract from it one tenth of it. What remains is the motion of the apogee in minutes. If we wish so, we <may> obtain their motions from the tables composed for them and we add them to their previously adjusted positions.

As for nodes, we do not need anything about them in this book, save their positions at the beginning of the Yazdigird era <i.e.,> Saturn: Cancer 10:45°; Jupiter: Cancer 0;45°; Mars: Taurus 3;15°; Venus: Pisces 18;31°; Mercury: Capricorn 17;44°. Their motions follow those of the apogees. For deriving their positions: we subtract 50 degrees from the apogee of Saturn, then we subtract 90 degrees from the remainder; we add 20 degrees to the apogee of Jupiter and subtract 90 degrees from the sum; we subtract 90 degrees from the apogees of Mars and Venus and 90 degrees from the <point> opposite to the apogee of Mercury. The result is the position of the nodes for this <given> time.

Chapter 5: On the equation of time.

There is a special adjustment for the time for which the true longitudes of the two luminaries are found, which is known as the "equation of time". If we want this, we subtract 10 signs and 16 degrees from the mean longitude of the sun at the <given> time. The remainder is <called> "the result of the mean longitude". We <also> subtract 10 signs, 22 degrees and 4 minutes from the right ascension of the true longitude of the sun. The remainder is <called> "the result of the right ascension". Then we take the excess of the result of the mean longitude over the result of the right ascension, and multiply it by 4. Then we take the degrees as minutes and the minutes as seconds. <The result> is the equation of time in minutes of an hour. We subtract it from the time adjusted for the difference between the two <geographical> longitudes. <The result> becomes the time adjusted for the equation of time. Another method: We add 6 degrees and 4 minutes to the mean longitude of the sun, and take the difference between it and the right ascension of its true longitude; we multiply <the result> by 4 and make it "lowered", i.e., we lower the place of degrees into minutes, minutes into seconds, and seconds into thirds. The result will be the equation of time in minutes of an hour and parts of the minutes of an hour. We always subtract it from the time adjusted for the difference between the two <geographical> longitudes. The result will be the time adjusted for the equation of time. <Based> on this calculation, we have compiled a table in which we have written the mean longitudes of the sun and, opposite to them, the equation of time in minutes and seconds of an hour, so that we do not need to find the true longitude of the sun twice. <The table is> based on <taking> the apogee in Gemini 24°. The motion of the apogee does not affect this equation sensibly, except in long time intervals. There is no need at all to apply this equation for finding the true longitudes of the five planets.

Chapter 6: On the true longitude of the sun.

We write the mean longitude of the sun in two positions and we subtract the adjusted apogee for the <given> time from one of the <numbers put in the two> positions. The result is the adjusted mean anomaly. We take the equation opposite to it <from the table>, and we interpolate it. Then we always add it (i.e., the equation) to the mean longitude, and the result is the true longitude <of the sun>.

Chapter 7: On the true longitude of the moon and its node<s>.

We write the mean longitude, the mean anomaly, and the double elongation. Then we take the first equation corresponding to the double elongation, and always add it to the mean anomaly. The result is the true anomaly (i.e., the adjusted position of the moon on the epicycle). Then we take the second equation corresponding to it, and keep it in mind. Then we take the difference <in epicyclic equation> at the lesser distance <of the epicycle center> corresponding to the double elongation and the sixtieths corresponding to the true anomaly. We multiply them one by another, and divide the product by 60. The result is the adjusted difference <in epicyclic equation>. If the true anomaly appears in the upper <part> of the table for the sixtieths, we add the adjusted difference to the second equation; and if the true anomaly appears in the lower <part> of the table for the sixtieths, we subtract the adjusted difference from the second equation. Then we always add this sum or remainder of the <second> equation to the mean longitude. The result is the true longitude <of the moon>. For all planets <and the moon>, the true anomaly is the same as the adjusted mean anomaly.

<For finding the true longitude of> **the** <ascending> **node**, we subtract its "mean longitude" from a <complete> rotation (i.e., 360°), and what remains is the true longitude of the ascending node. The descending node is always in opposition to the position of the ascending node.

Chapter 8: On the true longitude of the five planets.

We write the mean longitude and the mean anomaly, and <then> we subtract from the mean longitude the adjusted apogee for the <given> time. The remainder is the mean centrum. We take the first equation corresponding to it, and we always add it to the mean centrum and subtract it from the mean anomaly. The result of adding it to the mean centrum is the adjusted centrum. The remainder from the mean anomaly is the true anomaly (i.e., the adjusted position of the planet on the epicycle), and we take the second equation corresponding to it, and bear it in mind. Then we take the difference <in epicyclic equation> at greater or lesser distance <of the epicycle center from the earth> -whichever we <may> find- corresponding to the adjusted centrum, and the sixtieths corresponding to the true anomaly. We multiply them one by another, and divide <the product> by 60. The result is the adjusted difference. If the true anomaly appears in the upper <part> of the table for the sixtieths, we add the adjusted difference to the second equation. If the true anomaly appears in the lower <part> of the table for the sixtieths, we subtract the adjusted difference from the second equation. We always add this sum or remainder of the <second> equation to the adjusted centrum. We add to this sum the apogee; the result is the <desired> true longitude <of the planet>.

Section. This is not the true adjusted centrum, because it is according to the displacement of the equations in this *zīj*. If we want <to obtain> the true <adjusted centrum> in order to use it for the latitudes and the determination of the stations for the retrogradations and direct motions, we add to it 7 degrees for Saturn, 12 degrees for Jupiter, 47 degrees for Mars, 48 degrees for Venus, and 26 degrees for Mercury.

Chapter 9: On the latitude of the moon.

We subtract the true longitude of the node from the true longitude of the moon, or we add the "mean longitude" of the node to the true longitude of the moon. The remainder or the sum is the 'argument of the latitude'. Then we take the latitude corresponding to it. If the argument is less than 3 <zodiacal> signs, then the latitude is northerly, ascending, and

increasing. If it is greater than 3 and less than 6 <zodiacal signs>, then the latitude is northerly, decreasing, and descending. If it is greater than 6 and less than 9 <zodiacal signs>, then the latitude is southerly, descending, and increasing. If it is greater than 9 <zodiacal signs> up to an entire rotation, then the latitude is southerly, ascending, and decreasing.
Its calculation: We subtract the true longitude of the node from the true longitude of the moon. The remainder is the argument of the latitude. We multiply its Sine by the Shadow of the maximum latitude, lowered. The result is the Shadow of the latitude. The maximum latitude is 5 degrees.

Another method: We multiply the Sine of argument of the latitude by the Sine of the maximum latitude, lowered. The result is the Sine of the latitude of the argument. Then we multiply the Cosine of the argument by the Sine of the maximum latitude, lowered. The result is the Sine of the latitude of the complement of the argument. We find its arc, and obtain its Cosine, and divide by it the Sine of the latitude of the argument, lowered. The result is the Sine of the latitude. All people in the art <of astronomy> shorten this calculation, multiplying the Sine of the argument of the latitude by the Sine of the maximum latitude, lowered. They believe that the result is the Sine of the latitude. However, this is, not the Sine of the latitude of the moon, but the Sine of an arc close to the latitude <of the moon>.

Chapter 10: On the latitudes of the five planets.

The superior planets: We take the true adjusted centrum mentioned at the end of Chapter 8 of this section. For Saturn, we add 50 degrees to it; for Jupiter, we subtract 20 degrees from it; For Mars, we leave it as it is. Then we enter with it in the two rows of numbers (i.e., arguments), and take the corresponding <number of the> 'proportional minutes for the latitude', and write it down. If the centrum appears in the upper half of the two rows of numbers, we take the northern latitude of the planet corresponding to the true anomaly. If the centrum appears in the lower half, we take the southern latitude of the planet corresponding to the true anomaly. We multiply the resulting quantity by the 'proportional minutes for the latitude'. The result is the latitude of the planet in the direction which was found.

Venus and Mercury: We take the inclination and slant corresponding to the true anomaly, and we write down each of them separately. If for Mercury, specifically, the adjusted centrum appears in the upper half of the two rows of numbers, we subtract from its slant one-tenth of it. If it appears in the lower half, we add to its slant one-tenth of it. The result is the slant to be used, different from the initial one, and we keep it in mind. Then we add to the true adjusted centrum of Venus 3 <zodiacal> signs, and to that of Mercury 9 <zodiacal> signs. We use this for taking the proportional minutes for the latitude. We multiply it (i.e., the number of the proportional minutes for the latitude) by the inclination. The result is the first latitude <component>, which is the inclination of the epicycle. If the augmented centrum and the true anomaly both appear in the same half of the two rows of numbers, the first latitude <component> is southerly. If their positions are <in> different <halves>, the first latitude <component> is northerly. Then we take the true adjusted centrum for Venus as it is, and for Mercury <the true adjusted centrum> plus 6 <zodiacal> signs. We take the <number of the> proportional minutes for the latitude corresponding to it, and write it in two positions. We multiply <the quantity in> one <of these> positions by the slant. The result is the second latitude <component>, which is <called> the 'deflected latitude'. If this centrum from which we took the proportional minutes falls in the upper half and the true anomaly is less than 6 <zodiacal> signs, then the second latitude <component> is northerly, <but> if the true anomaly is greater than that, then it (i.e., the second latitude component) is southerly. If the centrum falls in the lower half, and the true anomaly is less than 6 <zodiacal> signs, then the second latitude <component> is southerly, <but> if the true anomaly is greater than that, the second latitude <component> is northerly. Then we take the proportional minutes written in the other position. We multiply them by 10 minutes for Venus, and by 45 minutes for Mercury. The result is the third latitude <component>, which is the inclination of the eccentric orb. For Venus, it is always northerly, and for Mercury, always southerly. Among these three latitude components, we add those in the same direction. When they have different <signs>, we subtract the smaller from the greater one, and we find the direction of the result (i.e., the direction of the greater one). That is the latitude of the planet in the direction which resulted.

Ascension and descension: We calculate the latitude for 10 days later. If it is northerly at the first <given date> and increases in the second <date>, it is ascending; if it decreases at the second <date>, it is descending. If it is southerly at the first <given date> and increases at the second <date>, it is descending; if it decreases at the second <date>, it is ascending. If it is northerly at the first <given date>, and southerly at the second <date>, it is 'descending in the north'. If it is southerly at the first <given date>, and northerly at the second <date>, it is 'ascending in the south'. The maximum latitude is <as follows>: for Saturn 3;2° northern, 3;5° southern; for Jupiter 2;5° northern, 2;8° southern; for Mars 4;21° northern, 7;50° southern; for Venus 6;22° in both directions; for Mercury 4;5° in both directions.

Chapter 11: On the retrogradation of the planets, their direct motion, and first and last visibility.

We take the first equation corresponding to the centrum, and keep it in mind. We add the mean <motion of the planet in> longitude for one day to the centrum, and take its <first> equation again. We subtract the smaller equation from the greater one. If the equation is additive, we add the difference to the mean <motion in> longitude for the day; if it is subtractive, we subtract <the difference from the mean motion in longitude for the day>. The remainder or the sum is the adjusted mean <motion in> longitude for the day. Then we take the second equation corresponding to the true anomaly, and bear it in mind. We add the mean <motion in> anomaly relating to one day to the true anomaly and take its <second> equation again. We subtract the smaller equation from the greater one. The remainder is the difference of the equation of the day. If the difference is less than the adjusted mean <motion in> longitude for the day, the planet is <in> direct <motion>. If it is greater, then the planet is retrograde. If it is equal to it (i.e. the adjusted mean motion in longitude for the day), the planet is stationary <before> retrogradation or direct motion.

Another method: We enter with the adjusted centrum in the table for the first station <column of the> table, and take the corresponding entry. We subtract the first station from a complete rotation (i.e., 360°). The remainder is the second station. Then we look at the true anomaly: If it is less than the first station and greater than the second station, then the planet is <in> direct <motion>. If it is greater than the first station and less than the second station, then the planet is retrograde. If it is equal to the first station, then it is stationary <before> retrogradation. If it is equal to the second station, it is stationary <before> direct motion. If <the difference> between them is a few degrees, we divide it by the daily <motion in the> anomaly of the planet. The result is the period of time until the planet retrogrades or since its retrogradation, or until it moves directly, or since it has moved directly. The daily <motions in> anomaly of the planets <are as follows>: For Saturn 0; 57°; for Jupiter 0;54°; for Mars 0; 28°; for Venus 0; 37°; for Mercury 3; 6°. We have written the retrogradation, direct motion, first and last visibility in their approximate positions in the table for the second equation. We take any of these situations (i.e., being in direct motion, stationary, or retrograde) corresponding to the true anomaly <from the table for the second equation>. If <the difference> between the true anomaly and one of these situations is a few degrees, we divide it by the daily <motion in> anomaly of the planet as we have mentioned before. The result is the period of time until it retrogrades, or since its retrogradation, or until it moves directly, or since it has moved directly, or until its apparition, or since its apparition, or until its last visibility, or since its last visibility. If the planet is seen rising before sunrise, it is 'eastward', and if it is seen setting after sunset, it is 'westward'. The limit of orientality (being eastward) and occidentality (being westward) for the superior planets is 60 degrees, for Venus 47 degrees, and for Mercury 26 degrees. This is their (i.e., the inferior planets') maximum elongation. The combustion of the superior planets <occurs> approximately in the middle of the days of their direct motion. Their opposition to the sun <occurs> approximately in the middle of the days of their retrogradation. The combustion of Venus and Mercury is approximately in the middle of the days of direct motion and the middle of the days of retrogradation.

Chapter 12: On the ascension and descension of the planets in their spheres.

The ascension and descension are meant to be <ascension and descension> in the <relevant> zones in the spheres of the apogee (i.e., the eccentric orb) and of the epicycle. On the sphere of apogee <is> the center of the epicycle, and on the epicycle <is> the body of the planet. The zones in the sphere of the apogee corresponding to <the positions of> the center are written in the table of the first equation. The zones in the epicycle corresponding to the true anomaly <are written down> in the table of the second equation. If the center and the true anomaly are found between the maximum and mean distances in the order of the <zodiacal> signs, then the center of the epicycle or the body of the planet on the epicycle is descending from maximum to mean distance. <If they are> between the mean and minimum distances, <they are> descending from mean to minimum distance. <If they are> between the minimum and the second mean distance, <they are> ascending from minimum to mean distance. <If they are> between the mean and the maximum distance, <they are> ascending from the mean to the maximum distance. As to the ascension of the planet and its descension, i.e., the ascension of the planet itself in the sphere of the apogee and its descension in it, they are clear if the position of the apogee is known.

Commentary

The term *aḥwāl* ("situations") in the title of this section refers to the apparent motion of the planets that may be direct, stationary, or retrograde (see Chapter 11).

I.4.1 According to Kūshyār, the geographical longitudes of Jurjān and Raqqa with respect to the Canary Islands are 90;0° and 73;15°, respectively (table II.54). By dividing their longitude difference, 16;45° by 15 degrees/hour, one obtains 1 hour and 7 minutes, in accordance with the time interval which Kūshyār uses to convert the positions of the celestial bodies from the local time at Raqqa into that at Jurjān.

Al-Battānī mentioned in his *Zīj al-Ṣābī* [1899-1907, III, 7] that he found Ptolemy's *Almagest* most reliable and followed it in his work.

I.4.2 In table II.13, the mean longitude of the sun is given to the nearest second for the years 1, 21, 41, ..., 581 of the Yazdigird era, and its motion is given for 1, 2, 3,..., 20 Persian years, 40, 60, 80, 100, 200, 300, 400, 500 Persian years, the 12 Persian months from Farwardīn-māh to Esfandārmadh-māh, 1, 2, 3,..., 30 days, and for 1, 2, 3,..., 60 hours. In table II.17 the same functions are given for the moon with the same precision. The same functions, to the nearest minutes are also given for the five planets: Saturn in II.22; Jupiter in II.25; Mars in II.28; Venus in II.31; and Mercury in II.34. For minutes and seconds of an hour, the relevant entries for the hours are lowered once or twice.

I.4.3 The planetary mean longitudes in this *zīj* are given for the geographical longitude 90°. In his table II.54 for the geographical coordinates of the localities, only Jurjān has this geographical longitude, and in I.1.2, he mentions Jurjān as a locality in which he lived. So, the tables must have been composed for Jurjān. Kūshyār says that for the localities west or east of the geographical longitude 90°, we add or subtract 1 hour for each 15 degrees, and 4 minutes for each degree of longitude difference. Thus the tables for the mean longitudes of the celestial bodies can also be used in other localities. For each celestial body, in tables II.13, II.17, II.18, II.19, II.21, II.22, II.23, II.25, II.26, II.28, II.29, II.31, II.32, II.34 and II.35, he provides in a final column the positive or negative corrections for the geographical longitudes 71, 72, 73,..., 100°.

I.4.4 In the margin of table II.12 (preliminaries for mean longitudes), Kūshyār gives these positions of the apogees for the beginning of the Yazdigird era. There he says that they are taken from the *zīj* of al-Battānī.

Kūshyār gives their motion, which is the same as the precession of the equinoxes, as one complete rotation per 24,000 years, or 54 seconds per year. The modern values are 25,770 years and 50.29 seconds respectively. Hipparchus, who described this motion first, evaluated it as "not less than 1° per 100 years", and Ptolemy as 1° per 100 years [Ptolemy 1984, 328]. The Mumtaḥan astronomers and al-Battānī had already improved the Ptolemaic value to 1° per 66 years [van Dalen 2006,]. Kūshyār computes this motion by subtracting one-tenth of the elapsed years in the Yazdigird era, and he takes the result as the motion of the apogee in minutes of arc. This computation is valid, because 0;0,54° = 0;1°.(1- 0.1). The motions may also be taken directly from table II.14.

I.4.5 Here Kūshyār provides two equivalent methods for finding the equation of time. In modern notation they are as follows:

$$E_t(\bar{\lambda}_d) = 0;4[(\bar{\lambda}_d - 10^s16°) - (\alpha(\lambda) - 10^s22;4°)]$$, and

$$E_t(\bar{\lambda}_d) = 0;4[(\bar{\lambda}_d + 6;4° - \alpha(\lambda)]$$.

Here $E_t(\bar{\lambda}_d)$ is the equation of time, $\bar{\lambda}_d$ the displaced mean longitude (see below) of the sun, λ the true longitude of the sun, and $\alpha(\lambda)$ the right ascension of the sun, for the given time. In both methods, Kūshyār applies "lowering" or "division by 60", however, he describes it in two different ways. In table II.15, Kūshyār gives the values of equation of time for each six degrees of mean longitude of the sun, with a column of increments for each degree that may be used for interpolations. This is the original form of the table, which is found in mss. F and Y; in mss. B and L, the entries are given for each degree of the argument. This chapter (in a rather abridged form) and the table are appended to the manuscript of Yaḥyā b. Abī Manṣūr's *Al-zīj al-Ma'mūnī al-mumtaḥan* [1986, 121-22], where Kūshyār's table is similar to the version found in B and L. Of course, since Kūshyār lived after Yaḥyā, this chapter had been added by someone who prepared the manuscript based on Yaḥyā's work, some time after Kūshyār composed his *Jāmiʿ Zīj.*

Prof. E. S. Kennedy recomputed Kūshyār's table for equation of time by computer [1988, 4]. In Y and L, there is an extra phrase to the effect that the 16° implied in the first method was originally 18°, and that Kūshyār subtracted 2° from it. Kūshyār used "displaced" mean solar longitudes, which were 2° less than mean longitudes, to avoid negative values for the solar equation [Van Dalen 1993, 138-139; 1996, 236-238]. As Dr. Benno van Dalen remarked [1993, 140], the 10^s 16° in the formula is close to the value of the displaced mean solar position for which the equation of time assumes its minimum value, and the 10^s 22°4' is approximately equal to the right ascension of the former solar position added to the displaced solar equation 3;38,21° to result the true solar position in the given time. See also the commentary on IV.4.1.

As Kūshyār says at the end of this chapter, since the argument of the table is mean longitude, there is no need to find the true longitude twice. If we have a table for the equation of time as a function of the true longitude, we in principle need to carry out an iteration, since the true longitude for the time adjusted for the equation of time is a little different from the true longitude for the original time. Then we would have to carry out the whole calculation again for the newly found time. See also the commentary to IV.4.1.

I.4.6 In table II.16 in four pages, the values of the solar equation are precise to the nearest second for each degree of anomaly. This table also includes columns of tabular differences to be used in interpolation.

I.4.7 In the relevant theory in the *Almagest*, the first equation is for converting mean anomaly into true anomaly. The second equation is the epicyclic equation at apogee. The difference in epicyclic equation accounts for the increment of maximum epicyclic equation (for any elongation). The sixtieths determine the portion of this increment to be applied for arbitrary true anomaly. Kūshyār's method of finding the true longitude of the moon is ultimately based on *Almagest* V.9 [1984, 237-39]. However, in the *Almagest* the increment in epicyclic equation is found as a function of the true anomaly and the sixtieths are found as a function of the double elongation, whereas we find the inverse in Kūshyār's *zīj*. This is due to an interesting innovation of Kūshyār who applies a different interpolation process for adjustment of the second equation. As demonstrated by Glen Van Brummelen [1998] for the planets, Kūshyār's attempt was conscious. His method is simpler and less accurate, but shows that Kūshyār "was no mere copyist".

Since the revolution of the nodes is opposite to the order of the zodiacal signs, "the mean longitudes" (or merely "the longitudes", because the motion is uniform) given in table II.21 are subtracted from an entire cycle. Among different methods of tabulating the longitude of the nodes (with positive or negative motions), Kūshyār chose to tabulate the supplement of the longitude. So, "the mean [longitude]" here actually means "the supplement of the longitude".

I.4.8 This method is also found in *Almagest* XI.1 [1984, 554]. But in Kūshyār's *zīj*, the first equation is always added to the mean centrum and subtracted from the mean anomaly, whereas in the *Almagest*, this is only for arguments from 180 to 360 degrees. For arguments less than 180 degrees, the equation is subtracted from the mean centrum and added to the mean anomaly. A similar difference is encountered in the calculation of the second equation. This is due to the displacement method used by Kūshyār in order to avoid negative values for the equation [Van Brummelen 1998, 268-270]. The first equation is the equation of centrum. The second equation is the equation of anomaly for the mean distance [Pedersen 1974, 279-294]. Here again, contrary to Ptolemy's procedure, Kūshyār uses the adjusted centrum and the true anomaly as arguments for obtaining the difference relating to greater or lesser distance (depending on the planet's position being between the mean distance and the apogee or the perigee), and the sixtieths, respectively. The reason is the same as in the case of the moon, i.e., application of a different interpolation process by Kūshyār [Van Brummelen 1998].

The tabular values of the mean centrum in the table for the first equation are not really the mean centrum, because Kūshyār shifts the mean centrum in order to compensate the above-mentioned displacement of the first and the second equation. For example, in the case of Mars, since he adds 12° to all tabular values of the first equation and 47° to all values of the second equation, he subtracts 59° from each tabular value of the mean centrum (to which the two equations are to be added). By adding the first equation, the result is

(mean centrum - 59°) + (first equation + 12°) =

(mean centrum + first equation) - (59° - 12°) = adjusted centrum - 47°

This is why Kūshyār adds certain amounts to the resulting centrum of each planet in order to obtain its real value. Of course, in the process of finding the true longitude of the planet, e.g., Mars, the remaining 47° is implicitly added later, in the calculation of the second equation [Van Brummelen 1998, 270].

I.4.9 The two methods for finding the argument of latitude are equivalent (see I.4.7). Note that the "mean longitude" of a node means a complete rotation minus its true longitude. Kūshyār provides a compact table for it in II.37. There is a similar table for the latitude of the moon in al-Battānī's *zīj* [1899-1907, II, 78-83, column 7]. Kūshyār's first method for calculating the latitude of the moon is:

$$tg\beta = \sin a_{\beta} tg\beta_m$$

where β is the latitude of the moon, α_β is the argument of latitude, and β_m is the maximum latitude of the moon. He also provides another method, i.e.,

$$\sin\beta' = \sin a_\beta \sin\beta_m \ , \ \sin\beta'' = \cos a_\beta \sin\beta_m \ , \ \sin\beta = \sin\beta' / \cos\beta''$$

where the auxiliary parameters β' and β'' are called 'the Sine of the latitude of the argument' and 'the Sine of the latitude of the complement of the argument', respectively. The second method can be derived from the first in the following way (not mentioned by Kūshyār):

$$\sin^2\beta=(1+\mathrm{cotg}^2\beta)^{-1}=(1+1/\sin^2\alpha_\beta tg^2\beta_m)^{-1}=\sin^2\alpha_\beta tg^2\beta_m/(1+\sin^2\alpha_\beta tg^2\beta_m)=$$
$$\sin^2\alpha_\beta\sin^2\beta_m/(\cos^2\beta_m+\sin^2\alpha_\beta\sin^2\beta_m)=\sin^2\alpha_\beta\sin^2\beta_m/[1-\sin^2\beta_m(1-\sin^2\alpha_\beta)]$$
$$=\sin^2\alpha_\beta\sin^2\beta_m/(1-\sin^2\beta_m\cos^2\alpha_\beta)=\sin^2\beta'/\cos^2\beta''$$

Kūshyār mentions the fact that some astronomers take β' as the latitude of the moon, whereas it is not equal to the latitude of the moon, but an approximation to it (cf. IV.4.8). Maybe Kūshyār is criticizing al-Battānī who did this [1899, III, 113]. Al-Battānī provides a method equivalent to the modern formula $\sin\beta=\sin\alpha_\beta\sin\beta_m$, which is the correct approach and produces Kūshyār's β'. It seems that Kūshyār misunderstood the geometrical concept of "argument of latitude" as meant by al-Battānī (see the commentary to IV.4.8). In Kūshyār's method the final result is divided by $\cos\beta''$. So he overestimates $\sin\beta$ by a factor whose maximum value is 1/cos 5°≈ 1.00382. Thus the maximal error in the calculation of $\mathrm{Sin}\,\beta$ is 0.382 percent and the maximum absolute error in Kūshyār's computation for β is about 1 minute and 9 seconds. Table II.37 gives the latitude of the moon, in degrees and minutes, for each degree of the argument of latitude.

I.4.10 The methods for finding the latitudes of the superior and inferior planets are taken from *Almagest* XIII.6 [1984, 635-36]. As remarked by Toomer in a footnote to his translation of the *Almagest* [1984, 635], the amounts to be applied to the true adjusted centrum of the superior planets represent the (rounded) distance between the apogee and the northpoint of the inclined orb. The tabular entries of *daqā'iq ḥiṣaṣ al-'arż* are interpolation coefficients used for calculating the latitudes of all planets. They should not be confused with the quantity *ḥiṣṣat al-'arż* (argument of latitude) used for the moon.

In the case of Mercury, the addition and subtraction of one tenth of the tabular value of the slant, relates to the fact that the maximum slant is taken as 2;30° in the table. However, Ptolemy found that the maximum slant actually differs from 2;30° by 13' in the negative direction at the apogee and by 16' in the positive direction at the perigee. He takes a middle value 15' or 1/4° for both these differences. Since 15' is one-tenth of 2;30°, he adds or subtracts one-tenth of each tabular value of the slant of Mercury [Ptolemy 1984, 630].

An explanation of the procedure for finding the third latitude component of the inferior planets is given by O. Neugebauer [1975, I, 224]. See also the commentary to IV.4.9.

I.4.11 The first method is a direct approach in which Kūshyār compares the motion in longitude and the epicyclic equation; so, in fact he computes the difference in true longitude. The second method is found in al-Battānī's *zīj*, chap. 46 [1899-1907, III, 173]. Ptolemy discusses it in detail in *Almagest* XII 7 & 8 [1984, 583-88]. Kūshyār gives the position of the first stations of each planet in degrees and minutes for each six degree of the argument in the same table for its latitude (tables II.38 to II.42). The different situations of each planet (direct motion, stagnation, retrogradation, occultation, and emergence west or east of the sun) depending on the value of the true anomaly are mentioned in the table for its second equation (tables II.24, II.27, II.30, II.33, II.36).

Kūshyār's definition of maximum orientality or occidentality for the superior planets (being 60° distant from the sun) is conventional, because they can have arbitrary distances from the sun. In this case, al-Bīrūnī assumes the maximum orientality (or occidentality) to be 30°. Then he calls the planet in the interval from 30° to 90° from the sun to be 'weakly oriental' (or 'weakly occidental'). Al-Bīrūnī also mentions that the lower limits of orientality (and occidentality) are conventional and the planets can be invisible after passing this limit [1934, 296-296a].

I.4.12 In this chapter, Kūshyār defines the meaning of the ascension and descension of the planets in their spheres, possibly because these may be interpreted in different ways [al-Bīrūnī 1934, 110-111]. Kūshyār's discussion of these terms is related to the variation of the distance of the planet from the earth between its maximum and minimum value.

Section 5: On the operations relating to the ascendants of the day and the night, <in> 22 chapters
Chapter 1: On the first declination.

We multiply the Sine of the degrees <of a point on the ecliptic> whose declination we want <to find> by the total declination, lowered: The result is the Sine of the first declination. Through successive observations, we found the total declination 23; 35°. Based on this calculation, a table for it (i.e., the first declination) has been compiled <in this *zīj*>.

Chapter 2: On the right ascensions of the <zodiacal> signs.

We divide the Cosine of the degrees <of a point on the ecliptic> of which we want <to find> the right ascension, by the Cosine of the <first> declination of the degrees, lowered: The result is the Cosine of the right ascension. We find the corresponding arc <from the Sine table> and subtract it from 90°.

Another method: We divide the Tangent of the degrees <of a point on the ecliptic> by the Tangent of the total declination: The result is the Sine of the right ascension of those degrees.

Another method: If <the values of> the second declination are known <by means of the relevant table>, then we find the arc corresponding to the first declination of those degrees in the table for the second declination. The result is the right ascension for those degrees. A table has been compiled for it.

Chapter 3: On the second declination.

We divide the Sine of the <first> declination of those degrees <of a point on the ecliptic> by the Cosine of the <first> declination of the complement of the degrees, lowered: The result is the Sine of the second declination.

Another method: We multiply the Sine of those degrees by the Tangent of the total declination, lowered. The result is the Tangent of the second declination, and its maximum <value> is <equal to> the maximum <value> of the first declination.

Another method: If <the values> of the right ascension are known <through the relevant table>, then we find the arc corresponding to the degrees in <the table for> the right ascensions: The result is the inverse of the right ascension. We take its first declination: <The result> is the second declination for those degrees. A table has been compiled for it.

Chapter 4: On the distance of the stars from the celestial equator.

If the latitude of the star and the second declination of its degree <on the ecliptic> are in the same direction, we add them; if they are in different <directions>, we subtract the lesser from the greater, and <on this basis> we know the direction of the remainder. Then we multiply its Sine by the Cosine of the total declination and divide it by the Cosine of the second declination obtained for the degree of the star (i.e., for its ecliptical longitude): The result is the Sine of the distance of the star from the celestial equator. Its direction is the <same> direction that we have found. This distance of the star is similar to the first declination of the sun.

Chapter 5: On the latitude of <any> locality.

We obtain the maximum <value> of the altitude of the sun on any day by one of the altitude <measurement> instruments. We know the <first> declination of the degree of the sun (i.e., of its ecliptical longitude). If the <first> declination is northern, we subtract it from the maximum <value> of the altitude. If it is southern, we add it to the maximum <value> of the altitude: The result is the complement of the latitude of the locality. Should <the result> become more than 90°, we subtract it from 180°. The remainder is the complement of the latitude of the locality.

Chapter 6: On the ortive amplitude of the sun and the star<s>.

We divide the Sine of the <first> declination of the degree of the sun (i.e., its ecliptical longitude), or the Sine of the distance of the star from the celestial equator, by the Cosine of the latitude of the locality, lowered: The result is the Sine of the ortive amplitude.

Another method: If half the day arc of the degree <on the ecliptic> or of the star is known, then we multiply the Cosine of the <first> declination of the degree <on the ecliptic> or the Cosine of the distance of the star from the celestial equator by the Sine of half the day arc of the degree <on the ecliptic> or of the star, lowered: The result is the Cosine of the ortive amplitude. We find the corresponding arc <from the Sine table> and we subtract it from 90°. Half the day arc is <discussed> in Chapter 10 of this section.

Chapter 7: On the equation of daylight of the sun and the star<s>.

We divide the Cosine of the ortive amplitude of the sun or the star by the Cosine of the <first> declination of the sun or the Cosine of the distance of the star from the celestial equator, lowered: The result is the Cosine of the equation of daylight.

Another method: We multiply the Sine of the <first> declination of the sun, or the Sine of the distance of the star from the celestial equator, by the Sine of the latitude of the locality, and divide it by the Cosine of the <first> declination or the distance. The result is <called> the 'base'. Then we divide the base by the Cosine of the latitude of the locality, lowered: The result is the Sine of the equation of daylight.

Another method: We multiply the Tangent of the <first> declination of the sun, or the Tangent of the distance of the star from the celestial equator, by the Tangent of the latitude of the locality, lowered: The result is the Sine of the equation of daylight. A table for the Tangent of the <first> declination has been compiled <in this *zīj*>.

Another method: For the degrees of the ecliptic if the equation of daylight for the first of Cancer or of Capricorn, i.e., the maximal equation of daylight is known: We multiply the Sine of the maximal equation of daylight by the Sine of the right ascension of the degree <of the ecliptic>, lowered: The result is the Sine of the equation of daylight of the degree <of the ecliptic>. A table for the equation of daylight for the latitude of 36° has been compiled <in this *zīj*>.

Chapter 8: On the ascensions for a locality (i.e., oblique ascensions).

<For> the northern degrees <of the ecliptic>, i.e., from the first of Aries until the end of Virgo, we subtract the equation of daylight <of the degree> from their right ascensions. <For> the southern degrees <on the ecliptic>, i.e., from the first of Libra until the end of Pisces, we add the equation of daylight <of the degree> to their right ascensions: The result is the oblique ascension of that degree for that locality. A table for the oblique ascensions of <the zodiacal signs for> the latitude of 36° has been compiled <in this *zīj*>.

Chapter 9: On the maximum altitude of the sun and the star<s>.

If the declination of the sun or the distance of the star from the celestial equator is northern, we add it to the complement of the latitude of the locality. If the declination or the distance is southern, we subtract it from the complement of the latitude of the locality. The sum or the remainder is the maximum altitude of the sun or the star. If the sum is over 90°, we subtract it from 180°. The remainder is the maximum altitude in the northern direction.

Chapter 10: On half the day arc of the sun and <any> star.

If the declination of the sun or the distance of the star from the celestial equator is northern, we add its equation of daylight to 90°. If the declination or the distance is southern, we subtract its equation of daylight from 90°. The sum or the remainder is half the day arc of the sun or the star.

Another method: We subtract the oblique ascension of the degree <of the ecliptic> from the oblique ascension of its opposite <degree>. The remainder is the day arc. If we subtract the day arc of the sun or the star from 360°, the remainder is the night arc.

Chapter 11: On the <equinoctial> day hours of the sun and the star<s> and the degrees of their <seasonal> hours.

We multiply the equation of daylight of the sun or the star by 8 minutes. Then, if the declination of the degree of the sun <on the ecliptic> or the distance of the star from the celestial equator is northern, we add it (i.e., the product) to 12 <hours>. If the declination or the distance is southern, we subtract it from 12 <hours>. The sum or the remainder is the <number of the equinoctial> hours of the daylight of the sun or the star.

We multiply the equation of daylight by 10 minutes. Then, if the declination or the distance is northern, we add it to 15 <degrees>. If the declination or the distance is southern, we subtract it from 15 <degrees>. The sum or the remainder is <the number of> degrees in one <seasonal> hour of the sun or the star.

Another method: We divide the day arc of the sun or the star by 15 <degrees>: The result is the <number of the> equinoctial hours of the day. We also divide it by 12 <hours>: The result is the <number of the> degrees in one seasonal hour of the day. If we subtract the <number of the equinoctial> hours of the day from 24, the remainder is <the number of> the hours of the night. If we subtract <the number of> the degrees in one <seasonal> hour of the day from 30, the remainder is <the number of> the degrees in one <seasonal> hour of the night.

If we add to <the number of> the equinoctial hours of the day one-fourth of it, the sum is <the number of> the degrees in one seasonal hour of the day. If we subtract from <the number of> degrees in one seasonal hour of the day one-fifth of it, the remainder is <the number of> the equinoctial hours of the day.

Chapter 12: On the <ecliptical> degree of the transit of a star through the meridian.

If the star has no (i.e., zero) latitude, the <ecliptical> degree of its transit is the same as its longitude. If it has <a non-zero> latitude, we multiply the Cosine of the latitude of the star by the Sine of the distance to the solstice nearest to it, either before it or after it, and we divide it by the Cosine of the distance of the star from the celestial equator: The result is the Sine of the adjusted distance from the solstice. We find the corresponding arc, and add it to the beginning of the <zodiacal sign of the> solstice if the star lies after it in the sequence of the <zodiacal> signs, and we subtract it from it (i.e., from the beginning of the zodiacal sign of the solstice) if the distance <of the star> from it is in the order opposite <to that of the zodiacal signs>: The result is the right ascension of the degree of transit <counted> from the beginning of Aries. We find the <ecliptical> arc corresponding to the right ascension: The result is that <ecliptical> degree which passes the meridian <simultaneously> with the star.

Chapter 13: On the <ecliptical> degree relating to the rising and setting of a star.

If the distance of the star from the celestial equator is northern, we subtract its equation of daylight from the right ascension of the degree of its transit. If the distance is southern, we add its equation of daylight to the right ascension of the degree of its transit: The result is the oblique ascension of the <ecliptical> degree that rises <simultaneously> with the star. We add the day arc of the star to the <right> ascension of the <ecliptical> degree of rising <simultaneously with the star>. We find the arc corresponding to the sum in <the table for> the oblique ascension. Then we take its opposite, and this is the <ecliptical> degree setting <simultaneously> with the star.

Chapter 14: On <finding> the arc of revolution of the celestial equator since the rising of the sun or the star<s> from the altitude of the <sun or the star at a given> time.

We multiply the Sine of the altitude of the <given> time by the Sagitta of half the day arc and divide it by the Sine of the maximum altitude: The result is <called> the "arrangement Sine" of the arc of revolution. We subtract it from the Sagitta of half the day arc. The remainder is the Sagitta of the excess of the arc of revolution. We find the corresponding arc which is the excess of the arc of revolution. If the altitude of the

<given> time is eastern, we subtract the excess of the arc of revolution from half the day arc. If the altitude is western, we add the excess of the arc of revolution to half the day arc: The result is the arc of revolution of the celestial equator.

Chapter 15: On <finding> the <elapsed> hours from the arc of revolution.

We divide the arc of revolution of the celestial equator by 15: The result is <the number of> the equinoctial hours <elapsed> since the rising of the sun or the star. We divide the arc of revolution by <the number of> degrees in the <seasonal> hours corresponding to the degree of the sun or the star: The result is <the number of> the seasonal hours since the rising of the sun or the star.

Chapter 16: On <finding> the ascendant from the arc of revolution during the day and at night.

We add the arc of revolution from the rising of the sun or the star to the oblique ascension of the sun or the oblique ascension of the <ecliptical> degree which rises <simultaneously> with the star: The sum is the oblique ascension of the ascendant. We find the corresponding arc in the table for the oblique ascensions, and thus the ascendant will be obtained.

Chapter 17: On <finding> the arc of revolution from the ascendant.

We subtract the oblique ascension of the sun or the oblique ascension of the degree which rises <simultaneously> with the star from the oblique ascension of the ascendant: The remainder is the arc of revolution of the celestial equator since the rising of the sun or the star.

Chapter 18: On <finding> the altitude of the <sun at a given> time from the arc of revolution.

We obtain the difference between the arc of revolution and half the day arc: The result is the excess of revolution. We subtract its Sagitta from the Sagitta of half the day arc: The result is the Sine of the altitude of the sun or the star at the <given> time corresponding to the given arc of revolution. We find the corresponding arc: It is the altitude.

Chapter 19: On <finding> the arc of revolution since sunset from the ascendant.

We subtract the oblique ascension of the degree opposite to the sun from the oblique ascension of the ascendant at the time of measurement: The remainder is the arc of revolution of the celestial equator since sunset.

Chapter 20: On <finding> the ascendant from the arc of revolution since sunset.

We add the arc of revolution of the celestial equator since sunset to the oblique ascension of the degree opposite to the sun: The sum is the oblique ascension of the ascendant. We find its <corresponding> arc in the table for the oblique ascensions: It is the ascendant.

Chapter 21: On a base <value> applying to most operations concerning day and night.

We multiply the Cosine of the declination of the degree of the sun by the Cosine of the latitude of the locality, lowered twice: The result is the base <value>.

<Finding> the arrangement Sine from the altitude of the <given> time: We divide the Sine of the altitude of the <sun at the given> time by the base value: The result is the arrangement Sine (i).

<Finding> the altitude from the arrangement Sine: We multiply the base value by the arrangement Sine of the arc of revolution: The result is the Sine of the altitude (ii).

<Finding> the Sagitta of half the day arc which is called the "day Sine": We divide the Sine of the maximum altitude by the base value: The result is the day Sine (iii).

<Finding> the meridian altitude from the day Sine: We multiply the base value by the day Sine: The result is the Sine of the meridian altitude (iv).

<Finding> the excess of the arc of revolution: We divide the difference between the Sine of the altitude of the <sun at the given> time and the Sine of the meridian altitude by the base value: The result is the Sagitta of the excess of the arc of revolution (v).

<Finding> the altitude of the <sun at the given> time from the Sagitta of the excess of the arc of revolution: We multiply the Sagitta of the excess of the arc of revolution by the base value. We subtract the remainder from the Sine of the meridian altitude: The remainder is the Sine of the altitude <of the sun> (vi).

<**Finding**> **the equation of daylight**: Half of the day arc is known from its Sagitta. The difference between half the day arc and 90° is the equation of daylight.

<**Finding**> **the arc of revolution of the celestial equator**: The excess of the arc of revolution is known from its Sagitta. Half of the day arc is <also> known from its Sagitta. If the altitude is eastern, we subtract the excess from half the day arc. If the altitude is western, we add the excess to half the day arc. The sum or the remainder is the arc of revolution of the celestial equator.

Chapter 22: On the equalization of houses.

We obtain the <number of the> degrees in the <seasonal> hours of the ascendant, double it, and keep it. We subtract this double from 60: The remainder is twice the <number of the> degrees in the <seasonal> hours of the degree opposite to the ascendant (i.e., the descendant). We keep it. Then we subtract 90° from the right ascension of the ascendant: The remainder is the right ascension of the tenth <house>. Then we write the right ascension of the ascendant in two positions. We subtract from one of the positions twice the <number of the> degrees in the <seasonal> hours of the ascendant. We add to the other <position> the <number of the> degrees in the <seasonal> hours of the degree opposite <to the ascendant>. The remainder is the right ascension of the twelfth <house> and the sum is the right ascension of the second <house>. We subtract the subtrahend from the remainder <again> and add the addend to the sum <again>: The result of the subtraction is the right ascension of the eleventh <house>, and that of the addition is the right ascension of the third <house>. We find the arcs corresponding to each of these right ascensions. The results are the <ecliptical> degrees of <the cusps of> the houses. Then, the fourth <house> is <diametrically> opposite to the tenth <house>; the fifth <house> is opposite to the eleventh <house>; the sixth <house> is opposite to the twelfth <house>; the seventh <house> is opposite to the ascendant; the eighth <house> is opposite to the second <house>; and the ninth <house> is opposite to the third <house>. If we want to check the operation to know if we have worked correctly or wrongly, we subtract from the right ascension of the eleventh <house> twice the <number of the> degrees in the <seasonal> hours of the ascendant, that we had subtracted, and we add to the right ascension of the third <house> twice the <number of the> degrees in the <seasonal> hours of the opposite <of the ascendant>. If the remainder is equal to the right ascension of the tenth

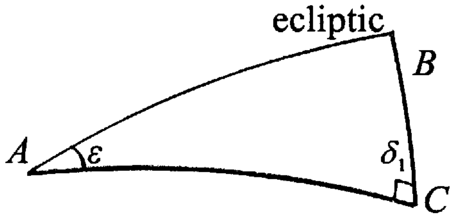
<house> and the sum is equal to its opposite, then we have worked correctly. If not, we have worked wrongly, and we repeat the operation.

Another method: We write the right ascension of the tenth <house> in two positions. We add to one position twice the <number of the> degrees in the <seasonal> hours of the ascendant. We subtract from the other <position> twice the <number of the> degrees in the hours of the opposite <of the ascendant>. The sum <is> the right ascension of the eleventh <house> and the remainder <is> the right ascension of the ninth <house>. We add the addend to the sum <again> and we subtract the subtrahend from the remainder <again>: The sum is the right ascension of the twelfth <house>, and the remainder is the right ascension of the eighth <house>. The arcs of these right ascensions are the <ecliptical> degrees of <the cusps of> the houses, and their opposites are <found> as mentioned before.

Commentary

I.5.1 In modern notation: $Sin\delta_1 = Sin\lambda Sin\varepsilon / R$, where δ_1 is the absolute value of the first declination, λ the true longitude, ε the total declination, and *R* the radius of the trigonometric circle (usually taken equal to 60 in medieval Islamic trigonometry). Ptolemy [1984, 69-70] explains this method by solving the problem for special cases. Al-Battānī gives the same formula as Kūshyār but in terms of the Chord function [1899-1907, III, 18]. A proof of the validity of the formula for the first declination is given in IV.5.1. A table for the first declination is given in II.43.

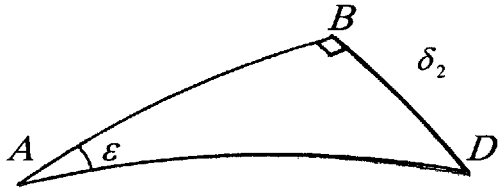
I.5.2 For a discussion of the right ascension see [Pedersen 1974, 99-101]. Kūshyār's methods for finding the right ascensions are equivalent to $CosA_0(\lambda) = Cos\lambda / (Cos\delta_1 / R)_1$ and $SinA_0(\lambda) = Tg\delta_1 / (Tg\varepsilon / R)$, where A_0 is the right ascension and the other symbols are as in I.5.1. The third method, in which we use the table for the second declination, is based on the symmetrical relation between the first and the second declinations. I will now explain Kūshyār's rather confusing description of the third method. Apparently, he provides the method for finding the right ascension by means of a table for the second declination, if the value of the first declination is known.



In the figure, *AB* is the true longitude of the ecliptical degree, *AC* the corresponding right ascension, δ_1 the first declination, and ε the total declination or the angle between the ecliptic and the celestial equator. Now, if we take *AC* as an arc on the ecliptic and *AB* as the celestial equator, then δ_1 will be the second declination of *AC*. So, if we find the argument of the tabular entry equal to δ_1 (which Kūshyār implicitly assumes to be known), in the table for the second declination, the argument will be equal to the required right ascension.

Ptolemy solves the problem for the special cases λ=30° and λ=60°, using a method equivalent to the second formula [1984, 71- 73]. Al-Battānī gives the second formula but instead of Sines and Tangents he uses the Chord function [1899-1907, III, 20]. Kūshyār gives a table for the right ascensions in II. 45 and proofs of the formulas in IV.5.2.

I.5.3 The formulas are equivalent to $Sin\delta_2 = Sin\delta_1 /[Cos\delta_1(90° - \lambda)/R]$ and $Tg\delta_2 = Sin\lambda Tg\varepsilon / R$, where δ_2 is the second declination, and δ_1(90°-λ) is the first declination of the complement of the true longitude (other symbols are as defined formerly). The third method in which we use the table for the rising times on the equator, is based on the symmetry between the first and the second declinations. Kūshyār's description in the third method is again confusing (cf. the third method in I.5.2). Apparently, he supposes that we have the table for the right ascensions and we can find the first declination (by computation or from the corresponding table).



In the figure, *AB* is the arc on the ecliptic, and δ_2 is the corresponding second declination. So, *BD* is perpendicular to *AB*. Now, if we take *AD* as an arc on the ecliptic, and *AB* as the celestial equator, then δ_2 may be regarded as the first declination of *AD*. Kūshyār calls *AD 'aks al-maṭāli'* ("the inverse of the right ascension", meaning: the ecliptical longitude where the right ascension is *AD*). Proofs of the three methods are given in IV.5.3. A table for the second declination is given in II.43.

I.5.4 In modern notation: $Sin(d) = Sin(\beta \pm \delta_2)Cos(\varepsilon)/Cos(\delta_2)$, where *d* is the distance from the celestial equator, and β the ecliptical latitude of the star (other symbols as defined above). In al-Battānī's *zīj*, *d* is found through a method equivalent to the first formula given below in I.5.6. Kūshyār gives a proof of his formula in IV.5.4.

I.5.5 In modern notation: $\varphi = 90° - (h_{max} \pm \delta_1)$, where φ is the geographical latitude of the locality, and h_{max} is the maximum altitude of the sun on the day of measurement, and δ_1 is the declination of the sun on that day. A proof of this formula is given in IV.5.5. If h_{max} is measured in the south, the + sign is for a southern declination and the – sign is for northern declination. If h_{max} is measured in the north, the + sign is for a northern declination and the – sign is for a southern declination. The above formula can be written as $90° - \varphi = h_{max} \pm \delta_1$. Now if $h_{max} + \delta_1$ exceeds 90° (In the northern hemisphere, this happens when northern δ_1 exceeds φ), then $90° - \varphi = 180° - (h_{max} + \delta_1)$. This is possible in those localities where

$\varphi<\varepsilon$. For example in Mecca, where φ=21.4°, this happens near the summer solstice, when $\delta_1 > 21.4°$.

I.5.6 The ortive amplitude (θ) is the arc on horizon between the East point and the rising point of the sun or the star [Kennedy & Sharkas 1962]. In modern notations, the first method is $Sin\theta = Sin\delta_1 /(Cos\varphi / R)$ for the sun, and $Sin\theta = Sind /(Cos\varphi / R)$ for the stars, where *d* is the distance of the star from the celestial equator. The second method may be expressed in modern notation as $Cos\theta = Cos\delta_1 Sin\frac{D}{2} / R$ for the sun and $Cos\theta = CosdSin\frac{D}{2} / R$ for the stars, where *D* is the day arc (see Chapter 10 of this section). Proofs of the validity of these formulas are given in IV.5.6. In the *Almagest*, the second method is used for finding the maximum ortive amplitude at Rhodes where φ=36° [1984, 76-77]. In al-Battānī's *zīj*, both methods for finding the ortive amplitudes are described [1989-1907, III, 29-30].

I.5.7 The equation of daylight (ΔD) is defined as $\Delta D = |D/2 - 90°|$. For the sun, the four methods are equivalent to the following formulas:
$Cos\Delta D = Cos\theta /(Cos\delta_1 / R)$,
$Sin\Delta D = Sin\delta_1 Sin\varphi /(Cos\delta_1 Cos\varphi / R) = Tg\delta_1 Tg\varphi / R = SinMSinA_0(\lambda) / R$.
In the first three formulas, we may substitute *d* for δ_1 to obtain the formulas for the stars (see the last sentence in I.5.4). In the last formula, *M* is the maximum value of ΔD. Proofs of the formulas are given in IV.5.7, where he also provides the proof for another method that is not mentioned here. A table of the function ΔD for the latitude 36° and a table of the function *M* for latitudes of 16°, 17°, 18°, ...,45° are given in II.48 and II.47 in F, respectively. However, the table in II.47 is left blank in F and it is also missing in the other manuscripts. In Y, L, and B, the table II.48 is given for the latitude of 35;30°. At the end of Y, there is a table for ΔD calculated for the latitude of 30;5°. In the *Almagest* we find an application of a method equivalent to Kūshyār's second method, for the latitude of 36° relating to Rhodes [1984, 78-79], without referring to the 'base'. Al-Battānī also uses the second method for finding the equation of the daylight arc [1989-1907, III, 48-49] in order to find the daylight arc itself (see I.5.10).

I.5.8 For a discussion of the oblique ascensions see [Pedersen 1974, 99-101]. A proof of the validity of Kūshyār's method for finding the oblique ascensions is found in IV.5.8. A table of the oblique ascensions for the latitude of 36° (possibly a rounded value for 36;50°, the latitude of Jurjān) is given in II.46 in F. The corresponding tables in Y, L, and B are given for the latitude of 35;30°. At the end of Y, corresponding tables for latitudes of 38° (possibly for Daylam in Gīlān) and 32; 23° (Isfahan) are given. In P, the relevant tables for latitudes of 38° and 36; 30° are given among the seven tables added in the sequel of Book I.

I.5.9 The method described in this chapter is equivalent to the modern formula $h_{max} = (90° - \varphi) \pm \delta_1$ where the positive sign is used for northern declinations, and the negative sign for southern declinations of the sun. For the planets and stars, we substitute the distance from the celestial equator for δ_1. A proof of the validity of this method is given in IV.5.9.

I.5.10 The method for finding half the day arc is equivalent to the formula $D/2 = 90° \pm \Delta D$. The positive and negative cases are for the northern and the southern declination of the sun or the distance of the planet from the celestial equator, respectively. A proof of this method is given in IV.5.10.

In the second method, if the oblique ascension of a certain ecliptical degree λ is $A(\lambda)$, then we have $A(\lambda+180°) = A(\lambda)+180° \pm 2\Delta D$ (this relation can be deduced from the rule for finding the oblique ascension given in I.5.8 above and the symmetries of the oblique ascension described in [Ptolemy 1984, 90-92]). So the difference is $180° \pm 2\Delta D$, which is the day arc. The plus sign is used for the first and fourth quadrants, and the minus sign is used for the second and third quadrants.

I.5.11 Since each degree of rotation of the celestial equator corresponds to 4 minutes of time, the length of a day is

$$4(180° \pm 2\Delta D) \text{ minutes} = 12 \text{ hours} \pm 8\Delta D \text{ minutes}$$

The number of degrees in a seasonal hour is

$$D/12 = (180° \pm 2\Delta D)/12 = 15° \pm (10/60)\, \Delta D$$

Dividing $2\Delta D$ by 15 or 12 is equivalent to multiplying $2\Delta D$ by 4 minutes (i.e., 4/60) or multiplying ΔD by 10 minutes (i.e., 10/60).

So the alternative methods are equivalent to those formulated above. The number of degrees in a seasonal hour of the night is

$$(360° - D)/12 = 30° - D/12, \text{ i.e.,}$$

30° minus the number of degrees in a seasonal hour of the day.

It is obvious that $\frac{D}{15}(1+\frac{1}{4}) = \frac{D}{12}$ and $\frac{D}{12}(1-\frac{1}{5}) = \frac{D}{15}$. There is no section corresponding to I.5.11 in Book IV.

I.5.12 The Sine of the 'adjusted distance' (d_a) of the star from the solstice (arc TE in the figure of IV.5.11), or the difference between the right ascensions of the ecliptical degree relating to the star, and the nearest solstice is found by a method equivalent to the formula $Sind_a = Cos\beta Sind_s / Cosd$, where β is the latitude of the star, d_s is its distance to the nearest solstice, and d is the distance of the star from the celestial equator. By adding or subtracting this auxiliary magnitude d_a to or from the longitude of the relevant solstice, we find the right ascension of the star, and hence, by the table of right ascensions, its ecliptical degree of transit. A proof of the validity of this method is given in IV.5.11. Kūshyār's method is different from and simpler than the method given by Ptolemy [1984, 411-12] and followed by al-Battānī [1899-1907, III, 48].

I.5.13 Proofs of these methods are given in IV.5.12. Kūshyār's methods are simpler than those provided by al-Battānī [1899-1907, III, 49-50] for this subject.

I.5.14 The arc of revolution is the distance covered by a point on the celestial equator due to the apparent revolution of the celestial sphere (1° per 4 minutes of time) between two moments of time. Usually these two moments are sunrise or sunset and the time of observation. In this chapter, Kūshyār wants to compute the arc of revolution from the altitude which he has found by observation (for a definition of the Sagitta function see I.2.4 and its commentary). In modern notation, first we obtain the so called 'arrangement sine' of the arc of revolution; then we obtain the excess of the arc of revolution, and then the arc of revolution, as follows:

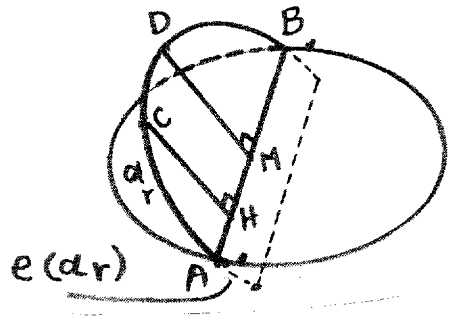
Let $$As(a_r) = Sinh_t(R - Cos\frac{D}{2}) / Sinh_{\max},$$

then $$R - Cose(a_r) = R - Cos\frac{D}{2} - As(a_r),$$

and $$a_r = \frac{D}{2} \pm e(a_r).$$

$As(a_r)$ is called the 'arrangement Sine' of the arc of revolution, h_t is the altitude of the sun or the star at the given time, $h_{\max}$ is the maximum altitude, and $e(a_r)$ is the excess of the arc of revolution.

A proof of the validity of this calculation is given in IV.5.13. Al-Battānī gives a similar method for this calculation [1899-1907, III, 45], but he does not use the term 'arrangement Sine'. Al-Kāshī used the term in the Book V of his *Khāqānī zīj* [Kennedy 1985, 45; 1998, 38]. In the introduction of Book V, he defines the 'arrangement Sine' as "the perpendicular from one end of an arc to a chord that passes through the other end of the arc". He adds that "if a perpendicular is drawn on the base of a circular segment and reaches the circumference and divides the arc into two [unequal] arcs, the perpendicular is regarded as the 'arrangement Sine' of either of the two arcs."



In the chapter entitled "A compendium of astronomy" in Book III, Kūshyār defines the 'arrangement Sine' in a similar way. In his definition $ACDB$ is the day arc and CH, the 'arrangement Sine' of the 'arc of revolution AC, is parallel to the Sagitta of half the day arc (DM).
Example:
Suppose the altitude of the sun is 53° eastern, the maximum altitude is 72;58°, and half the day arc is 104;24°.
Using the above formulas, we have:
$As(a_r)=Sin53°(60-Cos104;24°)=62.5807$,
$60\text{-}Cos\,e(a_r)=60\text{-}Cos104;24°\text{-}62.5807=12.3407$, so $Cos\,e(a_r)= 47.6593$,
$e(a_r)=37.24°$. Now the arc of revolution can be found by the last formula above, taking the minus sign (because the sun is in the eastern half of the sky): $a_r= D/2\text{-}37;24°= 67;0°$.
The example is taken from al-Nasawī's *al-Lāmiʿ fī amthilat al-Zīj al-jāmiʿ* (see the commentary on I.1.5), fol. 61v. Al-Nasawī finds a_r equal to 67;1°. The difference with the above result is only 1 minute that can be due to the rounding errors.
With the data of the example, the geographical latitude of the locality of observation can be found equal to 35;57.5°. This accords with the historical information that al-Nasawī worked in or around Rayy (with geographical latitude 35;40° N).
Having the arc of revolution, we can easily find the time of the day, as the equinoctial hours passed since sunrise (see I.5.15):

67;0/15= 4 hours + 28 minutes.

We can also find the local time:

12- 37;24°/15= 9;30.4 AM

I.5.15 Since a complete revolution of the equator occurs in 24 hours, by dividing the arc of revolution into 15=360:24, we obtain the number of the hours elapsed since the rising of the sun or the star. The method for finding the number of the seasonal hours elapsed since sunrise is obvious.

I.5.16 A proof of this method is given in IV.5.14. This method is also given by al-Battānī [1899-1907-, III, 45].

I.5.17 A proof of this method is also given in IV.5.14.

I.5.18 This is the inverse of the method given in I.5.14. Al-Battānī provides a similar method [1899-1907, III, 46].

I.5.19 This method is valid because the ecliptic is a great circle, and a semicircle of it is always above the horizon. Then, the setting of any point on the ecliptic is simultaneous with the rising of the diametrically opposite point on the ecliptic.

I.5.20 This is simply the inverse of the method mentioned in I.5.19.

I.5.21 In modern notation, the "base" B (not the same B mentioned in I.5.7) as well as the first six rules in the chapter are as follows:

$$B = Cos\delta_1 Cos\varphi / R^2$$

i) $As(a_r) = Sinh_t / B$

ii) $Sinh_t = BAs(a_r)$

iii) $R - Cos\frac{D}{2} = Sinh_{max} / B$

iv) $Sinh_{max} = B(R - Cos\frac{D}{2})$

v) $R - Cose(a_r) = (Sinh_{max} - Sinh_t) / B$

vi) $Sinh_t = Sinh_{max} - B[R - Cose(a_r)]$

The validity of formula i is proved in IV.5.15. The formulas ii, iii, iv, v, and vi can be derived from i, by applying the definition of the base and by the formulas given in I.5.14.

I.5.22 'Equalization of the Houses' is a method of division the ecliptic into 12 parts, different from the zodiacal signs, for astrological purposes. The beginnings of the first, fourth, seventh and tenth house were usually regarded to be the ascendant, the lower midheaven, the descendant, and the upper midheaven, respectively. These points are called the 'cardines' (Arabic: *autād*). Then the arcs between these four points were divided into three parts for finding the beginnings of the other houses. There were different methods for this division. Kūshyār's method was called "the well- known method" by al-Bīrūnī [1954-1956, III, 1357-59; 1985, 276] and "the Standard Method" in modern literature [North 1986, 4; Kennedy 1996, 538, 548]. It was actually the most popular one among the medieval Islamic authors. The origin of this method is pre-Islamic [North 1986, 6], however, some modern authors ascribe it to Alchabitius (Latinized form of al-Qabīṣī), the court astrologer of the Buyid Sayf al-Dawla [Kennedy 1996, 539-40].

In this method the cardines are projected onto the celestial equator along great circle arcs through the equatorial poles. Then the four resulting segments are trisected. The trisection points are projected back onto the ecliptic along great circle arcs through the equatorial poles to obtain the corresponding cusps (i.e., the beginnings of the houses). Kūshyār first finds $\Delta\alpha_d$, the equatorial arc corresponding to 2 diurnal unequal hours (for the sun at the ascendant). He also finds $\Delta\alpha_n = 60^\circ - \Delta\alpha_d$, the equatorial arc corresponding to 2 nocturnal unequal hours. By subtracting $\Delta\alpha_d$ from the right ascension of the ascendant and adding $\Delta\alpha_n$ to this right ascension, he finds the right ascensions of the cusps of the twelfth and the second houses. By repeating this process, he finds the right ascensions of the cusps of the eleventh and the third houses. Now he finds the ecliptical degrees corresponding to these right ascensions. He also finds the cusps of the other houses, which are opposite to the former houses correspondingly. In the second method, he starts from the tenth house and follows a process similar to the first one and differing in the order of the houses to be found.

Al-Battānī mentions the same method, and he follows the order given in Kūshyār's second method [al-Battānī 1899-1907, III, 110-11].

Section 6: On eclipses and what pertains to them, <in> 20 chapters
Chapter 1: On the motion of the two luminaries in <one> day and <one> hour.

The daily motion is the difference between the true longitude of <any> one of the two luminaries for any day and the true longitude for the next or the previous day. It is called 'daily rate'. The hourly motion is the result of dividing the daily motion by twenty-four. It is called 'hourly rate'. Or <using another method,> we find the true longitude of <any> one of the two luminaries for the given time; then <we find it> for six hours later or earlier. We take the difference between the two true longitudes and multiply it by 10 minutes (i.e., 10/60). A table is compiled for it (i.e., for the hourly rate) <in this *zīj*>. If the hourly rate of the sun is subtracted from the hourly rate of the moon, the remainder is the adjusted rate. It is called 'the lunar gain'.

Chapter 2: On the magnitude of the <apparent> diameter of the two luminaries and the diameter of the shadow <of the earth>.

<For finding> the diameter of the sun, we multiply its daily motion by 33 minutes (i.e. 33/60), or we multiply its hourly motion by $13\frac{1}{5}$: The result is its diameter according to its distance from the earth. As for the diameter of the moon, we multiply its daily motion by 2 minutes and 26 seconds (i.e., 2/60+26/60×60=146/3600), or we multiply its hourly motion by 58 minutes and 25 seconds (i.e., 58/60+25/60×60=3505/3600): The result is its diameter according to its distance from the earth. For the diameter of the shadow <of the earth>, we multiply the diameter of the moon by $2\frac{3}{5}$: The result is the diameter of the shadow according to the distance of the moon from the earth, the sun being at its maximum distance <from the earth>. If we want extreme precision, we take the excess of the hourly motion of the sun over 2 minutes and 23 seconds, multiply <the result> by 10, and subtract <this product> from the diameter of the shadow which was found: The result is the adjusted diameter of the shadow according to the distance of the sun also from the earth. A table is compiled for these diameters, with the hourly motions of the two luminaries <in this *zīj*>.

Chapter 3: On the <ecliptical> degree of a conjunction and opposition, their hours and ascendants.

We find the true longitudes of the two luminaries for noon of the day nearest to the conjunction or the opposition, and we take the distance between the two true longitudes from those true longitudes in the case of conjunction. But in the case of opposition, <we take the distance> after we have added 6 <zodiacal> signs to the position of the moon. We note which of the two <luminaries> precedes the other <in the ecliptic>. Then we multiply the distance by 5 minutes and we call the result 'the part of the distance', and we keep it; <then> we add it to the distance: The result is the distance plus 'the part of the distance'. Then we look: if the sun precedes <the moon>, we add the distance plus its part to the <true longitude of the> moon and we add the 'part of the distance' to the <true longitude of the> sun. If the moon precedes <the sun>, we subtract the distance plus its part from the <true longitude of the> moon, and we subtract 'the part of the distance' from the <true longitude of the> sun. <The sun and the moon> will be in conjunction or opposition in the same second <of a degree>. **The hours**: Then we find the hourly motions of the two luminaries. We subtract the hourly motion of the sun from the hourly motion of the moon. The remainder is <called> 'the lunar gain'. We divide the distance by 'the lunar gain'. The result is the <number of> hours corresponding to the distance. If the sun precedes <the moon>, we add the hours corresponding to the distance to the hours of the noon. If the sum is less than the hours of the entire day, then it is the <number of the> hours of the day elapsed <before conjunction>. If it is greater than the <number of the> hours of the day, we subtract the <number of the> hours of the day from it. The remainder is the <number of the> hours of the next night elapsed <before conjunction>. If the moon precedes <the sun>, and the <number of the> hours corresponding to the distance is less than the hours of the noon, we subtract the <number of the> hours corresponding to the distance from the hours of the noon. The remainder is the <number of the> hours of the day elapsed <before the conjunction>. If it is greater than <the number of> the hours of the noon, we subtract the <number of the> hours corresponding to the distance from the sum of the hours of the noon and the hours of the night. The remainder is the <number of the> hours of the previous night elapsed <before the conjunction>. Then we find the true longitudes of the two luminaries for the resulting hours (i.e., for the resulting time). If they coincide with the <ecliptical> degree that had been computed before, the <number of the> the hours is correct. If their positions are different, we take the difference between them, and we operate with them in the same way that we operated with the true longitude<s> at noon and the distance between the two luminaries. The result of this second time is the <ecliptical> degree of the conjunction and opposition and the time <of conjunction and opposition> with <more> precision. We find the ascendant for the resulting time. It is the ascendant of the conjunction and the opposition.

Chapter 4: On the absolute and adjusted magnitudes of a lunar eclipse in digits.

We consider the latitude of the moon at the <time of> opposition: If it is more than 63 minutes northern or southern, the moon will not be eclipsed, if it is less than this <limit>, it can be eclipsed. Then we find the diameter of the moon and the diameter of the shadow, we add them, and we halve the result. It is <called> half the <sum of the> diameters (i.e., the sum of the two radii). If the latitude of the moon is greater than the <sum of the> two radii or equal to it, the moon will not be eclipsed. If the latitude is less <than that, the moon> will be eclipsed. The excess of the <sum of the> two radii over the latitude is the <magnitude> of the eclipse in minutes. If it is greater than the diameter of the moon, <then the moon> will be totally eclipsed and will remain in it (i.e., in the eclipsed situation) for some time. If it is equal to the diameter of the moon, <then the moon> will be totally eclipsed, but it will not remain in the <total> eclipse. If it is less than the diameter of the moon, <then the moon> will be partially eclipsed. We multiply the <magnitude> of the eclipse in minutes by 12 and divide it by the diameter of the moon. The result is the absolute <magnitude> of the eclipse in digits, in which we take the diameter <of the moon equal to> 12 digits. For <finding> its adjusted <value>, we subtract the <magnitude> of the eclipse in minutes from the diameter of the moon, and <also> from the diameter of the shadow, and we add the remainders. Then we multiply the remainder of the diameter of the moon by the <magnitude> of the eclipse in minutes, and divide it by the sum of the two remainders. The result is the Sagitta of the shadow. We subtract it from the <magnitude> of the eclipse in minutes. The remainder is the Sagitta of the moon. Then we subtract the Sagitta of the moon from the diameter of the moon, multiply the remainder by the Sagitta of the moon, and take the square root of the result. The <final> result is the absolute Sine. We keep it. Then we multiply the absolute Sine by 60 and divide it by the radius of the moon. The result is the adjusted Sine. We find the corresponding arc. If the Sagitta of the moon is less than its radius, then this arc is <called> 'the arc of the moon<'s disk>'. If the Sagitta is greater than the radius <of the moon>, we subtract the arc from 180 <degrees>. The remainder is 'the arc of the moon<'s disk>'. Then we multiply the diameter of the moon by 22 and divide it by 7. The result is the circumference of the moon's disk. Then we multiply half of it by the radius of the moon. The result is the area of the moon's disk. Then we multiply the circumference of the disk by the arc <of the moon's disk>, and divide <the product> by 360 <degrees>. The result is half the 'arc of the sector'. We multiply it by the radius of the moon. The result is the <area of the> sector of the moon. Then we obtain the difference between the Sagitta and the radius, and multiply <the remainder> by the absolute Sine. The result is <the area of > the triangular <portion> of the moon. If the Sagitta is less than the radius <of the moon>, we subtract the <area of the> triangular <portion> from the <area of the> sector. If the Sagitta is greater <than the radius>, we add it <to the area of the sector>. The sum or the remainder is <the area of> 'the segment of the moon'. Then we repeat the operation for the shadow <instead of the moon> from the absolute Sine on. However, the Sagitta of the shadow does not reach the value of its radius. When we find the segment of the shadow, we add it to the segment of the moon. The result is the adjusted <magnitude> of the eclipse in minutes. We multiply it by 12 and we divide the result by the area of the moon's disk. The result is the adjusted <magnitude> of the eclipse in digits, based on taking the area of the <moon's> disk <equal to> 12 digits. <The procedure given in> this chapter is also sufficient for finding the adjusted magnitude of solar eclipses in digits, if we let the disk of the sun take the place of the disk of the moon in this <procedure>, and let the moon's disk take the place of the disk of the shadow. We follow the conditions that we laid down in the case of the moon, its Sagitta, its arc, and its triangular <portion>. A table for finding the approximate adjusted magnitudes of the two (i.e., solar and lunar) eclipses is compiled <in this *zīj*>.

Chapter 5: On the absolute and adjusted times of a lunar eclipse.

The time of the opposition is that of the middle of the lunar eclipse. The other times are <as follows:> the beginning of the lunar eclipse, the beginning of the duration <of totality>, the beginning of the emersion <of the eclipse>, and the end of the emersion. If there is no duration <of totality>, <then the times are:> the beginning of the lunar eclipse, and the end of the emersion. We subtract the square of the latitude of the moon at the middle of the lunar eclipse from the square of the <sum of the> two radii and obtain its square root. <The result> is the <magnitude> of immersion from the beginning of the lunar eclipse to its middle in minutes, whether or not it has a duration <of totality>. We divide it by the lunar gain. The result is the <number of the> hours of the immersion from the beginning to the middle <of the lunar eclipse>. We subtract it from the time of the middle of the lunar eclipse, and we <separately> add it <to the time of the middle>. The remainder is the time of the beginning <of the lunar eclipse>, and the sum is the time of the end of emersion. If <the moon> has a duration <of totality>, we subtract the radius of the moon from the radius of the shadow. Then we subtract from the square of the remainder the square of the latitude <of the moon> at the middle of the lunar eclipse, and obtain the square root of the <last> remainder. It is the <magnitude> of the immersion from the beginning of the duration <of totality> to its middle in minutes. We divide it by the lunar gain. The result is the <number of the> hours of the immersion from the beginning of the duration <of totality> to its middle. We subtract it from the time of the middle <of the lunar eclipse>, and we <separately> add it <to the time of the middle>. The remainder is the time of the beginning of the duration <of totality>, and the sum is the time of the beginning of emersion. For finding its adjusted value, we subtract the square of the latitude of the moon at the beginning of the lunar eclipse from the square of <the sum of> the two radii. We add what remains from the square of <the sum of> the two radii to the square of the difference between the latitude of the moon at the beginning of the lunar eclipse and its latitude at the middle of the lunar eclipse. We obtain the square root of the result. It is the adjusted <magnitude> of immersion from the beginning to the end <of the eclipse> in minutes. We divide it by the lunar gain. The result is the adjusted <duration of> the immersion in hours. We subtract it from the time of the middle of the lunar eclipse. The remainder is the adjusted time of the beginning <of the eclipse>. Then we also subtract the square of the latitude of the moon at the end of the emersion from the square of <the sum of> the two radii. We add the remainder to the square of the difference between the latitude of the moon at the end of the emersion and its latitude at the middle of the lunar eclipse. We obtain the square root of the result. It is the second adjusted <magnitude> of the immersion in minutes; it is <the duration> from the middle <of the lunar eclipse> to the end of emersion. We divide it by the lunar gain. The result is the second adjusted <duration of> the immersion in hours. We add it to the <time of the> middle of the lunar eclipse. The result is the adjusted time of the end of the emersion. Finding the adjusted <values> of the other times <involved in a lunar eclipse> is of no use.

Chapter 6: On drawing the figure of a lunar eclipse.

We draw a straight line <segment> of arbitrary length. We divide it by the number of the minutes of <the sum of> the two radii. Then we draw a circle the radius of which is equal to this line <segment>. It is the circle of <the sum of> the two radii. We take from the line <segment a part of it> equal to the radius of the shadow and we draw a circle with this segment as its radius and centered on the center of the first circle. It is the circle of the shadow. We draw the two diameters of the two circles, which intersect at the center at right angles. We write on their directions the four <geographical> orientations: east opposite to west, and north opposite to south. Then we take from the line <segment a part of it> equal to the latitude of the moon in the middle of the lunar eclipse. We put one leg of the compasses at the center of the two circles, and the other where it falls on the south-north line depending on the direction of the latitude, and we make a mark there. It is the center of the moon at the middle of the lunar eclipse. Then we take from the line <segment a part of it> equal to the radius of the moon and we draw a circle with this segment as its radius and centered on the center of the moon. It is the circle of the moon in the middle of the lunar eclipse. The portion of it inside the circle of the shadow is the eclipsed part of the moon.

Chapter 7: On <finding> the distance of the moon from the earth.

First we consider the double elongation <of the moon>. If it is <equal to> zero, then the distance of the center of the epicycle from the center of the earth is <put equal to> 60 parts. If the double <elongation> is exactly 6 <zodiacal> signs, then the distance of the center <of the epicycle from the center of the earth is> $39\frac{1}{3}$ parts. If the double <elongation> is exactly 3 or 9 signs, we subtract the square of $10\frac{1}{3}$ parts from the square of $49\frac{2}{3}$ parts, and obtain the square root of the remainder. The distance of the center <of the epicycle from the center of the earth> is <found to be> approximately $48+\frac{1}{3}+\frac{1}{4}$ parts. If the double <elongation> is between these <values>, we multiply both its Sine and Cosine by $10\frac{1}{3}$ minutes (i.e., $31/3\times60$), and we subtract the square of the product of the Sine <of the double elongation by $10\frac{1}{3}$ minutes> from the square of $49\frac{2}{3}$ parts. Then we obtain the square root of the remainder. <Now,> if the double <elongation> is less than 3 signs or greater than 9 <signs>, we add to the square root the product of the Cosine of the double <elongation by $10\frac{1}{3}$ minutes >. If the double <elongation> is greater than 3 signs and less than 9 signs, we subtract from the square root the product of the Cosine of the double <elongation by $10\frac{1}{3}$ minutes>. The <final> result is the distance of the center of the epicycle from the center of the earth. **The body of the moon**: Then we obtain from the equation tables the difference <in epicyclic equation> at lesser distance <of the moon from

the earth> corresponding to the double <elongation>, and the sixtieths corresponding to the true anomaly. We multiply them by each other. We add <the result> to 5 parts and 1 minute (i.e., 5+1/60 parts). We obtain the Sine of the result. The <last> result is the adjusted radius of the epicycle. Then if the true anomaly is <equal to> zero, we add the adjusted radius of the epicycle to the distance of the center of the epicycle. <The result> is the distance of the moon from the center of the earth. If the true anomaly is exactly 6 signs, we subtract the radius of the epicycle from the distance of the center of the epicycle. The remainder is the distance of the moon from the center of the earth. If the true anomaly is exactly 3 or 9 signs, we add the square of the adjusted radius of the epicycle to the square of the distance of the center <of the epicycle>. We obtain its square root, and <the result> is the distance of the moon <from the center of the earth>. If the true anomaly is between these <values>, we multiply both the Sine and the Cosine of the true anomaly by the adjusted radius of the epicycle, lowered. Then, if the true anomaly is less than 3 signs or greater than 9, we add the product of the Cosine <by the adjusted radius of the epicycle, lowered> to the distance of the center <of the epicycle from the center of the earth>. If the true anomaly is greater than 3 signs and less than 9, we subtract the product of the Cosine <by the adjusted radius of the epicycle, lowered> from the distance of the center <of the epicycle from the center of the earth>. We add to the square of the sum or the remainder the square of the product of the Sine <by the adjusted radius of the epicycle, lowered>. We obtain the square root <of the final result>. It is the distance of the moon from the center of the earth. A table is compiled <in this *zīj*> for the distance of the moon <from the earth> sufficient for what we need in <calculating> solar eclipses and the <lunar> crescent visibility. The distance of the moon is found in the table where the double elongation <is shown> in the <first row along the> width and the true anomaly, in the <first column along the> length <of the table>. This is sufficient for the calculation. Calculating the distance of the sun <from the earth> is not very necessary for us. Its calculation is like that of the moon, except that we use its (i.e., the sun's) mean anomaly instead of the true anomaly <used in the case of the moon>: <We use> 2 degrees and 1 minute instead of the adjusted radius of the epicycle, and <we use> 60 instead of the distance of the epicycle. Then we multiply what we find for the distance by $18\frac{4}{5}$. <The result> is the distance <of the sun> from the center of the earth. Its maximum distance is approximately 1,255 parts; its mean distance is approximately 1,208 parts; its minimum distance is approximately 1,161 parts.

Chapter 8: On the altitude of the pole of the ecliptic, which is called 'the latitude of the clime of visibility'.

We divide the Sine of the altitude of <the sun at a given> time by the Sine of the arc between the tenth <house> (i.e., mid-heaven) of <the given> time and its ascendant on the ecliptic, lowered. The result is the Cosine of the altitude of the pole. We find the <corresponding> arc <in the table of Sines> and subtract it from 90 <degrees>. The remainder is the altitude of the pole.

Chapter 9: On the altitude of any desired degree of the ecliptic.

We multiply the Sine of the arc between the <given> degree and the ascendant or the descendant by the Sine of the altitude of the tenth <house>. We divide <the product> by the Sine of the arc between the tenth <house> and the ascendant or descendant. The result is the Sine of the altitude of the <given> degree and <also> of the altitude of any planet of zero latitude.

Chapter 10: On the <equatorial> distance between the meridian and the <right> ascension of a known point of the ecliptic.

If the known point is between the tenth <house> and the ascendant, we subtract the right ascension of the tenth <house> from the right ascension of the <known> point. The remainder is the distance of <the ascension of> the point from the meridian. If the known point is between the seventh and tenth <house>, we subtract the right ascension of the <known> point from the right ascension of the tenth <house>. The remainder is the distance of the <ascension of the known> point from the meridian. If the remainder is greater than 90 <degrees>, we subtract it from 180 <degrees>. The remainder is the <desired> distance.

Chapter 11: On the parallax of the two luminaries in the altitude circle.

We obtain both the Sine of the altitude of the <ecliptical> degree of the moon and the Sine of the complement (i.e., the Cosine) of the altitude, lowered. We subtract the result of <lowering> the Sine of the altitude from the distance of the moon from the earth (as in Chapter I.5.7, the maximal distance is 60). We add to the square of the remainder the square of the result of <lowering> the Cosine of the altitude. We obtain the square root of the <final> result. Then we divide the result of

<lowering> the Cosine of the altitude by this square root, lowered. The result is the Sine of the parallax of the moon in the altitude circle. If the moon is on the horizon, we add to the square of the distance of the moon from the earth the square of the radius of the earth, which is <taken as> one part. We obtain the square root <of the sum>. Then we divide the radius of the earth by this square root, lowered. The result is the Sine of the parallax <of the moon>. If we subtract the parallax from the calculated altitude of the <ecliptical> degree of the moon, the remainder is the apparent altitude <of the moon, seen> from the surface of the earth.

Section: The parallax of the sun is calculated in a similar way, by using its mean distance from the earth. However, its parallax at different distances does not differ in a <noticeable> magnitude. Its maximum parallax is about 3 minutes. We need this to subtract it from the parallax of the moon. The remainder is the adjusted parallax of the moon in the altitude circle. This <is used> for <reaching> extra precision in <calculating> solar eclipses. A table is compiled <in this *zīj*> for obtaining it from the complement of the altitude of the sun.

Chapter 12: On the six angles which are needed in <the calculation of> solar eclipses.

The first angle: It is <in the case> when the position of the moon is the first <degree> of Aries or Libra, and <also> the <ecliptical> degree of the ascendant of a <given> time. It (i.e., the angle) is equal to the complement of the altitude of the beginning of Cancer or of Capricorn, whichever is on the meridian circle <above the horizon>. It is <called> 'the latitude angle,' and its complement <is called> 'the longitude angle.' **The second angle**: It is <in the case> when the position of the moon is the first <degree> of Aries or Libra, and <also> the <ecliptical> degree of the tenth <house> of the <given> time. It (i.e., the angle) is equal to the complement of maximum declination <of the sun>. It is <called> 'the latitude angle,' and its complement <is called> 'the longitude angle.' **The third angle**: It is <in the case> when the position of the moon is other than the first <degree> of Aries or Libra, and <also> the <ecliptical> degree of the ascendant of the <given> time. It (i.e., the angle) is equal to the altitude of the pole of the ecliptic at the <given> time. It is <called> 'the latitude angle,' and its complement is <called> 'the longitude angle.' **The fourth angle**: It is <in the case> when the position of the moon is the first <degree> of Cancer or Capricorn, and <also> the <ecliptical> degree of the tenth <house> of the <given> time. It (i.e., the angle) is a right angle. In this case, there is no longitude angle. **The fifth angle**: It is <in the case> when the position of the moon is other than the equinoxes or solstices, and <also> the <ecliptical> degree of the tenth <house> of the <given> time. Then we consider the declination of the <ecliptical> degree of the tenth <house> and the <geographical> latitude of the locality. If the declination is northern, we subtract the smaller <one of these two quantities> from the greater <one> (if $0<\varphi<\delta_1<\varepsilon$, then the tenth house is northern with respect to the zenith). If the declination is southern, we add it to the <geographical> latitude of the locality. The sum or the remainder is <equal to> the distance of the ecliptic from the zenith <along the meridian>. Then we divide the Sine of the altitude of the pole of the ecliptic by the Sine of the distance of the ecliptic from the zenith <along the meridian>, lowered. The result is the Sine of the latitude angle. We find the <corresponding> arc: it is the latitude angle; its complement is the longitude angle. **Another method**: We divide the Sine of the <right> ascension of the distance of the equinoctial point above the horizon from the meridian <,which is found> according to what <is described> in the relevant chapter (i.e., I.5.2), by the Sine of <the arc> between the tenth <house> and the equinoctial point of the ecliptic, lowered. The result is the Sine of the latitude angle. We find the <corresponding> arc: it is the latitude angle; its complement is the longitude angle. **The sixth angle**: It is <in the case> when the position of the moon is in an arbitrary <ecliptical> degree, between the ascendant and the descendant. Then we divide the Sine of the altitude of the pole of the ecliptic by the Cosine of the altitude of the <ecliptical> degree of the moon, lowered. The result is the Sine of the latitude angle. We find the <corresponding> arc: it is the latitude angle; its complement is the longitude angle.

Chapter 13: On <finding> the longitudinal and latitudinal parallax of the moon from these angles.

We multiply both the Sine of the latitude angle and the Sine of the longitude angle by the parallax in the altitude circle, lowered. The result from the latitude angle is the latitudinal parallax. The result from the longitude angle is the longitudinal parallax. If the distance of the moon from the zenith, when it reaches the meridian, is towards the south and the latitude of the moon is southern, or the distance is towards the north and the latitude of the moon is northern, we add the latitudinal parallax to the latitude. If they differ <in direction>, we subtract the smaller from the greater one. The result is the apparent latitude. Its direction is that of the sum of the latitude and the parallax, or the direction of the greater of the two. The place of the moon in most northern localities is southern with respect to the zenith.

Chapter 14: On <measuring> the absolute and adjusted magnitudes of a solar eclipse in digits.

If the latitude of the moon at the <time of> conjunction is southern, and more than 35 minutes, or northern and more than 95 minutes, the sun will not be eclipsed. If the latitude is less than that, it can be eclipsed. If it can be eclipsed, we find the time of the conjunction, its ascendant, the longitudinal parallax of the moon, and its apparent latitude <at the time>. Then we divide the longitudinal parallax by the lunar gain. The result is the <longitude> difference in hours. If the distance of the <ecliptical> degree of the <new> conjunction from the ascendant is less than 90 degrees, we subtract the <longitude> difference in hours from the time of the conjunction, and the longitude difference in minutes from the <ecliptical> degree of conjunction, and from the argument of the latitude, in order to learn the latitude from it. If the distance of the <ecliptical> degree of the <new> conjunction from the ascendant is greater than 90 <degrees>, we add the <longitude> difference in hours to the time of the conjunction, and the <longitude> difference in minutes to the <ecliptical> degree of the conjunction, and to the argument of the latitude. The sum or the remainder of the time of the conjunction is the time of apparent conjunction. The sum or the remainder of the <ecliptical> degree of the conjunction is the <ecliptical> degree of the apparent conjunction. We compute the ascendant for the time of the apparent conjunction. We obtain from this ascendant and from the <ecliptical> degree of the apparent conjunction, the apparent latitude of the moon and its longitudinal parallax. Then we divide <this> parallax by the lunar gain. The result is the second <longitude> difference in hours. If the distance of the first <ecliptical> degree from the ascendant of the apparent conjunction is less than 90 <degrees>, we subtract the <longitude> difference in hours from the first time of the conjunction, and the <longitude> difference in minutes <of arc> from the first <ecliptical> degree of the conjunction. If the distance of the first <ecliptical> degree of the conjunction from the ascendant of the apparent conjunction is greater than 90 degrees, we add the <longitude> difference in hours to the first time of the conjunction, and the <longitude> difference in minutes <of arc> to the first <ecliptical> degree of the conjunction. The sum or the remainder in hours is the time of the adjusted apparent conjunction and the time of the middle of the solar eclipse. The sum or the remainder in <ecliptical> degrees of the conjunction is the position of the moon in the middle of the solar eclipse (i.e., no further iteration is necessary). That is because if we derive this time of the conjunction from the ascendant and from the position of the moon in which the longitudinal parallax <is considered>, it (i.e., the observed time of the middle of the conjunction) will be equal to the second result or so close that <the difference> cannot be observed. When the time of the middle of the solar eclipse and its ascendant are found, we add the radii of the sun and the moon, which we call 'the <sum of the> two radii'. If the apparent latitude <of the moon> is equal to or greater than the <sum of the> two radii, the sun will not be eclipsed. If it is less <than the sum of the two radii>, the sun will be eclipsed. The excess of the <sum of the> two radii over the apparent latitude is the <magnitude of the> solar eclipse in minutes. We multiply the <magnitude of the> eclipse in minutes by 12 and divide it by the diameter of the sun. The result is the <magnitude of the> solar eclipse in digits. It is the eclipsed part of its diameter, based on <taking> the diameter <to be equal to> 12 digits. If the conjunction happens to be before sunrise, then the sun rises <in an> eclipsed <situation>. We <then> use the lower mid-heaven instead of the upper mid-heaven and we change the ascendant to the descendant in all the operations relating to the solar eclipse. Also, when we finish finding the <longitude> difference in hours and the <longitude> difference in minutes <of arc>, we always subtract the <longitude> difference in hours from the time of the conjunction and the <longitude> difference in minutes <of arc> from the <ecliptical> degree of the conjunction. Finding the adjusted magnitude of a solar eclipse is like finding the adjusted magnitude of a lunar eclipse, both in calculation and <in using> tables; the disk of the moon in this case plays the role of the disk of the shadow in that case, and the disk of the sun in this case plays the role of the disk of the moon in that case.

Chapter 15: On the absolute and adjusted times of a solar eclipse.

We subtract the square of the apparent latitude <of the moon> in the middle of the solar eclipse from the square of <the sum of> the two radii, and we obtain the square root of the remainder. The result is the <arc of> immersion in minutes. Then we divide it by the lunar gain. The result is the <duration of> immersion in hours. We subtract it from and <also> add it to the time of the middle of the solar eclipse. The remainder is the time of the beginning of the solar eclipse, and the sum is the time of the end of the emersion. Finding the adjusted magnitude of these two times is like finding the adjusted magnitudes of the times relating to the lunar eclipse, where we substitute the apparent latitude <of the moon> here for the absolute latitude <of the moon> there. The solar eclipse has no (i.e., zero) duration <of totality>.

Chapter 16: On drawing the figure of a solar eclipse.

We draw a straight line <segment> of arbitrary length. We divide it by the number of the minutes of <the sum of> the two radii. We draw a circle with a radius equal to that <line segment>, so that its radius will be equal to this line segment. It is <called> 'the circle of <the sum of> the two radii'. We draw two of its diameters which intersect in the center at right angles. We write around it the four directions: east opposite to west, and north opposite to south. Then we take <a part> from the line <segment> equal to the radius of the sun, and we draw a circle with a radius equal to that <part> and centered at the center of the circle of <the sum of> the two radii. It is <called> the circle of the sun. Then we take <a part> from the line <segment> equal to the apparent latitude. We put one arm of the compasses on the center of the two circles, and the other <arm> where it occurs on the north or south line, depending on the direction of its apparent latitude. We make a mark there to stand for the center of the moon in the middle of the solar eclipse. Then we take <a part> from the line <segment> equal to the radius of the moon. We take the mark as the center and draw the circle of the moon around it. The portion of the circle of the sun which falls in the circle of the moon is the amount of its (i.e., the sun's) eclipsed part.

Chapter 17: On <finding> the altitude of the moon taking account of its latitude.

Ptolemy and the experts in the art <of astronomy> who followed him, all calculated the magnitude of the parallax of the moon in the altitude circle and the measures of the six angles, which we have <already> mentioned, assuming the moon to have no latitude at all. They found the longitudinal and latitudinal parallax <of the moon> by substituting straight lines for the arcs of small circles. There is no noticeable disadvantage in what they did to <find> the latitude, except that precision has superiority to approximation, and exactness is more accepted <by people> than approximation. It was possible for us <to find> a method with proof which is not much different from the first <method> in difficulty and length, by which the altitude of the moon, its apparent latitude, and its longitudinal parallax is determined taking account of its latitude. It is <as follows:> We multiply the Cosine of the latitude by the Cosine of the distance of its <ecliptical> degree from the ascendant or descendant of the <given> time, whichever being less than 90 <degrees>, lowered. The result is a Sine. We find the <corresponding> arc, and subtract <the arc> from 90 <degrees>. The remainder is <called> 'the first arc'. Then we divide the Sine of the latitude by the Sine of 'the first arc', lowered. The result is <called> 'the Sine of the second arc'. We find the <corresponding> arc. If the latitude is northern, we add this arc to the complement of the altitude of the pole of the ecliptic. If the latitude is southern, we subtract it (i.e., the arc) from it. The sum or the remainder is <called> 'the result from the complement of the altitude of the pole'. Then we multiply the Sine of 'the first arc' by the Sine of 'the result from the complement of the altitude of the pole', lowered. The result is the Sine of the altitude <of the moon or the planet or star> taking account of the latitude of the moon or the other planet or star which has a <non-zero> latitude. The parallax in the altitude circle is obtained from this altitude.

Chapter 18: On <finding> the longitudinal and latitudinal parallax of the moon by a method <the validity of> which can be proved.

We have said in chapter 11 of this section that the altitude obtained by calculation is the true altitude that we would find if we observed from the center of the altitude circle. <If> the parallax is subtracted from it, <the remainder> is the apparent altitude <as observed> from the surface of the earth. Following what we have mentioned, <we add that the subject of> this chapter may occur in 5 cases. **First**: <The case in> which the altitude of the tenth <house> of the <given> time is 90 degrees, and the moon has no <non-zero> latitude. <Then> the parallax in the altitude circle is <the same> longitudinal parallax alone. It (i.e., the moon) has no latitudinal parallax. **Second**: <The case in> which the distance of the <ecliptical> degree of the moon from the ascendant of the <given> time is 90 degrees, the moon either having or not having a <non-zero> latitude. Then the parallax in the altitude circle affects the apparent latitude alone. It has no longitudinal parallax. **Third**: <The case in> which the altitude of the tenth <house> of the <given> time is 90 degrees, and the moon has a <non-zero> latitude. <For finding> the apparent latitude, we multiply the Sine of the latitude of the moon by the Cosine of the apparent altitude, and we divide <the product> by the Cosine of the true altitude. The result is the Sine of the apparent latitude. Its direction is the same as that of the latitude of the moon. As for the longitudinal parallax, we divide the Sine of the apparent altitude by the Cosine of the apparent latitude, lowered. The result is <considered> a Sine. We find the <corresponding> arc, and subtract it (i.e., the arc) from the distance of the <ecliptical> degree of the moon from the ascendant or descendant (whichever is closer). The remainder is the longitudinal parallax. **Fourth**: <The case in> which the altitude of the tenth <house> of the <given> time is less than 90

<degree>, and the moon has no <non-zero> latitude. <For finding> the apparent latitude, we multiply the Sine of the parallax in the altitude circle by the Sine of the altitude of the pole of the ecliptic, and divide <the product> by the Cosine of the true altitude. The result is the Sine of the apparent southern latitude. <For finding> the longitudinal parallax we divide the Cosine of the parallax in the altitude circle by the Cosine of the apparent latitude, lowered. The result is the Cosine of the longitudinal parallax. **Fifth**: <The case in> which the altitude of the tenth <house> is less than 90 <degrees>, and the moon has a <non-zero> latitude. <For finding> the apparent latitude, we multiply the Cosine of the latitude of the moon by the Cosine of the arc between its <ecliptical> degree and the ascendant or descendant of the <given> time, whichever is less than 90 <degrees>, lowered. The result is <called> 'the Sine of the first arc'. We multiply it by the Cosine of the apparent altitude. We divide <the product> by the Cosine of the true altitude. The result is <called> 'the Sine of the second arc'. We find the <corresponding> arc. Then we divide the Cosine of the apparent altitude by 'the Sine of the second arc', lowered. The result is <called> 'the Sine of the third arc'. We find the <corresponding> arc. We obtain the difference between it and the complement of the altitude of the pole of the ecliptic. The result is <called> 'the fourth arc'. We multiply the Sine of 'the fourth arc' by the Cosine of 'the second arc', lowered. The result is the Sine of the apparent latitude. If 'the third arc' is greater than the complement of the altitude of the pole of the ecliptic, the direction of the latitude is northern. If 'the third arc' is less <than that>, the direction of the latitude is southern. <For finding> the latitudinal parallax, we divide the Sine of the second arc by the Cosine of the apparent latitude, lowered. The result is <called> 'the Sine of the first arc'. We find the <corresponding> arc and keep it. Then we subtract <the distance> between the <ecliptical> degree of the moon and the ascendant or descendant, whichever is less than 90 <degrees>, from 90 <degrees>. We subtract the remainder from 'the first arc' which we kept. The <final> remainder is the longitudinal parallax.

Chapter 19: On extracting the longitudes of the localities.

We calculate a solar eclipse for the <geographical> longitude 90 <degrees>, and we find the time of its beginning or the time of the end of its emersion. Then we observe one of these two times in our locality, as accurately as possible. We obtain the altitude <of the sun> for this time. We find the <local> time from it. If the observed time is greater than the calculated <one>, then our locality is eastern with respect to <the meridian of> longitude 90 <degrees>. If the observed time is less <than

that>, then our locality is western with respect to <the meridian of> longitude 90 <degrees>. The difference between the calculated and the observed times is the <difference in> hours between the two longitudes. We multiply it by 15. The result is the longitude <difference> between the two localities. If our locality is eastern, we add it to the latitude 90 <degrees>. If it is western, we subtract it from the latitude 90 <degrees>. The sum or the remainder is the longitude of our locality. In a lunar eclipse, the altitude of the moon is not faultless because of its parallax, and it is difficult to obtain the precise altitude of the fixed stars, and we cannot rely on their true position. So, we may obtain the altitude of a planet whose true position we know. Then this planet will be <useful> like the sun in what we require. **Another method** is <as follows>. We find the true longitude of the sun for noon of the <given> day, relating to the <geographical> longitude 90 <degrees>. Then we observe its altitude at noon of that day <in our locality> with one of the reliable, precise devices for <measuring> the altitude. If the sun is in <one of> the northern <zodiacal> signs, we subtract the complement of the latitude of our locality from the altitude which was found. If the sun is in <one of> the southern signs, we subtract the altitude which was found from the complement of the altitude of the locality. The remainder is the declination of the sun. We find its <argument> arc in the table of declinations relating to the quadrant in which the sun <occurs>. The result is the position of the sun in our locality. We obtain the difference between it and the first true longitude. We enter it (i.e., the difference) in <the table for> the mean motion of the sun in <any number of> hours, and we obtain the corresponding <number of> hours. The result is the <difference> between the two longitudes in hours. We multiply it by 15. The result is the <difference> between the two <geographical> longitudes in degrees. If the position of the sun in our locality is <relating to a> smaller <zodiacal longitude> compared with its first position, our locality is eastern with respect to <the meridian of> longitude 90. Then we add <the difference> between the two <geographical> longitudes to 90 <degrees>. If its position in our locality is <relating to a> greater <zodiacal longitude>, then our locality is western with respect to <the meridian of> longitude 90 <degrees>. Then we subtract <the difference> between the two <geographical> longitudes from 90 <degrees>. The sum or the remainder is the <geographical> longitude of our locality. Whenever the sun is closer to the equinoctial points, <the result> is more correct because the declination is more distinct and its increments are greater here. **Another method**, used by the ancients by approximation, and from which are derived the <geographical> longitudes of most localities in the books and tables, is

<as follows>. We consider the <distance> between our locality and a locality of known <geographical> longitude and latitude in parasangs, and the <number of the> days <to travel> the road <between them>. Then we take one degree for each two days of <travelling> the road or for each 20 parasangs. We multiply <the number of the parts> by itself, bisect the result, and keep it. If the latitudes of the two localities are equal, we obtain the approximate square root of the bisected result. The <final> result is the longitude <difference> between the two localities. If the latitudes of the two localities are different, we subtract the less from the greater. We multiply the remainder by itself and subtract it from the bisected result. We obtain the square root of the remainder. The result is the longitude <difference> between the two localities. This is something obtained by approximation, not based on proof. A table is compiled <in this *zīj*> for the longitudes and latitudes of some localities. We have registered the famous localities in it, so that they (i.e., their coordinates) may be known approximately.

Chapter 20: On <determining> the visibility of the <lunar> crescent and the planets from <certain> arcs defined for them.

As for the <lunar> crescent visibility, none of the ancients spoke about it, because they knew (i.e., defined) the beginnings of the lunar months from the conjunctions. Each one of the moderns, when they needed the <anticipation of the lunar> crescent visibility for Islamic religious observances, worked out a chapter and a calculation <method> in this <matter> according to his own belief. For most inhabited <regions>, there is nothing general in it (i.e., their speculation about lunar crescent visibility) that can be relied upon. Their calculation thereof is not based on any valid rule and principle, and is not immune to the mistakes relating to the <degree of> clearness of the atmosphere and sharpness of the eyes <of the observer>. <Our method for> it is <as follows:> We obtain the distance between the two luminaries taking account of the latitude <of the moon>. This is <described> in chapter 5 of section 8 <of Book I of this *zīj*>. <We also obtain> the distance between the sun and the degree <on the ecliptic> which sets at the same time as the moon, <measured> in terms of descension degrees. The limit of the first arc is 10 degrees and of the second arc 8 degrees. Then we find the altitude of the moon at sunset or sunrise taking account of its latitude. We subtract from the result the parallax <of the moon> in the altitude circle. The remainder is <called> the 'visibility arc'. Its limit is 6 degrees. If the three arcs are <equal to> the above mentioned limits or greater than them, then the <lunar> crescent is visible. If they are less <than the

limits, then the lunar crescent> is not visible. If two of them witness to the visibility, then the judgment should be <based> on them. The difficult visibility will be related to the <third> deficient arc. If <the deficient arc> is the visibility arc, then <difficulty in visibility> is because of <small> altitude. If <the deficient arc> is the distance between the two luminaries, then it (i.e., the difficult visibility) is because of light insufficiency. If <the deficient arc> is the distance between them in terms of descension, then it (i.e., the difficult visibility) is because of the too small time it (i.e., the moon) remains above the horizon <after sunset>, and of the speed of its setting. **Another method**: We multiply the Sine of <the arc> between the <ecliptical> degree of the sun and the degree on the ecliptic which sets at the same time as the moon, by the Cosine of the pole of the ecliptic, lowered. The result is the Sine of the visibility arc under the earth (i.e., below the horizon). If <the sum of> the arc of visibility and the distance between the two luminaries taking account of the latitude <of the moon is> 18 <degrees> or more than that, then the <lunar> crescent will be seen. If it is less <than 18 degrees, then the lunar crescent> will not be seen.

The calculation of the visibility of the planets is similar to this. But in <the calculation of> their visibility we do not need <to consider their> parallax, and the distance between them and the sun. Their altitude is known (i.e., found) taking account of their latitudes. If that (i.e., the altitude) is <at least> equal to 'the visibility arc', <then the planet> is visible. If it is less, <then the planet> is not visible. The visibility arcs <for the planets> according to what was found in ancient <times is as follows:> For Saturn, 11°; for Jupiter, 10°; for Mars, 11; 30°; for Venus 5° and Mercury, 10°.

Commentary

I.6.1 The second method is equivalent to the first one, because multiplication by 10 minutes is equivalent to division by 6. The Arabic term *buht* for the daily motion of a planet is derived from the Sanskrit *bhukti* [al-Bīrūnī, 1910, II, 346; id., 1934, 105-106]. Table II.49 gives the hourly rate of the two luminaries as a function of their mean anomaly.

I.6.2 Multiplication of the daily motion by 0;33° is equivalent to multiplication of the hourly motion by $13\frac{1}{5}$, because 24 (0; 33°) = $13\frac{1}{5}$. The two methods are equivalent for the moon too, because 24(0; 2, 26°) = 0; 58, 25°. According to Kūshyār, the parameters 0; 33° and 0; 2, 26° are the constant ratios of the apparent diameter of the sun and the moon to their daily motion. Kūshyār's assumption of a constant ratio between diameter and daily motion is not accurate. According to Kepler's model, the ratio of $(\text{diameter})^2$ to daily motion is constant. Kūshyār's assumption of a constant ratio between the apparent diameter of the moon and the diameter of the earth's shadow is approximately true. The rules given in this chapter are found in al-Khwārizmī's *zīj* and may be traced beck to the Indian *zīj* Khandakhādyaka [al-Khwārizmī 1962, 57-59]. Table II.49 gives the diameter of the sun, the moon, and the shadow as functions of their mean anomaly.

I.6.3 Multiplication by 5 minutes is equivalent to division by 12. Here, for a first approximation of the ecliptical degree where the conjunction or opposition occurs, the hourly motion of the moon is taken 12 times that of the sun. Then if the distance of the sun and the moon on the ecliptic is d, we first suppose that they will be in conjunction after the sun has covered $d/12$ degrees and the moon has covered $d + d/12$ degrees. This method is described by Ptolemy [1984, 281-82], and by al-Battānī [1899-1907, III, 142]. However, they divide $d + d/12$ by the hourly motion of the moon, whereas Kūshyār divides d by the 'lunar gain'. The requirement that the sun and the moon should be in the same second is an exaggerated accuracy. By the expression "hours of the noon", Kūshyār means the number of the equinoctial hours elapsed since sunrise, at noon.

Worked example: On the day of conjunction, the sun is in Aries 24;59° and the moon is in Aries 26;39°. The distance between the two luminaries is: 26;39° – 24;59° = 1;40°.

The 'part of the distance' is: 1;40°×0;5=0;8,20°. The distance plus 'the part of the distance' is: 1;40°+0;8,20°=1;48,20°. Now we can find the ecliptical degree of the conjunction both by subtracting the 'part of the distance' from the longitude of the sun and by subtracting the distance plus 'the part of the distance' from the longitude of the moon:

24;59° - 0;8,20° = 24;50,40°,

26;39° - 1;48,20° = 24;50,40°.

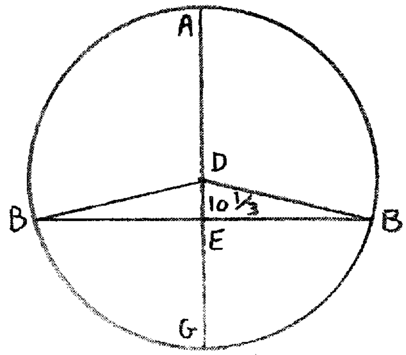
Then the ecliptical degree of the conjunction is Aries 24;50,40°.
Since in 30 days, the sun describes 30°, and the moon describes 30°+360° approximately, the lunar gain is about 12°/day or 30′/hour. We divide the distance by the lunar gain: 1;40°: 30′=3; 20. So the conjunction occurs 3 hours and 20 minutes before the noon, approximately. This example is based on al-Nasawī's example for this chapter (fol. 69 v). Al-Nasawī takes the lunar gain equal to 0;28,2°, so he finds the time of the conjunction 3 hours and 34 minutes before the noon (see the commentary on I.1.5). As can be seen in this example, the is no need to repeat the process of finding the time of the conjunction (as Kūshyār says at the end of this chapter).

I.6.4 Kūshyār expresses the magnitude of a lunar eclipse in two different ways. The 'absolute' magnitude of the eclipse is the length of the part of the line segment between the center of the moon and the center of the shadow which is inside both circles. The 'absolute' magnitude is expressed in (linear) digits, where 12 digits correspond to the diameter of the lunar disk. The 'adjusted' magnitude of the eclipse is the area of the common part of the two circles. The 'adjusted' magnitude is expressed in (area) digits, where 12 digits correspond to the area of the full moon [Kennedy 1956, 143]. The difference between 'the arc of the moon<'s disk>' and 'the arc of the sector' is only in the units of measurement. However, for finding 'half the arc of the sector', we should multiply "half" the circumference of the moon's disk (and not the whole circumference, as Kūshyār says) by the arc of the moon's disk, and divide the product by 360 degrees. Kūshyār proves his method in IV.6.1. Similar linear and area digits may be defined for a solar eclipse. Ptolemy used a similar method of finding the area digits for a special case [1984, 302-305]. Al-Battānī also used a similar method [1899-1907, III, 149-50]. Table II.52 of Kūshyār's *zīj* gives the adjusted digits as a function of absolute digits, from 1 to 12, for solar and lunar eclipses. It is a reproduction of the relevant table given by Ptolemy [1984, 308]. The same table is provided by al-Battānī [1899-1907, II, 890].

I.6.5 In IV.6.2, Kūshyār proves the validity of the calculation of the immersion time from the beginning of the partial eclipse and from the beginning of the total eclipse to the middle of the eclipse. In this chapter, 'adjusted value' means 'more precise value' for these times, in which the variation of the latitude of the moon is also considered. A proof of this more precise method is given in IV.6.3. The first method is provided by Ptolemy [1984, 300-301]. Both methods are presented by al-Battānī [1899-1907, III, 147-48].

I.6.6 A sample of such a drawing is provided in IV.6.4. A similar drawing is provided by al-Battānī [1899-1907, III, 154].

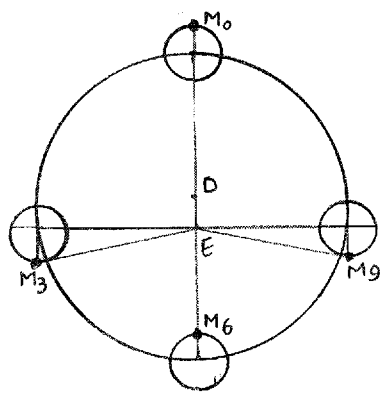
I.6.7 Kūshyār first finds the distance of the center of the moon's epicycle from the earth, where the maximal distance is assumed to be 60 "parts". Kūshyār's model for lunar motion is the same as that of Ptolemy; see [Pedersen 1974, 159-202] and [Ptolemy 1984, 173-254]. For the cases in which the double elongation of the moon is 0°, 6 zodiacal signs, and 3 or 9 zodiacal signs, the figure below justifies Kūshyār's calculation. The parameter 39⅓ is a rounding of the Ptolemaic one 39;22 parts [Ptolemy 1984, 251]. The parameter 5;1° is the maximum value of the epicyclic equation.



If the double elongation is between these values, in the figure drawn in IV.6.5, *ZD* is 'the product of the Sine' and *ZE* is 'the product of the Cosine'. Then it can be deduced from the figure that,

$$(BD^2 - ZD^2)^{\frac{1}{2}} = BZ\,,\ EB = BZ + ZE.$$

Again, if the true anomaly of the moon is 0°, 6 zodiacal signs, and 3 or 9 zodiacal signs, the figure below justifies Kūshyār's calculation of the distance of the body of the moon from the earth.



If the true anomaly is between these values, then in the figure of IV.6.5, *IH* is 'the product of the Sine' and *IB* is 'the product of the Cosine'.

The computation of an "adjusted radius of the epicycle" in I.6.7 seems to be superfluous; instead of this one can simply take the radius of the epicycle, as in the proof in IV.6.5. We may also use table II.50 instead of calculation. A simplified version of this method may be used for the sun; however, it is rarely necessary, according to Kūshyār. As Kūshyār mentions in Chapter 11 of this section, 'part' means the unit of measurement, which is equal to the radius of the earth. The coefficient $18\frac{4}{5}$ is the ratio of the mean distance of the sun from the earth to the maximum distance of the moon from the earth. Kūshyār mentions this value, as well as the maximum, mean, and minimum distance of the sun from the earth, in the chapter 'On the distances and sizes of the celestial bodies', in Book III of this *zīj*. Ptolemy [1984, 250-51] and al-Battānī [1899-1907, III, 81-82] provide similar methods but for concrete numerical values. Kūshyār tacitly assumes that the distance of the apogee of the center of the lunar epicycle to the earth is 60 earth radii (Ptolemy's value is 59 earth radii).

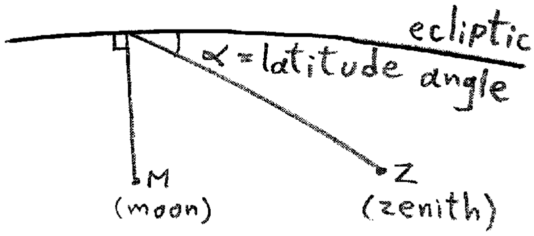
I.6.8 A proof for this method is given in IV.6.6. The "latitude of the clime of visibility" or "the latitude of visible climate" (*ʿarż iqlīm al-ruʾya*) was a standard term in Islamic spherical astronomy and probably taken over from the Hindus. It is the shortest distance from the zenith to the ecliptic, or the complement of the angle between the ecliptic and the horizon [Kennedy 1983, 168, 290].

I.6.9 A proof for this method is given in IV.6.7.

I.6.10 The case when the "remainder" in the last sentence is greater than 90° cannot occur. Kūshyār omits the cases where the point is between the ascendant and the fourth house, or between the fourth house and the descendant. Then the distance is between the right ascension of the point and the lower half of the meridian. There is no proof for this method in IV.6. The term 'ascension' in the title must refer to right ascension, because the latitude of the locality is not involved in this method. This chapter is not found in L. It is not mentioned in the table of contents at the beginning of B; however, the chapter belongs to the missing part of B. It is not mentioned in the table of contents of P, but the chapter itself is found in P. The term 'ascension' is missing in the title of the chapter in P.

I.6.11 A proof for this method is given in IV.6.8. Similar methods are provided by Ptolemy [1984, 256-64] and al-Battānī [1899-1907, III, 118]. The values for the solar parallax are given in table II.51.

I.6.12 This chapter provides the computation of the smaller angle between the ecliptic and the arc of the great circle passing through the zenith and the ecliptical position (perpendicular projection) of the moon (see the figure below). This angle is called the 'latitude angle' and its complement is called the 'longitude' angle. Kūshyār discusses six cases. He first treats the five special cases where the moon is situated on one of the equinoxes or solstices, and is in rising or setting position or at its culmination point, respectively. His sixth case is the general case. In 'the other method' provided in the fifth case, he first finds the complement of the right ascension of the equinoctial point (*KM* in the figure of IV.6.9) using the method provided in I.5.2. L and Y give only 'the other method' for the fifth case, but P gives both methods. Ptolemy [1984, 123-29] provides a special table for this angle, which is entitled "Table of zenith distances and ecliptic angles". Al-Battānī [1899-1907, III, 115] discusses different cases of this angle. See also [Pedersen 1974, 118-121, Neugebauer 1975, 48-50].



I.6.13 A geometrical proof of this method is given in IV.6.10. Al-Battānī presents a calculation method for this subject [1899-1907, III, 115], which is also given by Ptolemy [1984, 266]. This method presupposes that the relevant spherical triangles are considered as plane triangles. Kūshyār's rule for addition or subtraction of the latitudinal parallax is valid when the moon reaches the meridian. But when the moon is close to the horizon, especially near the equinoxes, so that a southern latitude may lead to an increase in the altitude, the rule is not valid.

I.6.14 In the description of the conditions in which the sun may be eclipsed, Kūshyār assumes that the case in which the distance of the moon from the zenith is northern (mentioned in Chapter I.6.13) will not occur. For finding the precise time of the middle of a solar eclipse, the time of conjunction is adjusted in two stages for the longitudinal parallax of the moon. The procedure for finding the magnitude of a solar eclipse in linear digits and area digits is like the procedure for lunar eclipses. Similar methods are provided by Ptolemy [1984, 310-13] and al-Battānī [1899-1907, III, 157-58, 164-65].

I.6.15 This method is similar to the method for lunar eclipses. See al-Battānī [1899-1907, III, 162].

I.6.16 An example of such a drawing is found in IV.6.11. Al-Battānī [1899-1907, III, 171] presents a similar drawing.

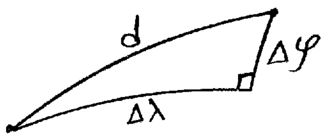
I.6.17 In IV.6.12, Kūshyār proves his precise method of finding the altitude of the moon. In the figure for Chapter IV.6.12, *IK* is the latitude of the moon, *IB* is 'the first arc', *HZ* is 'the second arc', and *ZA* is 'the result from the complement of the altitude of the pole <of the ecliptic>'. There is "another method" in this chapter which is not correct and authentic, so it has been omitted here. See the commentary to IV.6.12. The maximum difference in the parallax of the moon due to considering the latitude of the moon is equal to 5 minutes which is not negligible.

I.6.18 This is the continuation of the preceding chapter and contains Kūshyār's method for finding the precise values of the latitudinal and longitudinal parallax of the moon. This method that 'can be proved' is more precise than the method in I.6.13 which is approximate. By 'true altitude' he means the altitude of the moon if it is observed from the center of the earth. The first two cases are self-evident. In the third and fourth cases, F and C also give alternative methods for finding the longitudinal parallax, but the alternative methods are not correct and authentic. Y, B, and P give only the first method for finding this quantity, while L only gives the alternative method. So we have only quoted the first method here. F, C, V, Y, and even L provide the proof for the first method in the third and fourth cases. F, C and L give the proofs for the third, fourth, and fifth cases in IV.6.13.

I.6.19 Kūshyār presents three methods for finding the longitude difference between a given locality and another locality whose longitude and latitude are known. None of them is acceptable. The first method does not take into account the non-simultaneity of the solar eclipse phases as seen from different localities. Of course, finding the altitude of a planet at some moment (e.g., the beginning of emersion in lunar eclipses) in two localities is a practical way to calculate the geographical longitude difference. The second method does not consider that the change in the declination of the sun during a few hours is negligible. However, it is in principle correct, though not practical. As I will show below, the third method could be acceptable if we divided the distance in parasangs by 10 instead of 20, at the beginning. The length of an arc of one degree of a great circle on the earth is:

$$(40'000'000:6000)/360 \approx 18 \text{ parasangs}$$

For two localities with the same geographical latitude, we take the arc on the parallel circle for this latitude. Kūshyār's method in this case is equivalent to division by $10\sqrt{2}$. The length of an arc of one degree on the parallel circle is 18cosφ parasangs, where φ is the geographical latitude of the two localities. Then $10\sqrt{2}=18\cos\varphi$, and φ=38°13'. This is approximately for the geographical latitude for Kūshyār's position. For the case in which the geographical latitudes are different, Kūshyār simply applies the Pythagorean rule for the sides of a right triangle whose hypotenuse is the arc between the two localities and whose other sides are the arcs along the latitude parallel circle and the terrestrial meridian.



I.6.20 In this chapter, two methods for determining lunar crescent visibility are presented. The first method is based on the minimal values of three parameters: the distance between the two luminaries (10°), the distance between the sun and the ecliptical degree which sets simultaneously with the moon (8°), and the altitude of the moon at sunset in terms of its latitude (visibility arc) (6°). Kūshyār's second method is based on two parameters: the visibility arc below the horizon, and the distance between the two luminaries. The second method requires that the sum of these two parameters should not be less than 18°. Kūshyār presents here a rule for finding the visibility arc in the second method. A proof for this rule is given in IV.6.14. There Kūshyār says that the minimum values of the visibility arc have been found "from 6; 30° to 7°". B gives only the first method, and L mentions only the second method. Ptolemy dose not discuss lunar crescent visibility. The minimal values of the visibility arc for the planets in Kūshyār's *zīj* are the same as those given by Ptolemy [1984, 639-40]. Al-Battānī [1899-1907, III, 133] provides a method similar to Kūshyār's first method, but based on the first two parameters only. Al-Battānī does not use the visibility arc. He gives a greater minimal value for the two parameters: 13; 40° instead of 10° for the first parameter and 10; 50° instead of 8° for the second one. In his *Zīj-i Sanjarī* (no. 27 in Kennedy's *Survey*), Abū al-Fatḥ 'Abd al-Raḥmān al-Khāzinī who lived about a century after Kūshyār, follows Kūshyār's methods for lunar crescent visibility. But al-Khāzinī provides the minimal values of the parameters as intervals rather than single values, considering the position of the moon in its orbit around the earth: 10°-12° (instead of 10°) for the first arc; 8°-12° (instead of 8°) for the second arc; and 6°-8° (instead of 6°) for the third arc. For the visibility arc below horizon, he mentions the interval 8°-10° instead of Kūshyār's 6;30°-7° (MS 682, ex-Sepahsālār library, Tehran, fol. 18r).

Section 7: On the operations relating to astrology, <in> 6 chapters

Chapter 1: On <finding> the distance between the <ecliptical> degree of a planet and the cardines in <terms of> hours.

If the planet is above the earth, we obtain its distance from the <cusp of the> tenth <house>, <whether it is situated> before or after <the tenth house>, <measured> on the <basis of the> right ascensions. If it is under the earth, we obtain its distance from the fourth <house>, <whether it is situated> before or after <the fourth house>, <measured> on the <basis of the> right ascensions. Then, if the planet is above the earth, we divide the distance by the <number of the> degrees in each hour for the <ecliptical> degree of the planet. If the planet is under the earth, we divide the distance by the <number of the> degrees in each hour for the <degree> opposite the <ecliptical> degree of the planet. The result is the distance of the planet from the tenth <house> or the fourth <house> cardine, <whether it is situated> before or after <the cardine>, in seasonal hours. If these <number of> hours are subtracted from 6, the remainder is the distance from the ascendant or the descendant.

Chapter 2: On <finding> the projection of the ray by means of equal (i.e., ecliptical) degrees.

Projections of the rays <measured> in equal degrees are the arcs of the ecliptic whose magnitudes are 60°, 90°, 120°, and 180°. If the planet has <non-zero> latitude, these arcs are obtained from a circle passing through the planet. Then they are transferred to the circle of the ecliptic. This is <found as follows:> We divide the Cosine of 60°, i.e. the Sine of 30°, by the Cosine of the latitude of the planet, lowered. Then we find the arc <corresponding to the quotient>. The result is the Sine of the difference between 90° and the arc of the sextile <ray> or <that of> the trine <ray>. We subtract it from 90°; the arc of the sextile <ray> remains. We add it (i.e., the difference) to 90°; the sum is the arc of the trine <ray>. As for <the arc of> the quartile <ray>, it is always 90°, and the <arc of> opposition is always 180°.

Chapter 3: On <finding> the projection of the ray by means of ascension (i.e., equatorial) degrees.

This is <similar> to the calculation of the equalization of the houses, except that it is found for ascensions of the horizon of the planet (i.e., the great circle through the planet and the North and South points of the horizon), in the same way that the equalization of the houses is

<calculated> for the ascensions of the horizon of the locality. The astrologers unanimously accept this <method of> equalization. If that is correct, it is suitable to correctly compute the projections of the rays by this calculation <method>. For this <purpose>, we need to know the <number of> degrees in each hour for the <ecliptical> degree of the planet, based on its position <with respect to the horizon>. To do this, we check the <ecliptical> degree of the planet. If it is <the same as> the degree of the <cusp of the> tenth <house> or that of the <cusp of the> fourth <house>, then the <number of the> degrees in each hour for them is 15. If it (i.e., the ecliptical degree of the planet) is <the same as> the degree of the ascendant or descendant, then the <number of the> degrees in each hour for them is <equal to the number of the> degrees in each hour for the ascendant or the descendant (see I.5.11). If it is between the two cardines, we take the difference between <the number of> the degrees in each hour of its (i.e., the planet's) <ecliptical> degree and 15, multiply it by the distance of the <ecliptical> degree from the cardine of the <cusp of the> tenth <house> or the <cusp of the> fourth <house> (as described in I.7.1), and divide it by 6. The result is the equation. If the <ecliptical> degree is between the <cusp of the> tenth <house> and the ascendant, or in the quadrant opposite to this one, and if 15 is the greater value, we subtract the equation from it; otherwise, we add the equation to it (i.e., to 15). If the <ecliptical> degree is between the ascendant and the <cusp of the> fourth <house> or in the quadrant opposite to this one, and if the <number of> degrees in each hour for the <ecliptical> degree is the greater value, we subtract the equation from it; otherwise, we add the equation to it. The result is the <number of> degrees in each hour for the <ecliptical> degree of the planet based on its position. Then we obtain the right ascension of the <ecliptical> degree of the planet, and we subtract from it the <number of> degrees in each hour for it, multiplied by 4. We find the arc corresponding to the remainder in the <table for the> right ascensions. The result is the position of the left sextile. The right trine is opposite to it. We also subtract from the right ascension of the <ecliptical> degree of the planet the <number of> degrees in each hour for it, multiplied by 6. We find the arc corresponding to the remainder in the <table for the> right ascensions. The result is the position of the left quartile. The right quartile is opposite to it. Then, we subtract the <number of the> degrees in each hour for the <ecliptical> degree from 30. We multiply the remainder by 4 and add it to the right ascension of the planet. We find the arc corresponding to the result in <the table for> these ascensions. The <final> result is the position of the right sextile. The left trine is opposite to it. This expression, i.e. the projection of the ray, has no correct meaning other than one of <the meanings given in> these two chapters.

Chapter 4: On <finding> the prorogations (i.e., astrological progressions).

The prorogations are of four types. One of them is 13 <ecliptical> signs in a solar year. It is <called> the minor prorogation, because it has the highest speed. The second <type> is one <ecliptical> sign in a solar year. It is <called> the medium prorogation. The third <type> is one ascensional degree in a solar year. It is <called> the major prorogation, because it has the lowest speed. The fourth <type> is the prorogation of the transfer indicators, like the mean motion of the sun. It is <called> the transfer prorogation. We have compiled two tables for the minor and medium prorogations, from which the degrees in the table may be obtained for any given <number of> months and days; or the <number of> months and days <may be obtained> for any given <number of> degrees.

The transfer prorogation is known from the table for the mean longitudes of the sun. For the major prorogation, we need <to carry out some> operation. Its calculation is <as follows:> We check if the <ecliptical> degree to be moved by prorogation is the degree of the <cusp of the> tenth or fourth <house>; or, if the <ecliptical> degree of the planet is in them, we subtract the right ascension of the <cusp of the> tenth or fourth <house> from the <right> ascension of the <ecliptical> degree in which the prorogation ends. <We take> one year for each degree, and 6 days for each minute of the remainder. During such <number of> years and days, the <ecliptical> degree moved by prorogation will reach the <degree> in which the prorogation ends. If the <ecliptical> degree moved by prorogation is the <ecliptical> degree of the ascendant, or if the <ecliptical> degree of the planet is in it, we subtract the oblique ascension of the ascendant from the <oblique> ascension of the <ecliptical> degree in which the prorogation ends. <We take> one year for each degree, and 6 days for each minute of the remainder. If the <ecliptical> degree <moved by prorogation> is the <ecliptical> degree of the descendant, or if the <ecliptical> degree of the planet is in it, we subtract the oblique ascension of the ascendant from the <oblique> ascension of the opposite of the <ecliptical> degree in which the prorogation ends. <We take> one year for each degree, and 6 days for each minute of the remainder.

If the <ecliptical> degree moved by prorogation is between two cardines, we obtain the right ascension and the oblique ascension of that <ecliptical> degree. We multiply the difference between the two ascensions by the distance of the <ecliptical> degree of the planet from the cardine rising prior <to it> in hours (as described in I.7.1). We divide <the product> by 6. The result is the equation. If the <ecliptical> degree is between the <cusp of the> tenth <house> and the <cusp of the> fourth <house>, or in the quadrant opposite to this, and the right ascension has the greater value, we subtract the equation from it; otherwise, we add the equation to it. If the <ecliptical> degree is between the ascendant and the <cusp of the> fourth <house>, or in the quadrant opposite to this, and the oblique ascension has the greater value, we subtract the equation from it; otherwise, we add the equation to it. The result is the ascension of the <ecliptical> degree, based on its position.

Then we find the ascension of the <ecliptical> degree in which the prorogation ends, through a similar operation. However, we use in it the distance of the first <ecliptical> degree moved by prorogation (instead of the ecliptical degree of the planet) from the cardine which was used formerly, in hours. We use the ascension <here> as we used it in that <operation>. Then we subtract the ascension of the <ecliptical> degree moved by prorogation from the ascension of the <ecliptical> degree in which the prorogation ends. <We take> one year for each degree, and 6 days for each minute of the remainder.

If the <period of> time is known, and we want to know where the terminal point reaches from a given <ecliptical> degree during this <period of> time, <we carry out> **its calculation** <as follows>. If the given <ecliptical> degree is the <ecliptical> degree of the <cusp of the> tenth or fourth <house>, or if the <ecliptical> degree of the planet is in them, we add to its right ascension one degree for each year, and one minute for each 6 days, <considered> from a known time. We find the arc corresponding to the result in <the table for> right ascension. It will be the terminal point from that <ecliptical> degree. If the given <ecliptical> degree is the <ecliptical> degree of the ascendant or the descendant, or if the <ecliptical> degree of the planet is in them, we add to the <oblique> ascension of the ascendant one degree for each year and one minute for each 6 days, <counted> from a known time. We find the arc corresponding to the result in <the table for> oblique ascension. It will be the terminal point from the <ecliptical> degree of the ascendant. The opposite of this terminal point is the terminal point from the <ecliptical> degree of the descendant. If the given <ecliptical> degree is between two cardines, we add to the right ascension and the oblique ascension of the <ecliptical> degree one degree for each year, and one minute for each 6 days, <counted> from a known time. We find the arc corresponding to each one in <the table for> its ascension. Then we obtain the difference between the two arcs. We multiply it by the distance of the <ecliptical> degree from the cardine rising prior <to it> in hours, and divide it by 6. The result is the equation. If the <ecliptical> degree is between the <cusp of the> tenth and fourth <house>, or in the opposite <quadrant>, and the arc of the right ascension has the greater value, we subtract the equation from it; otherwise, we add the equation to it. If the <ecliptical> degree is between the ascendant and the <cusp of the> fourth <house> or in the

<opposite> quadrant, and the arc of the oblique ascension has the greater value, we subtract the equation from it; otherwise, we add the equation to it. The result is the terminal point from that degree. An example for this <operation:> The ascendant <is in> 4° of Pisces; the <cusp of the> tenth <house> <is in> 15° of Sagittarius; Venus <is> in 24° of Capricorn; and Mars <is> in 20° of Aquarius. We move Venus to the <ecliptical> degree of Mars by prorogation. It (i.e., Venus) reaches it in 23 years and 150 days. We want to know where the terminal point from Venus reaches at the completion of this <period of> time. It is <found> 20; 23° of Aquarius.

Chapter 5: On <finding> the transfers of the years and their ascendants.

In this chapter we need <to know> the mean longitude of the sun for the transfer and the time of the true longitude. It is the time for which we should find the true longitude of the planets for the transfer. <We also need> the time of the transfer and its ascendant. It should be known that when we subtract the <number of the> base year of the beginning, from the <number of the> year in which the transfer occurs, in the Yazdigird era, the remainder is the <number of the> entire years which followed this beginning. The transfer is the entering into the next year upon the sun's return to its original position. An example of this <follows:> The beginning occurred in the year 332. We want <to know> the transfer of the year in the year <3>89. We subtract 32 from 89. The remainder is 57. It is <the number of> the entire years which followed the beginning. The transfer is entering into the 58th year upon the sun's return to its original position. <Finding the> **transfer mean longitude**: We write down the true longitude of the sun for the base <year> somewhere on the dustboard to be known <during the operation>. Then we write it down in three positions, and subtract the adjusted apogee for the time of the transfer from the first position. The remainder is the <true> anomaly. We obtain the equation for it, and subtract it from the anomaly and from the second and third positions. Then we obtain the equation for this anomaly and add it to the second position. We check if it exceeds the true longitude of the epoch <year>. <If so,> we subtract the excess from the anomaly and from the third position. If it is less than the true longitude of the epoch <year>, we add the deficit to the anomaly and to the third position. We make the second <position> like the third <one>. Then we obtain again the equation for this anomaly and add it to the second position. We check if it exceeds the true longitude of the epoch <year>; <if it does,> we subtract the excess from the anomaly and from the third position. If it is less than the true longitude of the epoch <year>, we add the deficit to the anomaly and to the third position. We make the second <position> like the third

<position>. Then we obtain again the equation for this anomaly and add it to the second position. We check if it exceeds the true longitude of the epoch <year>; <if it does,> we subtract the excess from the anomaly and from the third position. If it is less than the true longitude of the epoch <year>, we add the deficit to the anomaly and to the third position. What results for the anomaly in this iteration, is the anomaly of the transfer. What results in the third position is the transfer mean longitude. <Finding> **the time of the true longitude** <is as follows>: For <finding> the time of the true longitude, we find, from the <relevant> table, the mean longitude of the sun for the beginning of the year in which the transfer occurs. Then <we find the mean longitudes> for the months and the days of the year, and the hours and their fractions, so that it will be equal to the mean longitude corresponding to the transfer. What results from the months, days and hours is the arc of revolution after midday in hours. <Finding the> **time of the true longitude from the arc of revolution in hours** <is as follows: If the <geographical> longitude of the locality is less than 90°, we obtain the difference between the longitude difference in hours and the equation of time in hours. If the longitude difference <in hours> is the greater value, we add it (i.e., the final difference) to the arc of revolution in hours. If the equation of time is the greater value, we subtract it from the arc of revolution in hours. The sum or the remainder is the time of the true longitude from the day or the night. Then we know its distance from the midday. If the <geographical> longitude of the locality is greater than 90°, we add the longitude difference in hours and the equation of time in hours, and subtract the sum from the arc of revolution in hours. The remainder is the time of the true longitude from the day or the night. Then we know its distance from the midday. **Section on the transfer time**: For <finding> the transfer time, if the <geographical> longitude of the locality is less than 90°, we subtract the <geographical> longitude difference in hours from the time of the true longitude. If the <geographical> longitude of the locality is greater than 90°, we add the <geographical> longitude difference in hours to the time of the true longitude. We add to the sum or the remainder the equation of time in hours. The result is the <time of> the transfer in hours after the midday. If it is less than the half-day hours, we add it to the time of the midday in hours. The result is the <number of the> hours elapsed <from the beginning of> the present day. If it is more than half-day hours, we subtract the half-day hours from it. The remainder is the <number of the> hours elapsed <from the beginning of> the next night. If it is greater than the sum of the half-day hours and the <number of the> hours of the next night, we subtract the <number of the> half-day hours and <the number of> the hours of the next night. The remainder is the <number of the> hours elapsed <from the beginning of> the next day. **The ascendant**:

When this (i.e., the transfer time) is found, we multiply it by 15. <The product> is the arc of revolution of the equator from the rising of the sun or from its setting until the time of the transfer. If it is <in> the day, we add it to the <oblique> ascension of the <ecliptical> degree of the sun. If it is <in> the night, we add it to the oblique ascension of the opposite of the <ecliptical> degree of the sun. We find the arc corresponding to the sum in the table for ascensions. The result is the ascendant. This operation <may be used> for finding the transfer of the sun into any ecliptical degree. The return of the sun to its <original> position <occurs> after <describing> a single cycle. The excess of the arc over a cycle of the equator is 86; 36°.

Chapter 6: On converting the ascendant of the world year from one locality to another.

We obtain the <geographical> longitude difference of the two localities in degrees. It is <equal to> the arc of revolution. If the second locality has a greater <geographical> longitude, we add the arc of revolution to the <right> ascension of the ascendant relating to the first locality. If the second <locality> has a smaller <geographical> longitude, we subtract the arc of revolution from the <right> ascension of the ascendant relating to the first locality. We find the arc corresponding to the sum or the remainder in the <table for> the <oblique> ascensions of the second locality. The result is the ascendant in the second locality.

Commentary

I.7.1 This is a method for finding the time interval that a point of the ecliptic needs for moving, by the daily motion of the universe, from its present position to one of the cardinal positions (in the horizon or meridian planes), or vice versa. The unit of measurement is the "seasonal hour"; that is one sixth of the time which the point needs to move from the last cardinal position before its present position to the first cardinal position after its present position. Kūshyār uses this computation in I.7.3 and I.7.4.

I.7.2 Ancient astrologers believed that each planet P casts seven visual rays to other points of the ecliptic whose positions are defined by the vertices of a regular hexagon, a square and an equilateral triangle with P as their common vertex. For a discussion of this theory see [Kennedy & Krikorian-Preisler 1972, 3-7; Hogendijk 1989, 170-72]. Kūshyār did not discuss this subject in his *Introduction to Astrology.* According to the *Jāmiʿ Zīj*, when the planet has a non-zero latitude, it casts its rays to the points on the ecliptic whose distances from the planet are 60°, 90°, etc. The ecliptical longitude of these points can be found by the method provided in this chapter, based on the Cosine theorem for spherical right triangles. For the sextile rays, we determine an arc $P(r)$ by:

$$\text{Cos}\, P(r) = R\text{Cos}\, 60° / \text{Cos}\, \beta,$$

where β is the latitude of the planet, and R = 60. If the ecliptical longitude of the planet is λ, it casts its two sextile rays to points with longitude $\lambda \pm P(r)$. A proof of this method is presented in IV.7.1. Al-Battānī provides another method for finding the projections of the rays [1899, III, 196-197], but his method is lengthy and complicated, as Kūshyār remarks in IV.7.1. Al-Bīrūnī provides two methods for finding the projection of the rays, taking the planet's latitude into account. His first method is similar to the method given by Kūshyār in this chapter [Kennedy & Krikorian-Preisler 1972, 6].

I.7.3 This alternative method can be shown to have the following geometrical rationale: first the position of the planet is projected onto the equator along arcs of great circles from the north point to the south point of the horizon. Kūshyār approximates this geometric determination by a computation which produces a very imperfect result, but which was nevertheless standard in his time. The method is described in terms of "hours" determined by the position of the planet in the ecliptic (compare I.7.1). When the planet is in its culmination point or the point opposite to it, each hour corresponds to 15 degrees of the celestial equator (i.e., the seasonal hour for geographical latitude 0°). When the planet is rising or setting, each hour is equal to the seasonal hour corresponding to the ecliptical degree of the planet (for the latitude of the locality; day hour [at rising] and night hour [at setting]). For other positions of the planet, the number of degrees on the celestial equator corresponding to each unequal hour may be found by linear interpolation. The results may be used for finding the whole series of the projections of the rays [Hogendijk 1989, 178-80].

I.7.4 The concept of 'prorogation' (or progression, in Arabic *Tasyīr*) was connected with an astrological method for anticipating important events in the life of a person, based on the positions of the celestial bodies at the moment of his or her birth [Yano and Viladrich 1991, 1-3]. Kūshyār discussed this subject in Chapter 20 and 21 of the third Book of his *Introduction to Astrology* [1997, 216-35]. M. Yano and M. Viladrich have discussed the content of Chapter 21 [Yano & Viladrich 1991]. Table II.53 of Kūshyār's *zīj* provides the minor and medium prorogations. The ms. F quotes a fragment "from another manuscript" under the misleading title "another method" after referring to the tables for minor and medium prorogations. But this is actually an explanation and example for the application of these tables, which does not sound authentic. The transfer prorogation is not mentioned in manuscript L. For the major prorogation, we should find the adjusted ascensions of the given point on the ecliptic. When the given point on the ecliptic to be moved by prorogation is in the tenth or fourth house, the adjusted ascensions are the same as the right ascensions. When the point is rising or setting, oblique ascensions are used for this purpose. If the given point is situated between the cardines, a combination of the right and oblique ascensions is applied, using linear interpolation. This is called the ascension of the ecliptical degree based on its position. The same process is carried out for the 'terminal point'. The required period of time is found by counting one year for each degree and 10 days for each minute of the difference between the initial and final adjusted ascensions. Inversely, for finding the terminal point relating to the major prorogation for a given period of time, a similar method is used. I checked the calculation of the example given for this case, using the right ascensions in table II.45 and the oblique ascensions for the latitude of 36° provided in table II.46: The result was in accordance with the terminal point given in the text with negligible deviation.

I.7.5 In this chapter Kūshyār wants to find the moment in a given year t, at which the true longitude of the sun has a given value c (equal to its true longitude at a definite moment a number of years ago). This problem is solved by an iterative procedure. Kūshyār first finds the position $A(t)$ of the apogee in year t. He uses $x_1 = c - A(t)$ as a first approximation of the mean centrum at *t*, and computes the equation $q(x_1)$. Then $x_2 = c \pm q(x_1)$ is a second approximation of the mean longitude, and so on. This method was also used by other Islamic period astronomers; see [Kennedy 1969, 249-50]. Al-Battānī deals with this matter in two places in his *zīj* [1899-1907, III,192-93, 223], but his approach is different from Kūshyār's. The mean longitude is then used for finding the hours relating to the arc of revolution after midday for the transfer. Then the number of the hours past midday is corrected for the geographical longitude difference and equation of time for finding the number of the hours relating to the true longitude in a locality with geographical longitude 90°. Then again the longitude difference and the equation of time are applied for finding the transfer times in different localities. The number of the hours past sunrise or sunset is used for finding the ascendant of the transfer time, using the ascension tables. The excess of revolution after a solar cycle is mentioned to be equal to 86; 36°. This magnitude divided by 360° gives the excess of a solar year over an integer number (365) of days, equal to 0.2405 of a day. The modern value for this magnitude is 0.2422 of a day.

I.7.6 The ascendant of the world year found for a locality can be used for finding the same ascendant for another locality with a different geographical longitude, using the oblique ascension tables, as described in this chapter. For a description of different systems of world years see [Kennedy 1962] and [Kennedy and van der Waerden 1963]. By the ascendant of the world year, Kūshyār apparently means the ascendant of the position of the prorogation (*Tasyir*) indicator on the celestial equator (see I.7.4 above) which makes a complete revolution on the celestial equator during one world year.

Section 8: On the operations which are less needed, <in> 10 chapters

Chapter 1: On <finding> the latitude of a locality from the hours (i.e., the duration) of <its> longest day.

We multiply half the <number of the> hours of the longest day by 15 <degrees>: <The result> is half the day arc. <This result> may be used to find the <maximum> ortive amplitude, which will be <for the sun> in the first of Cancer. Then we divide the Sine of the maximum declination <of the sun> by the Sine of the ortive amplitude, lowered: The result is the Cosine of the latitude of the locality. **Another method**: We find half the day arc and its deficit from 90° or its excess over 90°: This is the equation of daylight. Then we divide the Tangent of the declination of the <ecliptical> degree <of the sun> by the Sine of the equation of daylight, lowered: The result is the Cosine (this should be "Cotangent"; see commentary) of the latitude of the locality. This calculation <method> is generally valid for the <number of the> hours of any day of the year, if we use the declination of the sun on that day.

Chapter 2: On the altitude without a <non-zero> azimuth.

We divide the Sine of the declination of the sun or the distance of the planet from the celestial equator by the Sine of the latitude of the locality, lowered: The result is the Sine of the altitude corresponding to zero azimuth. This altitude can be found if the sun or the planet rises on the northern side of the celestial equator, i.e. <north of> the rising point of the first of Aries or Libra, and passes the meridian circle <at a point> south of the zenith.

Chapter 3: On <finding> the azimuth for any altitude which we assume.

We multiply the Cosine of the declination of the <ecliptical> degree of the sun by the Sine of the ascension of the distance of the <ecliptical> degree of the sun <from the meridian>, and we divide <the product> by the Cosine of the altitude <of the sun>: The result is the Cosine of the azimuth <of the sun>. If the sun is in the northern <zodiacal> signs, and the altitude of the <sun at the given> time is less than the altitude <of the sun> corresponding to its zero azimuth, then the azimuth is eastern or western towards north. If the altitude of the <sun at the given> time greater than the altitude <of the sun> relating to its zero azimuth, and the altitude <of the sun> is eastern or western, then the azimuth is southern. If the sun is in the southern <zodiacal> signs, then the azimuth is southern. <The method presented in> this chapter is less necessary for the planets. <However,> if it is needed

<for them>, <we take> the distance of the planet <from the celestial equator> instead of the declination of the sun, and its (i.e., the planet's) transit degree instead of the <ecliptical> degree of the sun. **Another method**: We multiply the Sine of the altitude <of the sun> by the Sine of the latitude of the locality, and divide <the product> by the Cosine of the latitude of the locality: The result is <called> 'the argument of the azimuth'. If the declination <of the sun> is southern, we add 'the argument of the azimuth' to the Sine of the ortive amplitude. If the declination <of the sun> is northern, we subtract the lesser <of these two values> from the greater <one>. The sum or the remainder is <called> 'the equation of the azimuth'. We divide it by the Cosine of the altitude, lowered. The result is the Sine of the azimuth. Now, if 'the argument of the azimuth' is greater than the Sine of the ortive amplitude, then the azimuth is southern; if it is less than it (i.e., than the Sine of the ortive amplitude), then the azimuth is northern.

Chapter 4: On <finding> the altitude from the azimuth.

We multiply the Cosine of the latitude of the locality by the Cosine of the azimuth, lowered: The result is <called> 'the Sine of the first arc'. We find the corresponding arc. Then we divide the Sine of the latitude of the locality by the Cosine of the first arc, lowered. The result is <called> 'the Sine of the second arc' which is <also> called 'the complement of the argument of the altitude'. Then we multiply the Sine of the declination of the sun by the Sine of the second arc, and divide <the product> by the Sine of the latitude of the locality: The result is <called> 'the Sine of the third arc'. We find the arc corresponding to it. <This arc> is called 'the equation of the altitude'. If the declination <of the sun> is southern, we subtract the third arc from the complement of the second arc. If the declination <of the sun> or the distance <of the planet from the ecliptic> is northern, we add the third arc to the complement of the second arc. The sum or the remainder is the <required> altitude. However, if the azimuth is northern, we always subtract the equation of the altitude from the argument of the altitude.

A <practical> use of these two chapters: If the birth of a baby occurred at a time during the day, and a line is drawn in the direction of the shadow of a vertical gnomon on a horizontal plane <at that time>, and if the return of this shadow to its first azimuth is noted in any <other> day, the altitude of the sun is obtained <by observation> at this <time>, and the azimuth relating to this altitude is computed, then this <azimuth> is the azimuth for the altitude <of the sun at> the time of birth. The altitude <of the sun> relating to this azimuth on the birthday and the position of the sun on that <day> can be computed. This will be the altitude of the sun at the time of birth; and the ascendant and what <else> that may be needed is computed from it.

Chapter 5: On the distance between two stars of which <only> one has a <non-zero> latitude.

We multiply the Cosine of <the longitude difference> in degrees between the two stars by the Cosine of the latitude of the star which has a <non-zero> latitude, lowered: The result is the Cosine of <the distance> between the two stars.

Chapter 6: On the distance between two stars both having <non-zero> latitudes.

We multiply the Cosine of the latitude of the star which has a smaller longitude by the Sine of <the longitude difference> between the two stars in degrees, lowered: The result is the Sine of the first arc. We find the corresponding arc. Then we divide the Sine of this latitude by the Cosine of the first arc, lowered: The result is the Sine of the second arc. We find the corresponding arc, and add <this arc> to the latitude of the star which has a greater longitude, if the two latitudes are in two different directions. If they are in the same direction, we obtain the difference between this latitude and the second arc: <The result> is the third arc. Then we multiply the Cosine of the first arc by the Cosine of the third arc, lowered: The result is the Cosine of <the distance> between the two stars.

Chapter 7: On the extraction of the meridian line.

We level a site on the ground so that its surface becomes parallel to the horizon. We draw a circle on it, we prick a straight needle and we measure its perpendicularity to the surface from three positions on the circumference, distant from each other. Then, near noon, we observe the tip of the shadow of the needle. <The shadow> will be diminishing as we make marks very close to one another by the tip of another needle on those positions <of the tip of the shadow>, while <the shadow> is turning. We check <the marks> carefully until the shadow begins increasing. Then we connect the mark nearest to the center <of the circle> and the center by a straight line. It will be the meridian line. **Another method** is <as follows>. We level the ground and <we take> the circle and the gnomon as we said <before>, except that the circle should be equal to the altitude circle on the back side of an available astrolabe. <Also> the length of the gnomon should be so that its shadow does not fall short of the circumference at noon. Then we extract the azimuth relating to its (i.e., the sun's) altitude on one of the two sides of the meridian <line>. We make a mark on the circumference where the shadow falls, when this altitude is reached <by the sun>. Using compasses, we obtain <the length> equal to the <chord of the> complement of the azimuth from the altitude circle of the astrolabe. We put one leg of the compasses on the mark, and the other on some point on the circumference in the direction of the altitude <of the sun>, which may be eastern or western <while the compass opening is the same>. From where it falls, we draw a line to the center of the circle. It will be the meridian line. If the altitude <of the sun> is <equal to> the altitude for a zero azimuth, the shadow will be on the east-west line. The line drawn from the middle of its two endpoints to the center of the circle is the meridian line. There are many ways to draw this line. However, all of them are less accurate and in practice <less> close to the correct <direction> than these two ways, but theoretically all of them are correct and can be proved.

Chapter 8: On the deviation of the <directions of> localities with known longitudes and latitudes from the meridian of our locality.

This deviation is called 'the azimuth of localities'. Let the locality whose azimuth is desired be Mecca. <To find this azimuth> we multiply the Cosine of the latitude of Mecca by the Sine of the <difference> between the two longitudes, lowered: The result is <called> 'the Sine of the equation of longitude'. We find the corresponding arc. Then we divide the Sine of the latitude of Mecca by the Cosine of the equation of longitude, lowered: The result is the Sine of the equation of latitude. We find the corresponding arc. If this arc is less than the latitude of our locality, we subtract it from the latitude of the locality. The remainder will be the adjusted latitude of the locality, <which latitude is> southern. If it (i.e., the adjusted altitude) is exactly equal to it (i.e., to the latitude of our locality), then the azimuth of Mecca is the east-west line. If it is greater <than the latitude of our locality>, we subtract from it the latitude of <our> locality: The result is the adjusted latitude of the locality, <which latitude is> northern. Then we multiply the Cosine of the equation of longitude by the Cosine of the adjusted latitude of the locality, lowered: The result is the Cosine of the distance between the two localities. Then we divide the Sine of the equation of longitude by the Sine of the distance between the two localities, lowered: The result is the Sine of the deviation of <the direction of> Mecca <from the local meridian>. <For finding> **the direction of the deviation**, we check <the arc> between the two localities and the adjusted latitude of the locality: If <the arc> between the two longitudes is situated in the east-south quadrant and the adjusted latitude of the locality is southern, then the deviation is towards the south-east; if the adjusted latitude of the locality is northern, then the deviation is towards north-east. If the <arc> between the two longitudes is situated in the south-west quadrant, and the adjusted latitude of the locality is southern, then the deviation is towards the south-west. If the adjusted latitude of the locality is northern, then the deviation is towards the north-west. When we carried out this operation for the locality of Rayy, taking its longitude from <Canary Islands in the> West (i.e., the Atlantic Ocean> to be 85°, and its latitude 36°, the longitude of Mecca 77°, and its latitude 21°, the deviation is <found to be> 27; 36° towards west.

Chapter 9: On the names of the fixed stars and their features in order to recognize them by seeing.

We have compiled in a table what we need <to know> about these stars in most <cases>. We have recorded their positions for the beginning of the year 301 of the Yazdigird <era>. <Their> adjustment <for other years> is <merely applying> the equation of the apogees. We have put in front of them <in the table> their latitudes, magnitudes, and their temperaments in relation to the planets. Since we need <to be able> to recognize a star and <sometimes> two stars by seeing <them> in each quadrant of the <sky> and all the quadrants of the ecliptic in order to obtain their altitudes <by observation> for knowing the ascendant and the time, we mention their features, so that the observer may recognize them. They are <as follows:> ***al-Kaff al-khaḍīb*** (lit. "the dyed palm [of the hand]"; Caph, β Cassiopeiae): a star in Aries, of the third magnitude, in the north, on the hump of the constellation known as *al-Nāqa* ("the she-camel") by the common people; there are two stars of the same magnitude under it, which, together with this star, form a triangle; ***'Ayn al-thaur*** (lit. "the eye of the bull"; Oculus Tauri, α Tauri): also called *al-Dabarān* (Aldeberan): a red star in Taurus, of the first magnitude, in the south, behind the Pleiades, between some stars which look like <the Arabic letter> *dāl*; ***al-'Ayyūq*** (Capella, α Aurigae): a big star in Gemini, of the first magnitude, in the north, on the edge of the Milky Way, behind three stars which are in a row; it rises <simultaneously> with the Pleiades; ***Mankib al-jawzā'*** (lit. "the shoulder of the Twins"; Betelgeuse, α Orionis): a red star in Gemini, of the first magnitude, in the south; it is in the place of the shoulder of a standing person; ***al-Shi'rā al-yamānīya*** (Sirius, α Canis Majoris): a white big star in the beginning of the Cancer, of the first magnitude, in the south, behind the stars of Gemini; ***al-Shi'rā al-shāmīya*** (Procyon, α Canis Minoris): a star in Cancer, of the first magnitude, in the south; it is smaller than Sirius, to the north of it, and in front of it; ***Qalb al-asad*** (lit. "heart of the lion"; Regulus, α Leonis): a star in Leo, of the first magnitude; it is approximately on the ecliptic, on the southern side of four stars standing from south to north in a crooked row; ***al-Ṣarfa*** (Cygnus, β Leonis) also called *Dhanab al-asad* (Denebola): a star in Virgo, on the tail of Leo, of the first magnitude, in the north; there are two bright stars called *al-Zubra* (Zubra, δ and θ Leonis) between it and Regulus; ***al-Simāk al-rāmiḥ*** (lit., "chest of the spearman"; Arcturus, α

Bootis): a star in Libra, of the first magnitude, in the north; there is a star smaller than it, called *al-Rāmiḥ* ("the spearman"), in front of it towards west; ***al-Simāk al-aʿzal*** (lit. "the chest of the unarmed <man>"; Spica, α Virginis): a star in Libra, of the first magnitude, in the south, in front of *al-Rāmiḥ*; ***al-Munīr min al-fakka*** (lit. "the luminous <star> of Coronae Borealis"; Alphacca, α Coronae Borealis): a star in Libra, of the second magnitude, in the north, between <a> circular <array of> stars behind *al-Simāk al-rāmiḥ*, the common people call it *Qaṣʿat al-masākīn* (lit. "the bowl of the poor"); ***Qalb al-ʿaqrab*** (lit. "heart of Scorpion"; Antares, α Scorpii): a red star in Scorpion, of the second magnitude, in the south, between two luminous stars on a curved line; ***al-Nasr al-wāqiʿ*** (lit. 'the falling eagle"; Vega, α Lyrae): a star at the end of Sagittarius, of the first magnitude, in the north; its path is close to the zenith; there are two small stars under it, which together with this star form a triangle called *athāfī* (lit. "andiron/trivet"; α, ε, ζ Lyrae or α, β, γ Lyrae) by the common people; ***al-Nasr al-ṭāʾir*** (lit. "the flying eagle"; Altair, α Aquilae): a star in Capricorn, of the second magnitude, in the north, between two luminous stars on a straight line; ***Dhanab al-dajāja*** (lit. "tail of the hen"; Deneb, α Cygni) <also> called *al-ridf*: a star in Aquarius, of the second magnitude, in the north, behind the luminous stars that cut through the Milky Way; ***Mankib al-faras*** (lit. "shoulder of the horse"; Scheat, Menkib, β Pegasi): a star in Pisces, of the second magnitude, in the north, northern in relation to another star of the same magnitude; they are called <together> as *al-fargh al-muqaddam* (α and β Pegasi), <which is> one of the lunar mansions

Chapter 10: On the names of the lunar mansions, and their rising days.

The 28 <lunar> mansions and their names <are as follows>:

1. *al-sharaṭain* 20th of Nīsān	2. *al-buṭain* 3rd of Ayār	3. *al-thurayyā* 16th of Ayār	4. *al-dabarān* 29th of Ayār	5. *al-haqʿa* 11th of Ḥazīrān
6. *al-hanʿa* 25th of Ḥazīrān	7. *al-dhirāʿ* 8th of Tammūz	8. *al-nathra* 20th of Tammūz	9. *al-ṭarfa* 2nd of Āb	10. *al-jabha* 15th of Āb
11. *al-zubra* 28th of Āb	12. *al-ṣarfa* 10th of Īlūl	13. *al-ʿawwāʾ* 23rd of Īlūl	14. *al-simāk* 6th of Tishrīn I	15. *al-ghafr* 20th of Tishrīn I
16. *al-zubānī* 2nd of Tishrīn II	17. *al-iklīl* 15th of Tishrīn II	18. *al-qalb* 28th of Tishrīn II	19. *al-shawla* 11th of Kānūn I	20. *al-naʿāʾim* 24th of Kānūn I
21. *al-balda* 6th of Kānūn II	22. *saʿd al-dhābiḥ* 19th of Kānūn II	23. *saʿd bulaʿ* 1st of Shabāṭ	24. *saʿd al-suʿūd* 14th of Shabāṭ	25. *saʿd al-akhbiya* 27th of Shabāṭ
26. *al-fargh al-muqaddam* 12th of Ādhār		27. *al-fargh al-muʾakhkhar* 25th of Ādhār		28. *baṭn al-ḥūt* 7th of Nīsān

The parts of the ecliptic corresponding to these mansions are equal. They are taken <subsequently, starting> from the point corresponding to the beginning of Aries. Their constellations are <formed> of fixed stars with different magnitudes and positions in the Zodiac. Their <heliacal> rising days, i.e. <the times of> their apparition <after they exit> from <being> under the rays <of the sun are as follows>: *al-sharaṭain* rises on the 20th of Nīsān around the year 1320 of the Two-Horned (i.e., Alexander) <era>. Then each next mansion rises 13 days later, until *al-Simāk* rises. We take the rising of *al-Ghafr* next to it, after 14 days for compensating the fractions which <are> with the 13 days. Then up to the end of the mansions <we take 13 days>, as before. After 66 years *al-Sharaṭain* rises on the 21st of Nīsān, and similarly all <other> mansions rise one day later. When a mansion <of the moon> rises, its opposite <mansion>, which is the fifteenth <mansion counting> from it, sets. Thus, when *al-Sharaṭain* rises, *al-Ghafr* sets. It is not impossible that there may be a difference of one or two days between the actual apparition <of these mansions> and what we have defined for them. Precise observations do not exist in this <connection> that may lead to major inconsistency, and there is no need to determine that <inconsistency>. <Now,> after we have completed the chapters <on elementary calculations> that we indicated in the preface to the <first> book, and we tried to make them close <to understanding>, and we did our best in making them precise, we finish the first book by this chapter. We ask God for help, and on Him is <our> reliance. This is followed by the second book on the tables.

Commentary

I.8.1 The ortive amplitude can be found from half the day arc by the formula given in I.5.6: $Cos\theta = (Cos\delta_1 Sin\frac{D}{2}) / R$, where θ is the ortive amplitude of the sun, δ_1 the declination of the sun, and D the day arc. Then, as mentioned here, we can find the latitude of the locality by the formula $Sin\theta_{max}\ Cos\varphi = RSin\varepsilon$, where θ_{max} is the maximum ortive amplitude of the sun, φ the geographical latitude of the locality, and ε the maximum declination of the sun. The second method can also be deduced from a formula given in I.5.7 for calculating the equation of daylight: $R\,Sin\Delta D\ =\ Tg\delta_1 Tg\varphi$, where ΔD is the equation of daylight. Kūshyār erroneously puts Cosφ instead of Cotgφ in the second method. Both methods are demonstrated in IV.8.1. The proof of the formula given in I.5.6 is repeated there. Ptolemy [1984, 77-78] provides a different method for finding the latitude of the locality from the length of the longest day. Al-Battānī [1899-1907, III, 30] first calculates the ortive amplitude in a similar way; then he obtains the latitude of the locality from the ortive amplitude, half the day arc and the excess of half the day arc, by a method equivalent to $Sin\varphi = RSin\Delta DCos\theta / Sin\theta Sin\frac{D}{2}$. This formula can be obtained from the above formulas.

I.8.2 Here Kūshyār calculates the altitude h_0 of the sun if it is due east or west (or has zero azimuth, as Kūshyār puts it) and if its northern declination (δ) is given, for a locality with geographical latitude φ. In modern notation, his method is equivalent to: $Sinh_0 = RSin\delta / Sin\varphi$. Kūshyār provides a proof of this formula in IV.8.2.

I.8.3 The two methods provided by Kūshyār in this chapter for finding the azimuth of the sun when its altitude is known, are equivalent to the following formulas:

(1) $\cos az = \cos\delta_1 \sin A_d(\lambda) / \cosh$,

(2) $\sin az = (\sin\theta \pm \sinh \tan\varphi) / \cosh$,

where az is the required azimuth, δ_1 is the declination of the sun, $A_d(\lambda)$ is the right ascension of the arc between the sun and the meridian, h is the known altitude, φ is the latitude of the locality, and θ is the ortive amplitude. Proofs of these two methods are given in IV.8.3. The second method is provided by al-Battānī [1899-1907, III, 33-34, 53-54] for the sun and the planets or stars.

I.8.4 The method for finding the altitude of the sun when its azimuth is known, is equivalent to the following formulas:

$$\cos\varphi \cos az = \sin\alpha_1$$
$$\sin\varphi / \cos\alpha_1 = \sin\alpha_2$$
$$\sin\delta_1 \sin\alpha_2 / \sin\varphi = \sin\alpha_3$$
$$h = 90° - \alpha_2 \pm \alpha_3$$

$\alpha_1, \alpha_2,$ and α_3 being auxiliary arcs, the last two of which are called 'the complement of the argument of the altitude' and 'the equation of altitude', respectively. Of course the third formula may also be presented in the simpler form, $\sin\delta_1 / \cos\alpha_1 = \sin\alpha_3$. However, the present form may better reflect the geometrical process of finding the unknown altitude. A proof of the validity of this method is provided in IV.8.4, where, for the case when the azimuth is northern, the above-mentioned simpler formula is actually demonstrated. As Kūshyār mentions in the text, this chapter together with the former chapter may be used to find the altitude of the sun at a certain daytime on the day when a person was born. For this we should have registered the azimuth of the sun at the time of his birth by drawing a line along the shadow of a gnomon at that time. It is interesting that Kūshyār also thinks of the astrological application in this chapter and the former one.

I.8.5 In modern notation, the distance between two stars whose longitude difference is $\Delta\lambda$ and whose latitudes are $0°$ and β, respectively, is found by the formula: $\cos\Delta\lambda \cos\beta = \cos d$, where d is the distance between the two stars. A proof of the validity of this method is provided in IV.8.5. Al-Battānī gives a more lengthy method for this [1899, III, 59].

I.8.6 In modern notation, the distance between two stars whose longitude difference is $\Delta\lambda$, and whose latitudes are β_1 and β_2, respectively, can be found by the following formulas:

$$\cos\beta_1 \sin\Delta\lambda = \sin\alpha_1$$
$$\sin\beta_1 / \cos\alpha_1 = \sin\alpha_2$$
$$\alpha_2 \pm \beta_2 = \alpha_3$$
$$\cos\alpha_1 \cos\alpha_3 = \cos d$$

α_1, α_2, and α_3 being auxiliary arcs and d being the distance between the two stars. Kūshyār proves this method in IV.8.6. Al-Battānī gives a different method in a more detailed way for this calculation [1899, III, 60].

I.8.7 In this chapter, Kūshyār describes two methods for finding the meridian line, which he mentions as the most accurate among several theoretically correct methods. His first method is based on the fact that the shadow of any gnomon is shortest at true local noon when the sun is on the local meridian. In practice, this method is not very accurate. In the second method, the chord of the angle between the azimuth of the sun and the meridian line is computed by the methods provided in I.8.3 or by the astrolabe, from the altitude of the sun at the time of measuring. The graduation for the altitudes on the rim of the astrolabe is then utilized for obtaining this angle. See also IV.8.7. The phrase in the second method for the case when the shadow is on the east-west line: "from the middle of its two endpoints" is unclear to me. Maybe there is a lacuna here. Ptolemy did not provide any method for drawing the meridian line in the *Almagest*, but he assumes that it can be drawn [1984, 62]. Diodorus of Alexandria (first century B.C.) devised a method for determining the meridian line from any three gnomon shadows [ibid., fn. 72]. His original work is lost, but an Arabic version of his method has recently been published with an English translation [Hogendijk 2001, 68-72]. Al-Battānī describes three methods in his *zīj* for determining the meridian line [1899, III, 35-38], the second of which is similar to Kūshyār's second method, except that al-Battānī finds the chord of the angle between the azimuth of the sun and the east-west line.

I.8.8 In this chapter Kūshyār provides a method for finding the angle between the southern direction and the great circle arc between his locality and another locality (he calls it *inḥirāf*, which means "deviation"), and discusses it for the example of Mecca, because its direction is needed for Islamic prayers. In modern notation, his method is equivalent to the following formulas:

$$\cos\varphi_m \sin\Delta\Lambda = \sin\alpha_1$$

$$\sin\varphi_m / \cos\alpha_1 = \sin\alpha_2$$

$$|\varphi_c - \alpha_2| = \alpha_3$$

$$\cos\alpha_1 \cos\alpha_3 = \cos l$$

$$\sin\alpha_1 / \cos l = \sin d$$

where $\varphi_m, \Delta\Lambda, \varphi_c, l$ and d are, respectively, the latitude of Mecca, the longitude difference between the locality and Mecca, the latitude of the given locality, the arc between the locality and Mecca, and the deviation of the great circle arc from the locality to Mecca from the meridian line, i.e., the angle between the direction of Mecca and the southern direction. The auxiliary parameters $\alpha_1, \alpha_2, \alpha_3$ are respectively, called 'the equation of longitude', 'the equation of latitude', and 'the adjusted latitude'. Kūshyār proves the validity of this method in IV.8.8. Kūshyār's method for finding the direction to Mecca is the same as al-Bīrūnī's fourth method in his *Taḥdīd* [al-Bīrūnī 1967, 253-55], which he calls "the method of the *zīj*es" [Berggren 1985, 1]. Kūshyār's terminology in this chapter from the mss. F, C, Y, B and P is like that of al-Bīrūnī. However, ms. L uses a different terminology in which 'the equation of longitude' is called 'the first arc', etc. and its method is slightly different from a geometrical point of view [ibid, 8]. Al-Battānī describes a simple method for finding the direction of Mecca [1899, III, 206]. Al-Bīrūnī [ibid., 199] says that al-Battānī's method for finding the direction of Mecca is erroneous, because he "treated the meridian circles as parallel straight lines, and the parallels of latitudes as parallel straight lines." At the end of this chapter, Kūshyār calculates the deviation for the locality of Rayy as 27; 36°. I have recalculated this value according to Kūshyār's method and coordinates, and found it to be 27; 8, 20, 20°. The difference may be due to rounding errors. However, the real value of the deviation may be calculated as 38; 40, 58°. In this calculation modern values are used: Mecca (long. 39.83°, lat. 21.41°), Rayy (long. 51.44°, lat. 35.60°. As we see, Kūshyār's obvious error is due to inaccurate coordinates of the localities, especially to incorrect longitudes (based on his data $\Delta\Lambda$ is 8° whereas the correct value is 11.60°). Since in this calculation only $\Delta\Lambda$ is involved, the fact that I take the longitudes with respect to Greenwich and Kūshyār takes them with respect to the Canary Islands is irrelevant.

I.8.9 Kūshyār describes 16 bright stars which occur in the 12 zodiacal signs. In table II.55, he provides the coordinates and characteristics of 48 stars including 16 stars occurring in the zodiacal signs, not exactly the same as those described in I.8.9. This table is for the year 301 of the Yazdigird era (932-33 A.D.). For other years, the positions of the stars are found by applying "the equation of the apogees", by which Kūshyār means the precession of the equinoxes that, according to him, is 54 seconds per year (see I.4.4 and its commentary). For each year, an arc of 54 seconds is subtracted from the ecliptical longitude of any star.

I.8.10 In this chapter Kūshyār mentions the names of the 28 lunar mansions and the days on which they rise just before the sun, for the year 1320 of the Seleucid (Two-Horned, Alexander) era, corresponding to the year 399/400 A.H (1008-09 A.D.). This may be the reason why Prof. Kennedy believes that Kūshyār wrote this *zīj* around 1010 A.D. The same lunar mansions are mentioned by al-Battānī [1899, III, 188-89] where the last mansion is missing. The idea of dividing the zodiac into 28 (or 27) parts comes from ancient Indian astronomy, and the names already existed in Arabic although they were used for unequal lunar mansions. Final remarks by the scribes indicate the date of copying of the final part of the manuscript of Book I. The ms. F bears Maḥmūd b. Aḥmad al-Ḥussain's date, 545 A.H., and C bears the date 1169 A.H.

In the name of God, the Merciful, the Compassionate

Book IV of the *Jāmiʿ Zīj*

Kiyā Abū'l-Ḥasan Kūshyār b. Labbān b. Bāshahrī al-Jīlī –may God illuminate his tomb! – says: When I got through with the third Book on astronomy, I started this fourth Book on proofs, following the order of the chapters of the first Book.

Geometrical demonstration is a reasoning that does not allow any excess or deficiency in <its> exactness, and all those who understand it, equally know what has been proved and learn it by this proof. This is the last Book of this treatise, and when finishing it, I begged God for infallibility, ability, success and guidance, <for> he is really a bestower of these.

List of the chapters <of Book IV> containing 8 sections and 70 chapters

Section 1: On Chords and Sines, <in> 11 chapters

1. On the description of the Chord and Sine.
2. On finding the quantity of the Chord of the complement of an arc when the Chord of the arc is known.
3. On finding the quantity of the Chord of a quarter <of a circle>.
4. On finding the quantity of the Chord of a third <of a circle>.
5. On finding the quantity of the Chord of one-tenth and one-fifth <of a circle>.
6. On a premise for what follows.
7. On finding the quantity of the Chord of the difference between two arcs whose Chords are known.
8. On finding the quantity of the Chord of half an arc whose Chord is known.
9. On finding the quantity of the Chord of the sum of two arcs whose Chords are known.
10. On a premise for what follows.
11. On measuring the Chord of 1° very accurately and the composition of the <table of the> Chords.

Section 2: On Tangents and Cotangents, <in> 3 chapters

1. On the description of Tangents and Cotangents.
2. On finding <the quantity> of the (i.e., any) Tangent.
3. On finding <the quantity> of the (i.e., any) Cotangent.

Section 3: On premises on which the proofs are based, <in> 7 chapters
1. On a general premise for most proofs.
2. On another premise which derives from the first one.
3. Notice on the properties of the proportional magnitudes.
4. On another premise which also derives from the first one.
5. On a premise concerning Tangent<s>, which is a substitute for the first premise in most proofs.
6. Notice on the properties of the Tangent<s>.
7. Another notice also on the properties of the Tangent<s>.

Section 4: On <finding> the true longitudes of the planets and their positions, <in> 10 chapters
1. On the equation of time.
2. On the equation of the sun.
3. On the first equation for the moon.
4. On the second equation for the moon and the planets.
5. On the difference between the <apparent> radius of the epicycle between its maximum and minimum distance <from the earth>.
6. On the first equation for Mercury.
7. On the first equation for the other planets.
8. On the latitude of the moon.
9. On the latitudes of the planets
10. On the retrogradation of the planets.

Section 5: On the operations relating to the ascendants of the day and night, <in> 16 chapters
1. On the first declination.
2. On rising times of the <zodiacal> signs on the equator.
3. On the second declination.
4. On the distance of the stars from the celestial equator.
5. On the latitude of the <given> locality.
6. On the ortive amplitude of the sun and the stars.
7. On the equation of the daylight of the sun and the star<s>.
8. On the rising times <of the signs> in any locality.
9. On the maximum altitude of the sun and the star<s>.
10. On half the day arc of the sun and the star<s>.
11. On the <ecliptical> degree of the transit of a star through the meridian.
12. On the <ecliptical> degree of the rising and setting of a star.
13. On <finding> the arc of revolution <of the celestial equator> since the rising of the sun and the star<s> from the altitude of the <sun or the planet at a given> time.

14. On <finding> the ascendant from the arc of revolution <of e.g., the sun> and <finding> the arc of revolution from the ascendant.
15. On the proof of <using> a base generally applicable to the arc of revolution and what relates to it.
16. On the equalization of the houses.

Section 6: On eclipses and what pertains to them, <in> 14 chapters
1. On the absolute and adjusted magnitudes of a lunar eclipse in digits.
2. On the absolute times of a lunar eclipse.
3. On the correction of times.
4. On drawing the figure of a lunar eclipse.
5. On the distance of the moon from the earth.
6. On the altitude of the pole of the ecliptic.
7. On the altitude of any desired degree of the ecliptic.
8. On the parallax of the two luminaries in the altitude circle.
9. On the six angles needed in <the calculation of> solar eclipses.
10. On <finding> the longitudinal and latitudinal parallax of the moon from these angles.
11. On drawing the figure of a solar eclipse.
12. On <finding> the altitude of the moon according to its latitude.
13. On <finding> the longitudinal and latitudinal parallax of the moon by a proven method.
14. On the visibility arc<s>.

Section 7: On what pertains to astrology, in one chapter
1. On <finding> the projection of the ray taking the latitude of the planet into account.

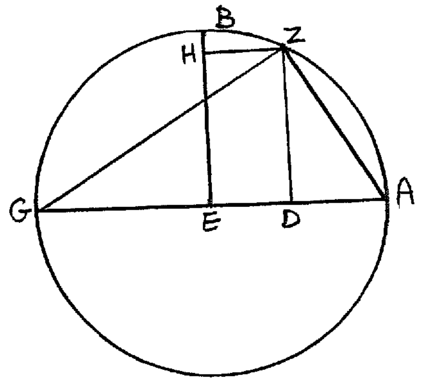
Section 8: On the operations which are less needed, <in> 8 chapters
1. On <finding> the latitude of a locality from the hours (i.e., the duration) of <its> longest and shortest days.
2. On <finding> the altitude without (i.e., with zero) azimuth.
3. On <finding> the azimuth of <a point of given declination and> any assumed altitude.
4. On <finding> the altitude from the azimuth <and the declination>.
5. On the distance between two stars, one of which has a <non-zero> latitude.
6. On the distance between two stars <both> having <non-zero> latitudes.
7. On the extraction of the meridian line.
8. On the deviation of <the directions of> the <other> localities from the meridian of our locality.

These chapters are sufficient to prove the <contents of> first Book, because what may be beyond this, can be proved for someone who has advanced in astronomy and geometry with a little effort and easy thinking. God grants success and help.

Section 1: On Chords and Sines, <in> eleven chapters

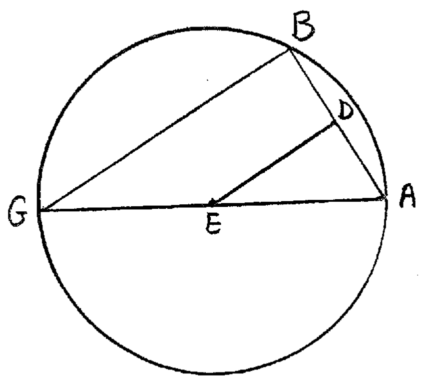
Chapter 1: On the description of the Chord and Sine.

ABG is a circle with *E* as its center, and *AG* its diameter. We draw *EB* at right angles <to the diameter>. We take the arc *AZ* and we draw the line <segment> *AZ*. We draw *ZD* perpendicular to *AG* and *ZH* perpendicular to *EB*. We draw the line <segment> *ZG*. Then the line <segment> *AZ* is the Chord of the arc *AZ* and <the line segment> *ZG* is the Chord of its complement. *ZD* is the Sine of the arc *AZ*, and *ZH* is its Cosine and equal to the line <segment> *DE*. *AD* is the Sagitta of the arc *AZ* and *BH* is the Sagitta of the arc *ZB*. The arc *ZB* is the complement of the arc *AZ* to a quarter of the circle and the arc *ZBG* is the supplement of the arc *AZ* to a semicircle. This is what we wanted to describe.



Chapter 2: On finding the quantity of the Chord of the supplement of an arc when the Chord of the arc is known.

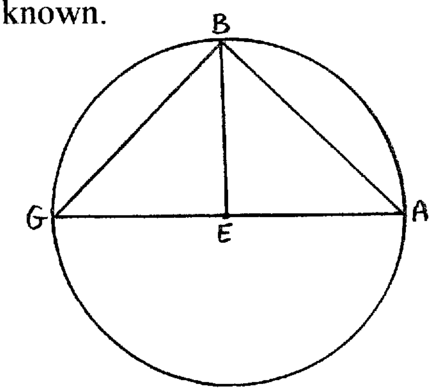
Let *ABG* be a circle and *AG* its diameter. We cut off the arc *AB* from it (i.e., from the circle) and we draw the line <segment>s *AB* <and> *BG*. We assume the Chord *AB* <to be> known; then I say that the Chord *BG* is <also> known.



Proof: The angle *ABG* is a right angle because it is <subtended> in a semicircle. Then the square of *AG* is equal to <the sum of> the squares of *AB* <and> *BG*. If we subtract the square of *AB* from the square of *AG*, the remainder <which is> the square of *BG* <will be> known. Its square root which is the Chord *BG* is <also> known. That is what we wanted to demonstrate. Now it has become clear that the ratio of any Chord to the diameter of the circle is equal to the ratio of the Sine of half the arc of the Chord to the radius of the circle. <For showing> this <we notice that> if we bisect the Chord *AB* at *D* and we draw *DE*, <where> *E* is the center of the circle, <then> *DE* will be parallel to *BG* and *AD* will be the Sine of half the arc *AB*. Then the ratio of *BA* to *AG* is equal to that of *DA* to *AE*. So all calculations that are made on <the basis of> the Chord and the diameter can be carried over to the Sine of half the arc of the Chord and the radius. This is what we wanted to demonstrate.

Chapter 3: On finding the quantity of the Chord of a quarter <of a circle>.

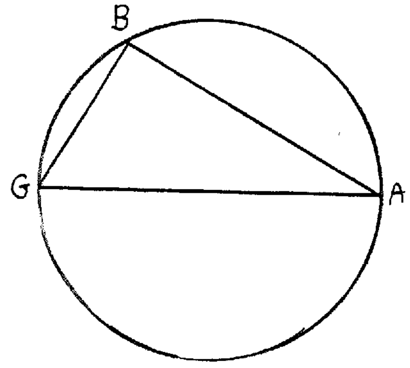
Let *ABG* be a circle centered at *E*, and *AG* its diameter. We draw *EB* at right angles <to the diameter> and we draw <the line segments> *AB* <and> *BG*. Then each of the arcs *AB* <and> *BG* is a quarter of the circle and each of the line <segment>s *AB* <and> *BG* is the Chord of a quarter <of the circle>. I say that they are known.



Proof: Angle *AEB* is right, so the square of *AB* is equal to the <sum of the> squares of *AE* <and> *EB*. Each of <the line segments> *AE* and *EB* is <equal to> the radius. Then the sum of their squares is known and <thence> its square root is known. Therefore, the Chord *AB* is known. That is what we wanted to demonstrate. Now it has become clear that the square of the Chord of a quarter <of a circle> is equal to twice the square of the radius, and that the square of the diameter is equal to four times the square of the radius. <This is> because the square of *AG* is equal to the <sum of the> squares of *AB* <and> *BG*, and each of the squares of *AB* <and> *BG* is equal to twice the square of *AE*. So the square of *AG* <is equal to> four times the square of *AE*. This is what we wanted to demonstrate.

Chapter 4: On finding the quantity of the Chord of a third <of a circle>.

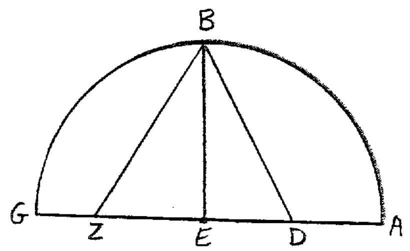
Let *ABG* be a circle and *AG* its diameter. We draw *BG* equal to the radius of the circle; it is <the Chord of> a sixth <of the circle>. We draw *AB*. I say that the Chord of a third <of a circle> is known.



Proof: Angle *ABG* is right because it is <subtended> in a semicircle. Then the square of *AG* is equal to <the sum of> the squares of *AB* <and> *BG*. The square of *AG* is known, and the square of *BG*, which is the Chord of a sixth <of a circle>, is known. So the square of *AB* remaining from the square of *AG* is known. So its square root is <also> known. It is the Chord *AB*. So the Chord *AB* is known. That is what we wanted to demonstrate. Now it has become clear that the square of the Chord of a third <of a circle> is <equal to> three times the square of the radius <of the circle>. The Chord *BG* is equal to the radius <of the circle>. If we subtract the square of *BG* from the square of *AG*, three times the square of the radius <of the circle> is the remainder of *AG*. It is the square of the Chord *AB*. This is what we wanted to describe.

Chapter 5: On finding the quantity of the Chord of one-tenth and one-fifth <of a circle>.

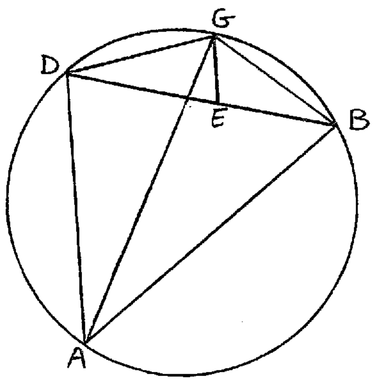
Let *ABG* be a semicircle centered at *E*, and *AG* its diameter. *EB* is perpendicular to *AG*. We bisect *AE* at *D* and we draw *BD*. We make *DZ* equal to *BD*. I say that *EZ* is equal to the Chord of one-tenth of the circle and *BZ* is equal to the Chord of one-fifth of it.



Proof: *AE* is bisected at *D* and *EZ* is added to it. So the product of *AZ* by *ZE* plus the square of *DE* is equal to the square of *DZ* (by *Elements* II.5). But *DZ* is equal to *DB*, and the square of *DB* is equal to <the sum of> the squares of *DE* <and> *EB*. So the product of *AZ* by *ZE* plus the square of *DE* is equal to the <sum of the> squares of *DE* and *EB*. We subtract the common square of *DE*. Then the remaining product of *AZ* by *ZE* is equal to the square of *EB*. But *EB* is equal to *EA*. So *AZ* is divided in mean and extreme ratio <at *E*>, and the greater portion is *AE*. But *AE* is the Chord of one-sixth <of the circle>. So (by *Elements* XIII.9) *EZ* is the Chord of one-tenth <of the circle>. Since the <sum of the> squares of *BE* <and> *EZ* is equal to the square of *BZ*, *BE* is the Chord of one-sixth <of the circle>, and *EZ* is the Chord of one-tenth <of the circle>, therefore (by *Elements* XIII.10) *BZ* is the Chord of one-fifth <of the circle>. This is what we wanted to demonstrate.

Chapter 6: On a premise for what follows.

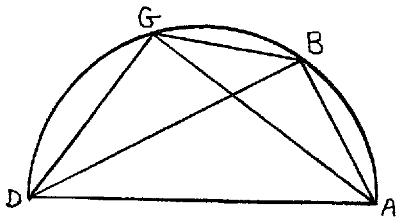
<In> any quadrilateral inscribed in a circle, if we multiply each side by its opposite side, the sum of the products will be equal to the product of the two diagonals. Let *ABG* be a circle and the quadrilateral *AGBD* is <inscribed> in it. I say that the product of *AB* by *GD* and <the product of> *AD* by *GB* when added <together> are equal to the product of *AG* by *BD*.



Proof: We make the angle *DGE* equal to the angle *BGA*. Since the angle *DGE* is equal to the angle *BGA* and the angle *AGE* is common, the angle *DGA* will be equal to the angle *BGE*. But the angle *GAD* is equal to the angle *GBD*, because they are <subtended> in the arc *GD*. So the remaining angle *ADG* is equal to the angle *BEG*. Therefore the ratio of *GB* to *BE* is equal to the ratio of *GA* to *AD*. So the product of *GB* by *AD* is equal to the product of *GA* by *BE*. Again, the angle *DGE* is equal to the angle *BGA*, and the angle *GDB* is equal to the angle *GAB*, because they are <subtended> in the arc *BG*. So the remaining angle *GED* is equal to the angle *ABG*. Therefore the ratio of *GD* to *DE* is equal to the ratio of *GA* to *AB*. Then the product of *GD* by *AB* is equal to the product of *GA* by *DE*. But it has been shown that the product of *GB* by *AD* is equal to the product of *GA* by *BE*. So the product of *AG* by *BD* is equal to the product of *GB* by *AD* and <the product of> *GD* by *AB* <added together>. This is what we wanted to demonstrate.

Chapter 7: On finding the quantity of the Chord of the difference between two arcs whose Chords are known.

Let *ABG* be a semicircle with <a known line segment> *AD* as its diameter. The Chords *AB* <and> *AG* in it (i.e., in the circle) are known. We draw *BG*. Then I say that *BG* is known.

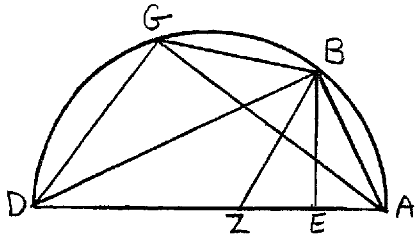


Proof: We draw *BD* <and> *GD*. Then they are both known, because they are the Chords of the supplements of *AB* <and> *AG* <to a semicircle>. So according to what was demonstrated in the premise, the product of *AG* by *BD* is equal to the sum of the product of *AB* by *GD* and <that of> *AD* by *GB*. But the product of *AG* by *BD* is known, and the diameter *AD* is known. So the Chord *BG* is known. This is what we wanted to demonstrate.

Chapter 8: On finding the quantity of the Chord of half an arc of whose Chord is known.

Let *ABGD* be a circle and *AD* is its diameter. We assume the Chord *AG* <to be> known. We bisect the arc *AG* at *B*. We draw *AB* <and> *BG*. Then I say that *AB* is known.

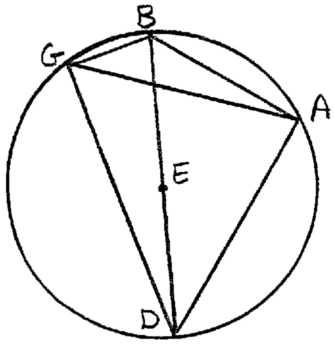
Proof: We draw *GD* and we make *DZ* equal to *GD*. We draw *BD* <and> *BZ*, and we draw *BE* perpendicular to *AZ*. Then *GD* is equal to *DZ* and *DB* is common. Thus, *GD* <and> *DB* are equal to *ZD* <and> *DB* <respectively>, and the angle *ZDB* is equal to the angle *BDG* because they are <subtended> in two equal arcs. So the base *BG* is equal to the base *BZ*. But *AB* is equal to *BG*. So *AB* is equal to *BZ*. So the triangle *ABZ*



is isosceles. The perpendicular *BE* is drawn from <the vertex of> the angle *ABZ*, so *AE* is equal to *EZ*. Since the triangle *ABD* is right-angled and the perpendicular *BE* is drawn from its right angle, the triangles *ABD* <and> *ABE* are similar. So the ratio of *DA* to *AB* is equal to the ratio of *BA* to *AE*. So the product of *DA* by *AE* is equal to the square of *AB*. Each of *DA* <and> *AE* are known, so the square of *AB* is known. So, its square root, i.e. the Chord *AB*, is known. This is what we wanted to demonstrate.

Chapter 9: On finding the quantity of the Chord of the sum of two arcs whose Chords are known.

Let *ABG* be a circle with *E* as its center. We assume the two known Chords *AB* <and> *BG* in it. We draw *AG*. Then I say that *AG* is known.

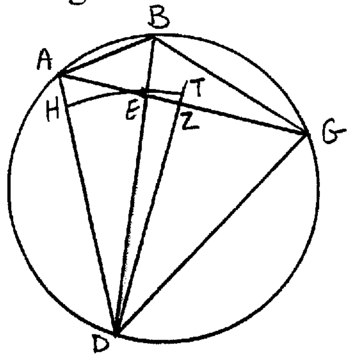


Proof: We draw the diameter *BD* from *B*, and draw *AD* <and> *DG*. Then *AD* is the Chord of the supplement of *AB* <to a semicircle> and *GD* is the Chord of the supplement of *BG* <to a semicircle>, and <hence> they are known. So the product of *AB* by *GD* and <the product of> *BG* by *AD* <added together> is equal to the product of *BD* by *AG*. But each of *AB*, *GD*, *BG*, <and> *AD* is known and the diameter *BD* is known. Then the Chord *AG* is known. This is what we wanted to demonstrate.

Chapter 10: On a premise for what follows.

If there are two unequal chords in a circle, then the ratio of the greater chord to the smaller chord is less than the ratio of the arc of the greater chord to the arc of the smaller chord. Let *ABGD* be the circle circumscribing them (i.e., the chords). <Inscribed> in it are the chords *AB* <and> *BG*, and *BG* is the greater <chord> of the two. I say that the ratio of the chord *BG* to the chord *BA* is less than the ratio of the arc *BG* to the arc *BA*.

Proof: We bisect the angle *ABG* by the line <segment> *BD*. We draw the <line segments> *AG*, *AD*, and *GD*. Since the angle *ABG* is bisected by the line <segment> *BD*, the line <segment> *GD* is equal to the line <segment> *AD*. But the line <segment>

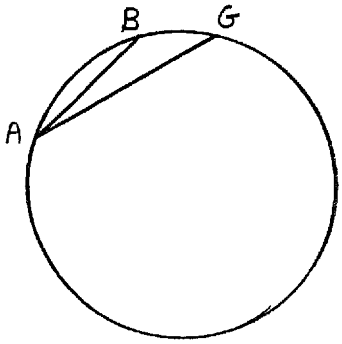


GE is longer than the line <segment> *EA*. We draw from *D* to the line <segment> *AG* the perpendicular *DZ*. Since *AD* is longer than *ED*, and *ED* is longer than *DZ*, the circle drawn with the center *D* and radius *DE* intersects *AD* and goes beyond *DZ*. We draw <an arc such> as *HET* and we extend *DZ* to *T*. Since the sector *DET* is greater than the triangle *DEZ* and the triangle *DEA* is greater than the sector *DEH*, the ratio of the sector *DET* to the sector *DEH* is greater than the ratio of the triangle *DEZ* to the triangle *DEA*. The ratio of the triangle *DEZ* to the triangle *DEA* is equal to the ratio of the line <segment> *EZ* to *EA*. The ratio of the sector *DET* to the sector *DEH* is equal to the ratio of the angle *ZDE* to the angle *EDA*. Then the ratio of the line <segment> *ZE* to the line <segment> *EA* is less than the ratio of the angle *ZDE* to the angle *ADE*. Componendo, the ratio of the line <segment> *ZA* to the line <segment> *EA* is less than the ratio of the angle *ZDA* to the angle *ADE*. The ratio of halves is equal to the ratio of <their>doubles. So, the ratio of the double of *AZ*, which is *GA*, to *AE*, is less than the ratio of the double of the angle *ZDA*, which is the angle *GDA*, to the angle *ADE*. Separando, the ratio of the line <segment> *GE* to *EA* is less than the ratio of the angle *GDE* to the angle *EDA*. The ratio of *GE* to *EA* is equal to the ratio of the chord *GB* to the chord *BA*, and the ratio of the angle *GDB* to the angle *BDA* is equal to the ratio of the arc *GB* to the arc *BA*. So, the ratio of the chord *GB* to the chord *BA* is less than the ratio of the arc *GB* to the arc *BA*. This is what we wanted to demonstrate.

Chapter 11: On measuring the Chord of 1° very accurately and the composition of the <table of the> Chords.

It was shown in Chapter 7 how to find the Chord of the difference between a sixth and a fifth of a circle, which is <equal to> the Chord of

12°. From Chapter 8, <we can find> the Chord of its half and half of its half, up to the Chord of one and half degrees, and the Chord of a half plus a quarter of a degree. After this <introduction>, we draw a circle <with the points> A, B, <and> G on it. First, we take the line <segment> AB as the Chord from the circle of the arc of a half plus a quarter of a degree, and AG as the Chord of 1°. Then the ratio of the Chord of AG to the Chord of AB is less than the ratio of the arc AG to the arc AB. The arc AG is equal to the arc AB plus one-third of it. So the Chord AG is less than the Chord AB plus one-third of it. The Chord AB plus one-third of it <is> 0; 1, 2, 49, 52. Again we take in this circle the line <segment> AB as the Chord of 1° and the line <segment> AG as the Chord of one and a half degree. Then the arc AG is equal to the arc AB plus half of it. So the Chord AG is less than the Chord AB plus half of it. So the Chord AB is



greater than two thirds of the Chord AG. Two thirds of the Chord AG <is> 0; 1, 2, 49, 48. Since the Chord of 1° is once <found> less and another time more than the same thing exactly, without a <noticeable> magnitude difference, if half the difference is added to the smaller value, the Chord of 1° is found with the closest approximation <equal to> 0; 1, 2, 49, 50. After knowing this, <I add that> in Chapter 7 <finding> the Chord of the sum of two arcs has been explained. <Since> the Chord of 1° is known, the Chord of 2° is <also> known. Again, the Chord of 1° and the Chord of 2° are known, so the Chord of 3° is <also> known. Again, the Chord of 1° and the Chord of 3° are known, so the Chord of 4° is known as well. On this basis we compose the Chords of <any number of> degrees up to 90° and put them in the table. This is what we wanted to demonstrate.

Commentary

Book IV actually has 70 chapters as found in F, V, L, and M. However, F mentions the number of the chapters to be 60, whereas L, M and V mention it to be 66 in the contents list provided in the opening of the Book. The number of the chapters in Y and A, which lack chapters IV.3.4, IV.3.6, IV.3.7, and IV.4.1, is equal to 66. Most parts of this section are direct quotations from the *Almagest*. The contents of IV.1 are also found in al-Battānī's *zīj* [1899, III, 13-14] without any proof.

IV.1.1 This introductory chapter defines the terms Chord, Sine, Sagitta, and complement or supplement of any arc.

IV.1.2 This method is found in Ptolemy's *Almagest* I.10 [1984, 50].

IV.1.3 In Ptolemy's *Almagest* I.10 [1984, 49-50], the approximate value of the Chord of 90° is found by this method.

IV.1.4 Again, the approximate value of the Chord of 120° is found by this method in Ptolemy's *Almagest* I.10 [1984, 49-50].

IV.1.5 This proof is found in Ptolemy's *Almagest* I.10 [1984, 48-49].

IV.1.6 This is usually called Ptolemy's theorem. Kūshyār presents a proof similar to that in Ptolemy's *Almagest* I.10 [1984, 50-51].

IV.1.7 Kūshyār's proof is found in Ptolemy's *Almagest* I.10 [1984, 51].

IV.1.8 This proof is found in Ptolemy's *Almagest* I.10 [1984, 52-53].

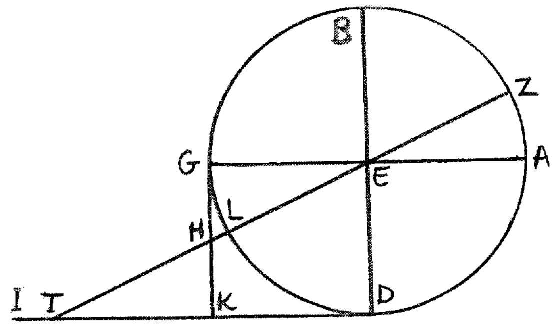
IV.1.9 This theorem is also proved by using Ptolemy's theorem in Ptolemy's *Almagest* I.10 [1984, 53]. However, Kūshyār's proof is simpler.

IV.1.10 When speaking about [the ratios of] sectors or triangles, Kūshyār means [the ratios of] their areas. This proof is found in Ptolemy's *Almagest* I.10 [1984, 54-55].

IV.1.11 Ptolemy presented the same method in *Almagest* I.10 [1984, 55-56] with a less accurate result (1; 2, 50) compared to Kūshyār's result 1; 2, 48, 50). In I.2.1 Kūshyār takes Sine 1° equal to 1; 2, 49, 38, 31. Both Kūshyār's values are correct to 2 sexagesimal digits. Sin 1° is 1; 2, 49, 43, 11 correct to 4 sexagesimals.

Section 2: On Tangents and Cotangents, <in> three chapters
Chapter 1: On the description of Tangents and Cotangents.

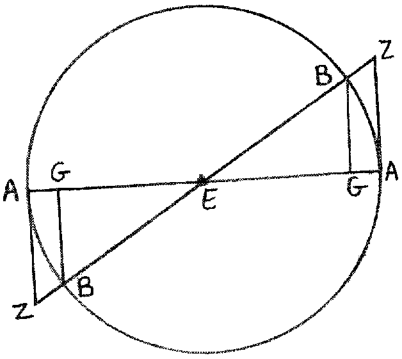
Let *ABGD* be the altitude circle with *E* as its center, and <let> *DI* be the intersection of the plane of the altitude circle and the horizon circle, and <let> *DE* be the vertical gnomon perpendicular <to the horizon> at the point *D*, and <let> *GK* be the intersection of the plane of the altitude circle and the plane perpendicular to the horizon <plane>, and <let> *GE* be the gnomon parallel to the horizon plane, perpendicular to the above mentioned (i.e., the vertical) plane at the point *G*.



We assume *AZ* <as> the altitude arc. We draw *ZET* which is the ray joining the tip of the gnomon and the endpoint of the shadow. *DT* is the shadow of the gnomon *DE*, and it is <called> the Horizontal Shadow and <also> the Cotangent (lit., "Second Shadow") of the altitude *AZ*. *GH* is the shadow of the gnomon *GE*, and it is <called> the Reversed Shadow and <also> the Tangent (lit., "First Shadow") of the altitude *AZ*. If we assume *BZ* as the altitude <arc>, then *GE* will be the gnomon for the Horizontal Shadow (Cotangent) and *DE* will be the gnomon for the Reversed Shadow (Tangent). So, *DT* will be the Tangent of the altitude *BZ* and *GH* will be its Cotangent. But *BZ* is the complement of *AZ*. So the Tangent of any altitude is the Cotangent of the complement of this altitude. The Reversed Shadow (Tangent) is called the First <Shadow> because it begins to appear and to increase <simultaneously> with the appearance and increase of the altitude of the sun. The Second Shadow (Cotangent) decreases with increasing altitude. This is what we wanted to demonstrate.

Chapter 2: On finding the quantity of the (i.e., any) Tangent.

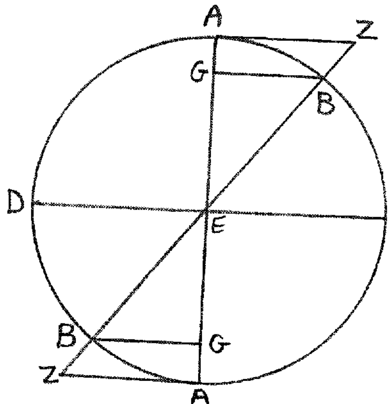
Let *ABGD* be the altitude circle centered at *E* and <let> *AEA* be its diameter, and <let> *AB* be the altitude arc. We draw *EBZ*, and we draw *AZ* perpendicular to *AE*. We also draw *BG* perpendicular to *AE*. Then *AZ* is the Tangent of the altitude *AB*. I say that it is known.



Proof: *ZA* and *BG* are both perpendicular to *AE*, so they are parallel. So the ratio of *ZA* to *AE* is as the ratio of *BG* to *GE*. But *AE* is the radius <of the circle> and it is equal to the gnomon <which may be> assumed <as divided> into any <number of> parts, and *BG* is the Sine of the arc *AB* and *GE* is equal to its Cosine. Therefore *AZ* is known. This is what we awnted to demonstrate.

Chapter 3: On finding the quantity of the (i.e., any) Cotangent.

Let *ABGD* be the altitude circle centered at *E*, and *AEA* and *DED* its two <perpendicular> diameters. We assume arc *DB* as the altitude. We draw *EBZ* and we draw *AZ* perpendicular to *AE*. We also draw *BG* perpendicular to *AE*. Then *AZ* is the Cotangent of the altitude *DB*. I say that it is known.



Proof: *ZA* and *BG* are both perpendicular to *AE*, so, they are parallel. So the ratio of *ZA* to *AE* is as the ratio of *BG* to *GE*. But *AE* is the radius <of the circle> and it is equal to the gnomon <which may be> assumed <as divided> into any <number of> parts, and *BG* is the Cosine of the altitude and *GE* is equal to the Sine of the altitude. Therefore *AZ* is known. This is what we wanted to demonstrate.

Commentary

IV.2.1 In the Islamic-period trigonometry the function $60\text{tg}\theta$ was called الظل الاول (*al-ẓill al-awwal*, lit. "the first shadow"), الظل المعكوس (*al-ẓill al-ma'kūs*, lit. "the reversed shadow"), الظل المنكوس (*al-ẓill al-mankūs*, lit. "the inverted shadow"), or الظل المنتصب (*al-ẓill al-muntaṣab*, lit. "<vertically> erected shadow"). Similarly, the function $60\text{cotg}\theta$ was called الظل الثاني (*al-ẓill al-thānī*, lit. "the second shadow"), الظل المستوي (*al-ẓill al-mustawī*, lit. "the horizontal shadow"), الظل المبسوط (*al-ẓill al-mabsūṭ*, lit. "the <horizontally> extended shadow"), or الظل البسيط (*al-ẓill al-basīṭ*, lit. "the plain shadow") [Kennedy 1956, 140; al-Bīrūnī 1954, I, 332-54; al-Bīrūnī 1985, 127-29]. Whenever the term "shadow" was used without any adjective, it meant the First Shadow (Tangent) [al-Bīrūnī 1985, 129]. In this work, I always translate "the first shadow, meaning $60\text{tg}\theta$, as the Tangent, and "the second shadow", meaning $60\text{cotg}\theta$, as the Cotangent.

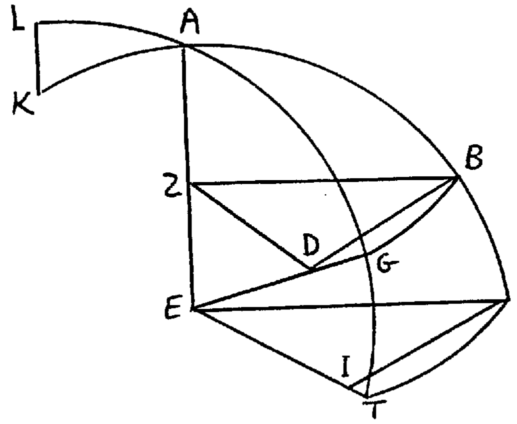
I do not know why Kūshyār uses the expression "horizon circle" and not merely "horizon", while al-Bīrūnī [1985, 127] uses the correct expression "horizon plane". Ptolemy [1984, 80-82] discusses the noon shadows at equinoxes and solstices, but he does not mention "Shadows" as trigonometric functions. Al-Battānī [1899, III, 31-33] discusses the horizontal and vertical shadows in their proper meaning and describes their calculation in terms of the Chords and vice versa. In the commentary to this and the following section, I always refer to al-Bīrūnī's *Maqālīd 'ilm al-hay'a* ("The keys to astronomy") [al-Bīrūnī 1985], because this was a standard work on spherical trigonometry, and the first independent treatise on the subject, whose author was a contemporary of Kūshyār and was aware of Kūshyār's work [al-Bīrūnī 1985, 101, 103, 143, 145].

IV.2.2 A similar method using a different figure is provided by al-Bīrūnī [1985, 129].

IV.2.3 A similar method using a different figure is provided by al-Bīrūnī [1985, 127].

Section 3: On premises on which the proofs are based, <in> 7 chapters
Chapter 1: On a general premise for most proofs.

<In> any triangle consisting of arcs of great circles on the sphere, in which one angle is right and another angle is assumed, the ratio of the Sine of the side subtending the right angle to the Sine of the side subtending the given angle, is equal to the ratio of the greatest Sine to the Sine of the assumed angle. Let the triangle be *ABG*, and its right angle be *G*, and the assumed <angle> be *BAG*. I say that the ratio of the arc *AB* to the Sine of the arc *BG* is equal to the ratio of the greatest Sine to the Sine of the angle *BAG*.

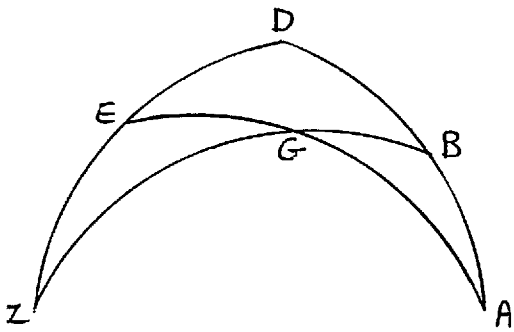


Proof: The center of the sphere is *E*. We draw *AE*. We complete each of the arcs *AB* and *AG* to quadrants, *AH* and *AT*. We take *A* as the pole and we draw the arc *HT* with radius equal to the side of an <inscribed> square. Then angle *HTG* is right. We draw *GE* and *TE* both equal to the radius of the circle *AGT*. Then they are in the plane of that circle. We draw *BD* perpendicular to *GE*, and *HI* perpendicular to *TE*. Then they are perpendicular to the plane of the circle *AGT*. We draw *BZ* perpendicular to *AE* and similarly, *HE* perpendicular to it. Then they are in the plane of the circle *ABH*. We draw *DZ*. Then *BZ* is the Sine of the arc *AB*, *BD* is the Sine of the arc *BG*, *HE* is the greatest Sine, and *HI* is the Sine of the arc *HT*, and it is <also> the Sine of the angle *BAG*. Since *BD* and *HI* are perpendicular to the plane of the circle *AGT*, all lines drawn <in the plane of circle *AGT*> from <any of> the two points *D* and *I* make a right angle with the perpendicular. So, the angles *D* and *I* are right. Thus, *BZ* and *HE* are parallel, *BD* and *HI* are parallel; then, *ZB* and *BD* are parallel to *EH* and *HI*<, respectively>. So, the angle *ZBD* is equal to the angle *EHI*. The angles *D* and *I* are right, then the angles *Z* and *E* of the two triangles are equal. So the two triangles *ZBD* and *EHI* are similar, so the ratio of *ZB* to *BD* is equal to the ratio of *EH* to *HI*. It has previously been said that *ZB* is the Sine of the arc *AB*, *BD* is the Sine of the arc *BG*, *EH* is the greatest

Sine, and *HI* is the Sine of the angle *HAT*. Therefore, the ratio of the Sine of the arc *AB* to the Sine of the arc *BG* is equal to the ratio of the greatest Sine to the Sine of the angle *HAT*. This is what we wanted to demonstrate.
Now it has become clear that in any two triangles in the sphere with two <correspondingly> equal angles, and two right angles, the ratio of the Sine of the side subtending the right angle in one triangle to the Sine of the side subtending the <other> equal angle is equal to the ratio of the Sine of the side subtending the right angle in the other triangle to the Sine of the side subtending the angle corresponding to the first one.
<Proof:> On the basis of this rule (i.e., the general premise), if the angle *L* in the triangle *AKL* is right, we may mount the arc *AL* on the arc *AG* as the arc *AK* may be mounted on the arc *AB*, because the angles *A* <in the two triangles> are equal, and the ratio of the Sine of the arc *AK* to the Sine of the arc *LK* will be equal to the ratio of the Sine of the arc *AH* to the Sine of the arc *HT*. Similarly, if the angle *K* is right, and we mount the arc *AK* on the arc *AG*, as the arc *AL* may be mounted on the arc *AB*, then the ratios are those ratios <which we just described>.

Chapter 2: On another premise which derives from the first one.

<In> any triangle consisting of arcs of great circles on the sphere, in which one angle is right, the ratio of the Cosine of one of the two sides encompassing the right angle to the Cosine of the hypotenuse of the right angle is equal to the ratio of the greatest Sine to the Cosine of the third side. Let the angle *B* in the triangle *ABG* be right; then I say that the ratio of the Cosine of *BG* to the Cosine of *GA* is equal to the ratio of the greatest Sine to the Cosine of *AB*.



Proof: We take *A* as the pole and we draw the circle *DHZ* with distance equal to side of an <inscribed> square. We complete the quadrants *DEZ*, *AGE*, *ABD*, and *BGZ*; then the angle *E* in the triangle *ZGE* is right. So, according to what was demonstrated in the first premise, the ratio of the Sine of *ZG* to the Sine of *GE* is equal to the ratio of the greatest Sine to the Sine of the angle *Z*. But *ZG* is the complement of *BG*, *GE* is the complement of *AG*, and *BD* is the arc of the angle *Z*, and it is the complement of *AB*. Thus the ratio of the Cosine of *BG* to the Cosine of *AG* is equal to the ratio of the greatest Sine to the Cosine of *AB*. This is what we wanted to demonstrate.

Chapter 3: Notice on the properties of the proportional magnitudes.

If there are four proportional magnitudes, and <also> four others making another ratio, so that the ratios are not in continued proportion, and the two means of the first <proportion> are equal to the two means of the other <respectively> (First example), then by composition <of ratios>, the ratio of the <first> antecedent to the <second> antecedent is equal to the ratio of the <first> consequent to the <second> consequent, in inverse order. Also, the ratio of the <first> antecedent to the <second> consequent is equal to the ratio of the <second> antecedent to the <first> consequent, in inverse order. If the two antecedents of the first <proportion> are equal to the two antecedents of the second (Second example), <respectively,> then the ratio of the <first> consequent to the <second> consequent of the first <proportion> is equal to the ratio of the <first> consequent to the <second> consequent of the other <proportion>. If the two consequents of the first <proportion> are equal to the two consequents of the other <proportion> (Third example), then the ratio the <first> antecedent to the <second> antecedent of the first <proportion> is equal to the ratio of the <first> antecedent to the <second> antecedent of the other <proportion>. This is what we wanted to mention.

First example

A	**B**	**G**	**D**
2	**4**	**3**	**6**
E	**W**	**Z**	**H**
1	**4**	**3**	**12**

Second example

A	**B**	**G**	**D**
2	**4**	**3**	**6**
E	**W**	**Z**	**H**
2	**8**	**3**	**12**

Third example

A	**B**	**G**	**D**
2	**4**	**3**	**6**
E	**W**	**Z**	**H**
1	**4**	**1; 30**	**6**

Proof: <In the first example,> the ratio of **A** to **B** is equal to the ratio of **G** to **D**, the ratio of **E** to **W** is equal to the ratio of **Z** to **H**, and **B** is equal to **W** and **G** is equal to **Z**. <So> the product of **B** by **G** is equal to the product of **A** by **D**, and the product of **W** by **Z** is equal to the product of **E** by **H**. We cast out the equal <product of the means>. It follows that the product of **A** by **D** is equal to the product of **E** by **H**. Thus the ratio of **A** to **H** is equal to the ratio of **E** to **D**.

<In> the second example, the product of **B** by **G** is equal to the product of **A** by **D**, and the product of **W** by **Z** is equal to the product of **E** by **H**. But **A** is equal to **E** and **G** is equal to **Z**. Thus the product of **B** by **Z** is equal to the product of **E** by **D** and the product of **G** by **W** is equal to the product of **A** by **H**. We cast out the equal <product of the means>. It follows that the product of **B** by **H** is equal to the product of **W** by **D**. Thus the ratio of **B** to **W** is equal to the ratio of **D** to **H**.

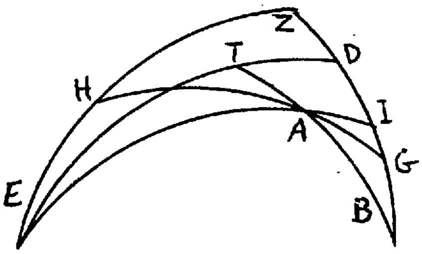
<In the> third example, the product of **B** by **G** is equal to the product of **A** by **D**, and the product of **W** by **Z** is equal to the product of **E** by **H**. <Now,> **B** is equal to **W**, and **D** is equal to **H**. Then the product of **B** by **Z** is equal to the product of **E** by **D**, and the product of **G** by **W** is equal to the product of **A** by **H**. We cast out the equals. There remains that the product of **A** by **Z** is equal to the product of **E** by **G**. Then the ratio of **A** to **E** is equal to the ratio of **G** to **Z**. That is what we wanted to demonstrate.

What <we proved> in this notice on proportions <may be> summarized <as follows:> If the second <term>s are equal and the third <term>s are equal, the ratio of the first <term of the first proportion> to the first <term of the second proportion> will be equal to the ratio of the fourth <term> to the fourth <term>, in inverse order. If the first <term>s are equal and the third <term>s are equal, the ratio of the second <term of the first proportion> to the second <term of the second proportion> will be equal to the ratio of the fourth <term> to the fourth <term>, in the same order. If the second <term>s are equal and the fourth <term>s are equal, the ratio of the first <term> to the first <term> is equal to the ratio of the third <term> to the third <term>, in the same order, and the ratio of the first <term> to the third <term of the first proportion> is equal to the ratio of the first <term> to the third <term of the second proportion, in the same order>.

Chapter 4: On another premise which also derives from the first one.

<In> any triangle consisting of arcs of great circles, the ratio of the Sine of an angle of it to the Sine of another angle <of it> is equal to the Sine of the side subtending the first angle to the Sine of the side subtending the

other angle. <Let> the triangle *ABG* have different sides and angles; then I say that the ratio of the Sine of the angle *B* to the Sine of the angle *G* is equal to the Sine of the arc *AG* to the Sine of the arc *AB*.

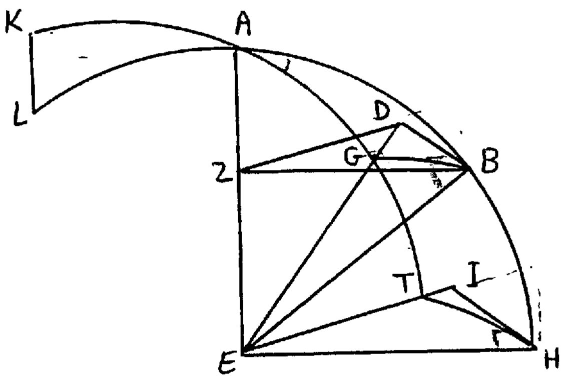


Proof: We take *B* as the pole and we draw <the circular arc> *DTE* with a distance equal to side of an <inscribed> square. We take *G* as the pole and we draw *ZHE* <similarly>. We complete each of <the arcs> *BGZ*, *BAT*, and *GAH*, and we draw *EAI*. Since *B* is the pole of *ETD*, *BT* and *BD* are quadrants. Since *E* is the pole of *BDZ*, *EZ*, *ED*, and *EI* are quadrants. Since *G* is the pole of *ZHE*, each of <the arcs> *GE*, *GZ* are quadrants. Then, in the triangle *BAI* the angle *I* is right. So, the ratio of the Sine of *BA* to the Sine of *AI* is equal to the ratio of the greatest Sine, which is the Sine of *BT*, to the Sine of *TD*. Also the angle *I* in the triangle *GAI* is right. So the ratio of the Sine of *GA* to the Sine of *AI* is equal to the ratio of the greatest Sine, which is the Sine of *GH*, to the Sine of *HZ*. Since the means of the first <proportional> magnitudes, i.e., <the Sines of> *AI* and *BT*, are equal to the means of the other <proportional> magnitudes, i.e., <the Sines of> *AI* and *GH*, therefore the ratio of the Sine of *BA*, which is the side subtending the angle *G*, to the Sine of *AG*, which is the side subtending the angle *B*, is equal to the ratio of the Sine of *HZ*, which is equal to the Sine of the angle *G*, to <the Sine of> *TD*, which is the Sine of the angle *B*. Thus the ratio of the Sine of an angle to the Sine of <another> angle <in any triangle> is equal to the Sine of the side subtending the <first> angle to the Sine of the side subtending the <other> angle. This is what we wanted to demonstrate.

Chapter 5: On a premise concerning the Tangent<s>, which is a substitute for the first premise in most proofs.

<In> any triangle consisting of arcs of great circles, in which an angle is right and another angle is assumed, the ratio of the Sine of the side between the right angle and the assumed angle to the Tangent of the side subtending the assumed angle is equal to the ratio of the greatest Sine to the Tangent of the assumed angle. Let *ABG* be the triangle where angle *B* is right and *BAG* is the assumed angle. Then I say that the ratio of the

Sine of the arc *AB* to the Tangent of the arc *BG* is equal to the ratio of the greatest Sine to the Tangent of the angle *BAG*.

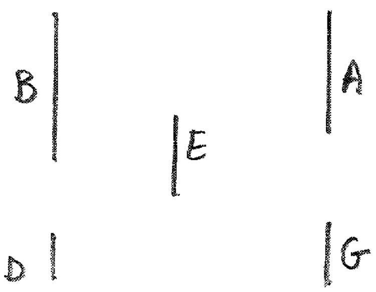


Proof: With *E* as the center of the sphere, we draw *AE* and we complete each one of <the arcs> *AB* and *AG* into the quadrants *AH* and *AT*. We draw *HE* and *BZ* perpendicular to *AE*. We take *A* as the pole and we draw the arc *HT* with a distance equal to the side of an <inscribed> square. We draw *EG* and *ET*, the radii of the circle *AGT*, and we extend them to *D* and *I*. We draw *EB*, a radius of the circle *ABH*. We draw *BD* and *HI* perpendicular to the diameters <containing> *EB* and *EH* at *B* and *H*. We draw *DZ*. Then *BZ* is in the plane of *ABH*, so it is the Sine of the arc *AB*. *HE* is also in that plane, and it is the greatest Sine. *ZB* and *HE* make right angles with the perpendiculars *BD* and *HI*, <respectively>. So the two planes *BZD* and *HET* are parallel (see commentary). *BD* is perpendicular to the diameter <containing> *EB*, and therefore, perpendicular to the plane *ABH*. Then all lines drawn in the plane *ABH* make right angles with the perpendicular *BD*. So the angle *DBZ* is right and the two angles *HEI* and *BZD* are equal. Then the two triangles *HEI* and *ZBD* are similar. So the ratio of *ZB* to *BD* is equal to the ratio of *EH* to *HI*. But *ZB* is the Sine of the arc *AB*, *BD* is the Tangent of the arc *BG*, *EH* is the greatest Sine, and *HI* is the Tangent of the angle *HAT*. So the ratio of the Sine of the arc *AB* to the Tangent of the arc *BG* is equal to the ratio of the greatest Sine to the Tangent of the angle *BAG*. This is what we wanted to demonstrate.
Now it has become clear that in any two triangles in a sphere having two <respectively> equal angles and two right angles, the ratio of the Sine of the side between the right angle and the <other> equal angle to the Tangent of the other one of the two sides containing <the right angle> is equal to the ratio of the Sine of the corresponding <side> to the Tangent of the <other> corresponding side in the other triangle. This rule is valid,

because in the triangle *ALK*, the proof is equal to the ratio of the first premise, whether the angle *K* or the angle *L* is right.
The spherical right triangles will be known by these premises in <the following> three cases.
First <case>: An angle with one of the sides, either the side subtending the right angle or the side subtending the known angle, <are given. Solution:> The ratio of the Sine of the side subtending the right angle to the Sine of the side subtending the known angle is equal to the ratio of the greatest Sine to the Sine of the known angle.
Second <case>: Any two of its sides <are given. Solution:> The ratio of the Cosine of one of the sides containing the right angle to the Cosine of the side subtending the right angle is equal to the ratio of the greatest Sine to the Cosine of the third side; the ratio of the Sine of the side subtending the right angle to the Sine of the other <known> side is equal to the ratio of the greatest Sine to the Sine of the angle <opposite to> the other known side.
Third <case>: An angle with a side adjacent to it, namely one of the two sides containing the right angle, <are given. Solution:> The ratio of the Sine of this side to the Shadow of the other side containing the right angle is equal to the greatest Sine to the Shadow of the known angle.

Chapter 6: Notice on the properties of the Tangent<s>.

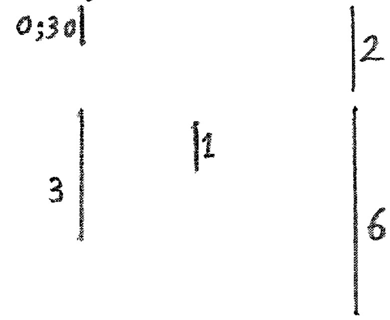
<Given> any two different arcs, their Tangent is the inverse of their Cotangent. Let **A** and **B** be the Tangents of two different arcs, **G** and **D** be their Cotangents, and **E** be the gnomon. For the arc whose Tangent is **A**, let its Cotangent be **G**, and for the arc whose Tangent is **B**, let its Cotangent be **D**; then I say that the ratio of **A** to **B** is equal to the ratio of **D** to **G**.



Proof: The ratio of **A** to **E** is equal to the ratio of **E** to **G**, and the ratio of **B** to **E** is equal to the ratio of **E** to **D**. Then the product of **A** by **G** is equal to the product of **B** by **D**. So, the ratio of **A** to **B** is equal to the ratio of **D** to **G**.

Chapter 7: Another notice also on the properties of the Tangent<s>.

Something divided by the Tangent or Cotangent of any arc is equal to the <same> thing multiplied by its (i.e., the arc's) Cotangent or Tangent <, respectively>. <Example:> Let the Cotangent of a given arc be 2, its Tangent be 0; 30, and <the length of> the gnomon be 1, that is, unity. <If> the magnitude 6 is divided by 2, <the result> is 3, and then I say that 3 is equal to the product of 6 by 0; 30.



Proof: 6 divided by 2, is 3. Then the product of 2 by 3 is 6, and the product of 2 by 0; 30, i.e. a half is 1, because the ratio of 2 to 1 is equal to that of 1 to 0; 30. Then the ratio of 0; 30 to 3 is equal to that of 1 to 6. The product of 0; 30 by 6 is equal to that of 1 by 3. The product of 1 by 3 is 3, because 1 is <the length of> the gnomon which is taken <equal to> unity. Then the product of 0; 30 by 6 is 3. Since the Cotangent of any arc is <equal to> the Tangent of its complement, something divided by the Tangent of any arc is equal to the <same> thing multiplied by the Tangent of its complement. This is what we wanted to demonstrate.

Commentary

This chapter has already been translated into English and published with an introduction, summary and commentary based on ms. L in [Berggren, 1987]. I have mentioned the differences of that translation with mine, whenever they are significant. There are also some minor differences mostly due to the differences between the two mss.

IV.3.1 This is a special case of the Sine Theorem for right spherical triangles. Al-Bīrūnī [1985, 101,103] notes that Kūshyār took this theorem from Abū Maḥmūd Khujandī, named it *al-Mughnī* (lit., "making [one] able to dispense" [with Menelaus' Theorem]), and abridged Khujandī's proof of it. Al-Bīrūnī later quotes Kūshyār's abridged version of the proof [*op. cit.*, 143, 145] and adds that Kūshyār did not find the generalized form of this theorem. However, we find a generalization of it in IV.3.4. The second proof at the end of this chapter seems to be a later addition which exists in F but is not found in A and Y, and has later been added to the mss. V and L.

IV.3.2 This is equivalent to the Cosine Theorem for right spherical triangles. According to al-Bīrūnī [*op. cit.*, 151], Abū'l-ʿAbbās al-Nayrīzī and Abū Jaʿfar al-Khāzin in their non-extant commentaries on Ptolemy's *Almagest* presented this theorem. Al-Bīrūnī then quotes the proof provided by them, which is more complicated than Kūshyār's.

IV.3.3 Here Kūshyār states the following theorems. If A:B=G:D and E:W=Z:H, then:

(a) If B=W, G=Z, we have A:E=H:Z and A:H=E:Z
(b) If A=E, G=Z, we have B:W=D:H.
(c) If B=W, D=H, then A:E=G:Z.

Kūshyār supposes that "the ratios are not in continued proportion", i.e., A:B≠E:W. If A:B=E:W, we would have A=E, B=W, G=Z, and D=H in all three cases (a), (b), and (c).
In the ms. L, the part on the proofs of the three propositions which comes before the final part, the summary of the propositions of this notice, is transferred to Section 5 of this chapter, just before the proof of the Tangent Theorem. Prof. Berggren who has based his English translation of this chapter on ms. L, has restored the misplaced fragment to the end of Chapter 3 [1987, 24, 28]. He shows that this is a piece skipped by the scribe who, after he had noted his error, simply put it at the first available

place. However, the proof for the third example is missing in Prof. Berggren's translation [*op.cit.*, 24].

IV.3.4 This is the general case of the Sine Theorem for a general spherical triangle. Kūshyār's proof is similar to the proof which al-Bīrūnī presents for this theorem [1954, 355-56]. This chapter is missing in the mss. A and Y.

IV.3.5 This is the Tangent Theorem for right spherical triangles which, according to al-Bīrūnī, was discovered by Abū al-Wafā' al-Būzjānī. Kūshyār's proof is similar to al-Būzjānī's as presented by al-Bīrūnī [1985, 131]. The second part of this chapter, which discusses the solution of right spherical triangles, was presented by al-Bīrūnī [*op. cit.*, 169-91] in more details: In proving that the planes *BZD* and *HET* are parallel, it must also be noticed that the spherical angles *B* and *H* are right angles, so the planes *HEI* and *BEG* are perpendicular to the plane of *ABH.* Therefore *BD* and *HI* are both perpendicular to the plane *ABH.* Prof. Berggren has translated the title of this chapter differently: "On a premise called 'the Shadow' established on the basis of the first premise in many of the proofs". The Arabic title in L is like what we read in this edition but written in a less clear form, and this title implies that the Tangent Theorem may be used instead of the first premise (the Sine theorem) in many proofs. At the end of the proof of this chapter, before mentioning the three cases of spherical right triangles, the sentence "This rule is valid, because ..." which involves a reference to a later addition in Chapter 1, also seems to be a later addition to F, and is not found in other mss. So it is not found in Berggren's translation either.

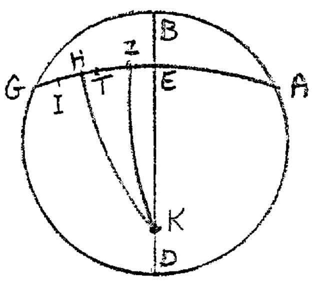
IV.3.6 and IV.3.7 Since the Tangent and Cotangent functions are inversely proportional by definition, these propositions are trivial. It is quite interesting that Kūshyār takes the length of the gnomon equal to unity, as we do now.

IV.3.7 In [Berggren 1987, 27] the Arabic characters involved in this chapter are supposed to denote magnitudes rather than their numerical values in the *Abjad* numeral system, possibly due to the misleading diagram in L, which was the only ms. accessible to Berggren. However, the present interpretation is more consistent and leaves no ambiguity about the content of the chapter.

Section 4: On <finding> the true longitudes of the planets and their positions, <in> 10 chapters

Chapter 1: On the equation of time.

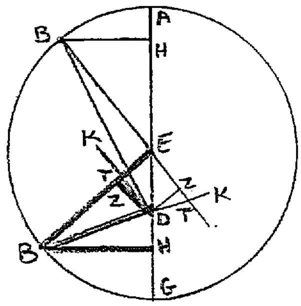
As has been said in Book III, this equation is the difference between the mean day and the true day. The mean day is <the duration of one> rotation of the celestial equator from the (i.e., any) meridian to the <same> meridian, plus the arc of it (i.e., the equator) equal to the mean daily motion of the sun. The true day is <the duration of one> rotation of the celestial equator from the (i.e., any) meridian to the <same> meridian, plus the part of it (the equator) which rises together with the varying <daily> motion of the sun (i.e., the difference between the right ascensions of the true solar longitudes at the end and the beginning of the day). <The maximum magnitude of> (i.e., an upper bound of) this equation is the sum of twice the difference between the <solar> ecliptical degrees and the <corresponding> right ascensions, plus twice the difference between the sun's mean longitude and its <corresponding> true longitude. That part relating to the difference of the <right> ascension <and the true longitude> is <at most> about 5 degrees, and that part relating to the difference of the <positions of the mean and the true> sun <on the ecliptic> is <at most> 4 degrees, approximately. Then the sum of the two <maximum> differences is approximately 9 <time->degrees, which is three-fifths of an equinoctial hour, minus a small amount. <However,> this equation never reaches the total <amount>, because while one of the two differences is maximum, the other is somewhat less than its maximum, except when the apogee is in the middle or in the last decan of Leo, because <the variation of> this equation in one or two days is not noticeable. We may define any position on the ecliptic as the base (i.e., the point of reference or zero point). But if we define the middle decan of Aquarius (as the zero point of the equation of time), the mean days will always exceed the true days, until it (i.e. the sun) reaches the aforesaid apogee. If another <position> is defined as the base (i.e., the zero point), the mean days sometimes exceed the true days and sometimes are less than them. Now I say that the <amount in> hours of the excess of the mean days over the true <days> is known.



Proof: *ABGD* is the horizon circle, *DB* the meridian, and *AEG* the celestial equator with *K* as its pole. Let point *E* be the mean longitude of the base, which is one of the degrees in the middle decan of Aquarius, and *Z* the <right> ascension of its true longitude. We draw *ZK*. Let point *T* be another mean longitude. The <right> ascension relating to its true longitude may be less or more than it. First we let it be more, as *H*. We draw *HK*. In the situation which we described, the difference between the two mean longitudes <here represented as arcs on the celestial equator> is greater than the difference between the <right> ascensions of the two true longitudes. Thus, the arc *ET* is greater than the arc *ZH*. *ZT* is common <to them> so *EZ* is greater than *TH*. Therefore, the time in which the arc *ET* passes the meridian is greater than the time in which the arc *ZH* passes the meridian, in the amount of the excess of *EZ* over *TH*. Each one of the arcs *ET*, *ZH*, *EZ*, and *TH* is known, so the excess of *ET* over *ZH* is known. Each 15 degrees of the equator <corresponds to> one hour. So the magnitude of this excess with respect to 15 degrees is known. So the excess of the mean days over the true days is known. It is the deficit of the true days from the mean days, if we want <to find> the mean days. Also let the point *I* be a mean longitude and the point *H* the right ascension of its true longitude, being less than it (i.e., the mean longitude). Thus *EI* is greater than *ZH* in <the amount of the sum of> the two arcs *EZ* and *HI*. The two arcs *EI* and *ZH* are known, so the sum of the two arcs *EZ* and *HI* is known. The time in which the arc *EI* passes the meridian is greater than the time in which the arc *ZH* passes <the meridian> in the amount of the sum of the arcs *EZ* and *HI*. But their amount with respect to 15 degrees is known. So the excess of the mean days over the true days is known. It is the deficit of the true days from the mean days, if we <to find> the mean days from the true days. This is what we wanted to demonstrate.

Chapter 2: On the equation of the sun.

<Let> *ABG* <be> the circle of the eccentric orb with *E* as its center and *AG* as its diameter, and <let> *D* <be> the center of the orbit representing the ecliptic (i.e., parecliptic; *D* is the center of the earth). Then *DE* is the eccentricity. It has been found to be <equal to> 2 parts and 4 minutes plus half and a quarter <of a minute>, based on <taking> *EA* <equal to>

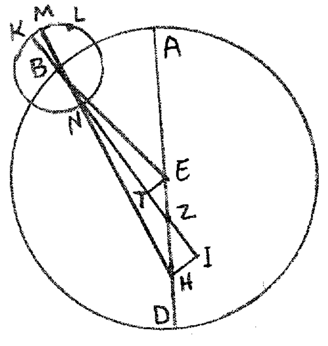


The bold lines are for the case in which the anomaly exceeds 90°

sixty parts. *A* is the position of the apogee, *B* is the body of the sun, and *AB* the solar mean anomaly. We drop *BH* perpendicular to *AE*. It is the Sine of the arc *AB*. <We drop> *DZ* perpendicular to *BZ*. The angle *ZED* is equal to the angle *HEB*, and the two angles *Z* and *H* are right. So the ratio of *EB* to *BH* is equal to the ratio of *ED* to *DZ*. *EB* is <equal to> sixty parts. *BH* and *ED* are known. So *DZ* is known, and *ZE* is known because *HE* is the Cosine of the mean anomaly (and *EB:HE=ED:ZE*). So *BZ* is known. The <sum of the> squares of *BZ* and *ZD* is equal to the square of *BD*. So *BD* is known. The ratio of *BD* to *DZ*, which is known (i.e., which has been computed) based on <taking> *BE* as the radius, is equal to the ratio of sixty to *DZ* in the magnitude in which it (i.e., *DZ*) is desired (i.e., we want *DZ* for *BD* = 60). So *DZ* based on <taking> *BD* as the radius is known. It is the Sine of the angle *ZBD*. So the angle *ZBD* is known, and it is the angle of the equation. That is what we wanted to demonstrate. Since *AEB* is an exterior angle of the triangle *BDE*, the angle *AEB*, which is the value of the mean anomaly, is greater than the angle *EDB*, which is the angle of the true longitude, in the amount of the angle *EBD* which is the angle of the equation. If the equation is to be subtracted from the mean anomaly or the mean longitude, <the angle of> the true longitude (i.e., *EDB*) and the mean anomaly are less than 180 <degrees>. If the mean anomaly is greater than 180 <degrees>, <we do> the opposite (i.e., we add the equation to the angle of the true longitude and the mean anomaly).

Chapter 3: On the first equation for the moon.

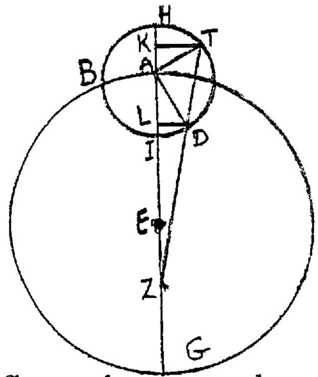
<Let> *ABG* <be> the circle of the eccentric orb (=deferent) with *E* as its center, *AD* as its diameter, *Z* as the center of the inclined orb (i.e., the lunar orbit; *Z* is the earth), *H* as the (prosneusis) point towards which the line joining the apogee and perigee of the epicycle, i.e. *M* and *N* is pointing, *LKG* as the epicycle centered at *B*, and *L* as the body of the moon. The angle *ZBH* is the angle of equation. Then *AZB* is the angle of double elongation. *EZ* and *ZH* are equal, each of them being <equal to> 12 parts and half based on <taking> *AE* <equal to> 60 parts. *ET* and *HI* are perpendicular to *BI*. The angle *EZT* is known, and the angle *T* is right. Then the remaining angle *E* as well as the sides of the triangle *EZT* are known.



EB is <equal to> 60 parts and its square is equal to the <sum of the> squares of *BT* and *TE*. So, *BT* and, therefore, the whole *BZ* are known. The angles of the triangle *EZT* are equal to those of the triangle *ZIH*. Then the ratio of *EZ* to *ZH* is equal to the ratio of *ZT* to *ZI* and to the ratio of *ET* to *HI*. *EZ* and *ZH* are equal. Then *IZ* and *ZT*, and *ET* and *HI* are equal. Then the whole *BI* is known and its square plus the square of *IH* is equal to the square of *BH*. Then *BH* is known. If we take the point *B* as the center and draw a circle with its radius equal to *BH*, then *HI* is the Sine of the angle *IBH* based on <taking> *BH* as the radius whose value is known (i.e., 60). Then *HI* is known, based on <taking> *BH* <equal to> 60 parts. Then the angle *IBH* is known (so *ZBH*, the angle of the first equation, is known). The angle *IBH* is equal to the angle *MBK*; so, the arc *MK* is known. *KL* is the adjusted anomaly based on <taking> *AB*, the double elongation, less than 90 <degrees>. If the double elongation is greater than 90 <but not greater than 180 degrees>, we find the angle of equation in the same way. If it is greater than 180 <degrees>, the equation is subtracted from the mean anomaly. That is what we wanted to demonstrate.

Chapter 4: On the second equation of the moon and the planets.

<Let> *ABG* <be> the circle of the eccentric orb centered at *E*, *Z* the center of the inclined orb, and *HTD* the epicycle centered at *A* (supposed to be the apogee). Let *T* <be> the position of the moon, because the motion of the moon is in this direction (i.e. clockwise on the epicycle). We join *TA* and *TZ*, and <we draw> *TK* perpendicular to *AH*. The angle *TZH* is the angle of equation. *TK* is the Sine of the adjusted mean anomaly, i.e. the arc *TH*, and *KA* is its Cosine. Each one of them is known based on <taking> *TA* <equal to> 5 parts and a quarter. Since the ratio of *AT* to *TK* is equal to that of the greatest Sine to the Sine of the <adjusted> anomaly and *ZA* is <assumed equal to> 60 parts, the whole *ZK* is known. Then its square plus that of *KT* is equal to *TZ* squared. Then *TZ* is known. If we take *Z* as the center and draw a circle with its radius <equal to> *ZT*, then *TK* is the Sine of the arc of equation angle in terms of the known value. But *TK* is known based on <taking> *TZ* <equal to> 60 parts. It is the Sine of the arc of the equation angle. In the same way, if we take the position of the moon at *D*, then *DI* is known, *DL* is its Sine and *AL* is its Cosine. Then *DL* is found by the previous method based on <taking> *ZD* <equal to> 60 parts. It is the Sine of *DZL*, the arc of the equation angle. That is what we wanted to demonstrate.

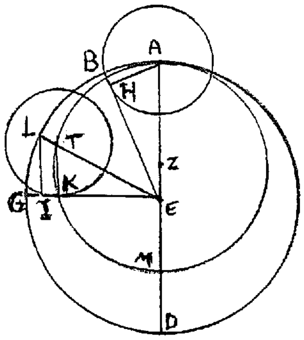


It is apparent from this figure that when the adjusted anomaly is less than 180 <degrees>, the arc of equation is subtracted from the mean longitude of the moon. When the <adjusted> anomaly is greater <than 180 degrees>, it (i.e., the arc of equation) is added to it (i.e., the mean longitude of the moon). For the other planets, if the adjusted anomaly is greater than 180 <degrees>, this equation is subtracted from the adjusted center. If the adjusted anomaly is less <than 180 degrees>, it (i.e., the arc of equation) is added to it (i.e., the adjusted center). This is because the motions of their bodies in the epicycles are in the opposite direction of that of the moon. That is what we wanted to describe.

Chapter 5: On the difference between the <apparent> radius of the epicycle between its maximum and minimum distances <from the earth>.

<If> the center of the epicycle of the moon is supposed <to be> at the maximum distance, and its distance from the center of the inclined orb is <taken as> 60 parts, <then> the radius of the epicycle in terms of this value is <equal to> 5 parts and a quarter. The maximum value of the second equation depends on the radius of the epicycle. The apparent value <of the equation> varies between the maximum and minimum distances, because the angle at the center of the inclined orb and subtended by the radius of the epicycle becomes greater when the center of the epicycle gets nearer to the center of the inclined orb. The same holds for the radius of the epicycles of the planets, however, <in this case> the centers of their orbs (i.e., epicycles) are supposed to be at mean distance when <the distance> between them and the center of the inclined orb is <taken to be> 60 parts. Between the mean distance and the maximum distance, the radius of their epicycles is less than the supposed value <of the radius>. Between the mean distance and the minimum distance, it (i.e., the radius of the epicycle) is greater than the supposed value <of the radius>. This is because the maximum distance for each planet is 60 <parts> plus half the eccentricity, and the minimum distance is 60 <parts> minus half the eccentricity. The ratio of each of these two (i.e., the minimum and the maximum) distances to the Sine of the <maximal> second equation at the mean distance, is equal to the ratio of 60 <parts> to the Sine of the <maximal> second equation at that

<minimum or maximum> distance. The same holds for other distances (i.e., the Sine of the maximal value of the second equation for any distance is inversely proportional to the distance).

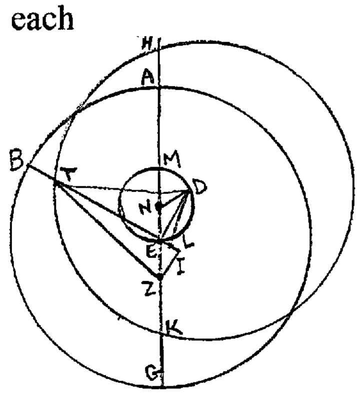


Let *ABGD* centered at *E* be the circle of the inclined orb; *ATM* centered at *Z* be the circle of the eccentric orb; *A* be the center of the epicycle at the maximum distance; and *T* be its center at another distance. We draw *EB* tangent to the circle at *H* and, we join *AH*. We draw *EG* tangent to the circle at *K* and we join *TK* and *LI* both perpendicular to *EG*. The angle *LEG* is greater than the angle *AEB*, because *ET* is smaller than *EA*. If we place it on *EA*, *TK* falls outside the line *EH*. *AH* is the radius of the epicycle at the maximum distance and its arc is *AB*. It subtends the angle *AEB*. Then *AB* is the maximum equation at maximum distance. *TK* is the radius of the epicycle at this <arbitrary> distance. The angle of equation is *LEG* and its arc is *LG*. Then *LG* is the maximum equation at this distance. The angle *LEG* is greater than the angle *AEB*. Then the arc *LG* is greater than the arc *AB*. The ratio of *ET* to *TK* is equal to that of *EL* to *LI*, because the two triangles *TEK* and *LEI* are similar. *ET* is known from the figure relating to the first equation (i.e., IV.4.3). It is <the distance> between the center of epicycle and the center of the inclined orb. The magnitude of *TK* is like *AH*, and *EL* is equal to *EA*. Then *LI* is known. It is the Sine of the arc *LG*. Then *LG* and its excess over *AB* are known. It is the total difference at this distance depending on <the length of> the line <segment> *ET*. The total difference at other distances is known in this <same> way. That is what we wanted to demonstrate.

We have written down the difference for the moon and the planets based on this calculation. We incorporated the approximate <magnitudes> of this equation (i.e., the difference, both for the moon and the planets) in a single table in a manuscript, but we do not intend <to compute a table like this here>. For the moon it (i.e., the difference) is uniformly additive from the maximum distance to the minimum distance. But for the planets <the difference is> diminutive from the maximum distance to the mean distance, and additive from the mean distance to the minimum distance. It is clear that this difference for the moon depends on the double elongation, which corresponds to *AL* in the figure. For the other planets, it depends on the adjusted centrum. Then we look for the sixtieths, their ratio to 60 minutes being equal to the partial second equation to the total <second> equation. If we multiply these sixtieths by the <total> difference for the moon, the amount of the <partial second> equation in that position will result. Since there is no equation at the apogee of the epicycle, no difference is necessary for it, and at maximum equation, the total difference is necessary. That is why the difference is taken in terms of double elongation for the moon, <but> for other planets <it is taken> in terms of the adjusted center. The sixtieths are found from the <corresponding values of> the mean anomaly. It has become clear for us from this proof that the epicycle radius difference of Mars at maximum distance is one part and a fifth less than what is written in Ptolemy's tables. At minimum distance, it is 2 parts and a fifth <less than Ptolemy's value>. This is something that we used in our calculation of the tables. But the value in the <present> treatise is correct and this difference in the calculations is inevitable.

Chapter 6: On the first equation for Mercury.

<Let> *ABG* <be> the circle of the equant orb; *E* its center; *AG* its diameter; *Z* the center of the inclined orb; *N* the center of the small circle carrying the center of the deferent for the center of the epicycle; and *M* the center of the deferent. We imagine that *M* moves and describes the arc *MD* <in the direction> opposite to the succession <of the zodiacal signs> equal to the <amount of the> motion of the sun <from *A*>, and the center of the epicycle moves <simultaneously> with *M* in the direction <of the zodiacal signs> until it moves from *H* to *T* and describes the arc *AB* of the circle *ABG* similar to the arc *DM*. We take *D* as the center and we draw the deferent <equal> to the equant in magnitude. It is *HTK*. We join *ETB*, *ZT*, *DT*, *DN*, *DE*, <and we draw> *DL* and *ZI* perpendicular to *BI*. *ITZ* is the angle of equation. The two angles *MND* and *AEB* are equal, because their arcs are similar and each

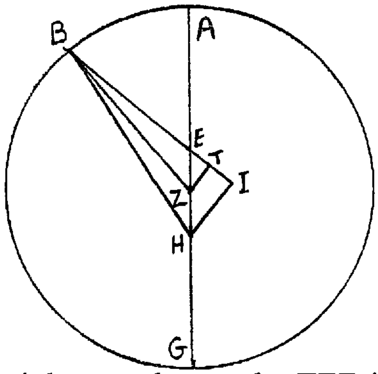


of them is <equal to> the angle of the centrum. So they are known. Each of the two arcs *MD* and *DE* are known. So, the chord *DE* is known in

terms of the greatest diameter. Its ratio to it (i.e., to the greatest diameter) is like the chord *DE* to the diameter *EM*. *EM* is equal to 6 parts and a third. Then the chord *DE* is known. The angle *DEN* is half the angle *DNM*. Then the angle *DEN* is half the angle *AEB*. Then the whole angle *DEB* is known. The angle *DLE* is right. Then the angle *LDE* is known. *DE* is known, so, the sides of the triangle *LDE* are known. *DT* is equal to 60 parts and its square is equal to the <sum of the> squares of *DL* and *LT*. Then *LT* and *LE* are known. Then *TE* is known. The angle *ZEI* is also known, because it is equal to the angle *AEB*. The angle *I* is right. Then the angle *IZE* is known. *ZE* is known to be 3 parts and a sixth. Then the sides of the triangle *ZEI* are known. *TE* is known, so, *TI* is known and its square plus the square of *IZ* are equal to the square of *ZT*. Then *ZT* is known. If we take *T* as the center and draw a circle with its radius <equal to> *TZ*, *ZI* is the Sine of the arc of the angle *ITZ* in terms of the radius *ZT*. Then *ZI* is known based on <taking> *ZT* <equal to> 60 parts. It is the Sine of the angle of equation. This is what we wanted to demonstrate.
In this way, we obtain the equation for all sides of the circle. It is found by calculation that the line <segment> *TZ* is equal to 60; 30° for the centrum being <equal to> zero; it is <equal to> 60° for the centrum being <equal to> 66°; it is <equal to> 56; 50° for the centrum being <equal to> 90°; it is <equal to> 55; 20° for the centrum being <equal to> 120° and in this case the line *DT* coincides with the line *ET*; it is again <equal to> 56; 50° for the centrum being <equal to> 180°. Its maximum <value occurs> at the maximum distance. Its mean <value occurs> at the distance <equal to> 66°. Its minimum <value occurs> at the distance 120°. <Its values> are the same at the distances 90°and 180°. Since the angle *AZT* in this figure, being the angle of the centrum, is less than the angle *AET* and the difference between them is <equal to> the angle *ITZ*, it is necessary that we subtract the equation from the centrum and add <the equation> to the mean anomaly, if the centrum is less than 180°. If the center is greater than 180°, we should add <the equation> to the centrum and subtract <the equation> from the mean anomaly.

Chapter 7: On the first equation for the other planets.

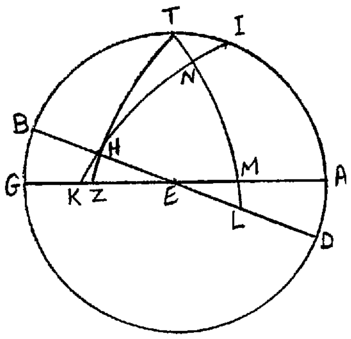
<Let> *ABG* centered at *Z* <be> the circle of the deferent; *AG* its diameter; *E* the center of the equant; and *H* the center of the inclined orb. *EZ* and *ZH* are equal. Each of them is <equal to> 3 parts plus a quarter and a sixth for Saturn, 2 parts and a half and a quarter for Jupiter, 6 parts for Mars, and one part plus 2 and half minutes for Venus <if *AZ* is 60 parts>. *B* is the center of the epicycle. We join the line segments *EB*, *ZB*, and *HB*. *ZT* and *HI* are perpendicular to *BI*. The angle *EBH* is the angle of equation. *AEB* is the <given> angle of the centrum; so the angle *TEZ* is



known. The angle *T* is right; so the angle *TZE* is known. *ZE* is <also> known. So, both *ET* and *TZ* are known. *BZ* is <taken equal to> 60 parts. Its square is equal to the <sum of the> squares of *ZT* and *TB*. Then *TB* is known. Since the triangles *IHE* and *TZE* are similar and *ZE* is half *EH*, then *ZT* is half *HI* and *ET* is half *EI*. Then *TI* and *TB* are known. So, the whole *IB* is known. Its square plus the square of *IH* are equal to the square of *HB*. Then *HB* is known. If we take *B* as the center and draw a circle with *HB* as its radius, *HI* is the Sine of the arc of the angle *IBH* based on <taking> the magnitude of *BH* as the radius. Then *HI* is known based on <taking> *BE* <equal to> 60 parts. It is the Sine of the arc of the equation angle. That is what we wanted to demonstrate.
In this way, we obtain the equation for each side of the circle. Since the angle *EBH* is the difference between the two angles *AEB* and *AHB*, the equation is subtracted and added as explained for Mercury. If the center is less than 180°, <the equation> is subtracted from the center and added to the <mean> anomaly. If the center is greater than 180°, <the equation> is added to the center and subtracted from the <mean> anomaly.

Chapter 8: On the latitude of the moon.

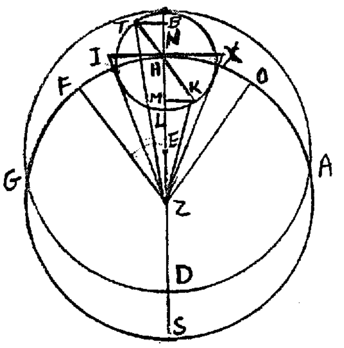
<Let> *ABGD* centered at *E* <be> the circle passing through the poles of the inclined <lunar> orb and the ecliptic; *AEG* the circle of the inclined <lunar> orb with *T* as its pole; *DEB* the circle of the ecliptic with *I* as its pole; the point *E* a lunar node; *H* the position (i.e., the orthogonal projection) of the moon on the ecliptic; and *K* the body of the moon on the inclined orb which is not different from its position on the epicycle, because the plane of the epicycle is in (i.e., it coincides with) the plane of the inclined orb. Then *EH* is the argument of latitude. We pass the arcs *THZ* and *IHK* through *H*. *HK* is the latitude of the moon. Those engaged in the art <of astronomy> take the arc *HZ* <for it> according to their calculations. <However,> *HZ* is not the latitude of the moon. It is actually an arc close <in magnitude> to the latitude of the moon. <Now> I say that *HK* is known.



Proof: As it was demonstrated in the fourth premise, <since> the angle *H* in the triangle *EHK* is right, *HEK* is the total latitude angle, i.e., the arc *BG*; thus the ratio of the Sine of *EH* to the Shadow of *HK* is equal to the ratio of the greatest Sine to the Shadow of the angle *HEK*. *EH* is the <given> argument of latitude, the angle *E* is the total latitude, and the greatest Sine is known. Then the Shadow of *HK* is known, and so is *HK*.
But <we may also provide the proof> merely based on Sines: The angle *Z* in the triangle *EHZ* is right, and the angle *E* is the total latitude. As it was demonstrated in the first premise, the ratio of the Sine of *EH* to the Sine of *HZ* is equal to the ratio of the greatest Sine to the Sine of the angle *E*. *EH* is the argument of latitude, the angle *E* is the total latitude, and the greatest Sine is known. Then *HZ* is known. We take the point K as the center and draw the quadrant *NML* with its radius equal to the side of an <inscribed> square. *L* is the center of the circle *IHK*. Then each of <the arcs> *NK* and *HL* are quadrants. Then *EL* is the complement of *EH*. So, in the triangle *ELM*, the angle *M* is right, and the angle *E* is the total latitude angle. Then the ratio of the Sine of *EL* to the Sine of *LM* is equal to the ratio of the greatest Sine to the Sine of the angle *E*. *EL* is the complement of the argument of latitude and the angle *E* is known. Then *LM* is known. So, its complement *MN* is known. It is the magnitude of the angle *HKZ*. <In> the triangle *HKZ*, *Z* is a right angle and the angle *K* is known. Then the ratio of the Sine of *KH* to the Sine of *HZ* is equal to the ratio of the greatest Sine to the Sine of the angle *K*. *HZ* and the angle *K* are known. So, *HK* is known and it is the latitude of the moon. That is what we wanted to demonstrate.

Chapter 9: On the latitudes of the planets.

It was said in Book III that for each of the superior planets there are two anomalies in the latitude. One of them is <due to> the inclination of the inclined orb from the ecliptic and the other is the inclination of the apogee and perigee of the epicycle from the inclined orb. The inclination of the apogee of the epicycle is towards the ecliptic, and the inclination of the perigee is <in the direction> opposite to it. For Venus and Mercury, there are three anomalies. The first and second are those mentioned for the superior planets. The third is <due to> the inclination of the diameter passing through the two mean distances of the epicycle. The magnitudes of these inclinations as found by observation are provided in their (i.e., the planets') descriptions.



Let the circle *ABG* centered at *E* be the ecliptic circle; *AHGS* the circle of the inclined orb centered at *Z*; *A* the ascending node; and *G* the descending node. *AHG* is northward except for Mercury. *BIL* is the epicycle centered at *H*. We suppose *HB* <to be> towards the ecliptic and *HL* in its opposite <direction>. We take *HI*, the radius of the "epicycle", equal to the Sine of the maximum inclination of the apogee or the perigee of the epicycle. Let the circle intersect at right angles the plane of the inclined orb, so that the half on which <falls> *BTL*, is towards the ecliptic and the other half is towards the inclined orb. The angle *HZI* is the maximum inclination of the apogee of the epicycle from the inclined orb towards the ecliptic. The angle *IZF* is the excess of the inclination of the inclined orb <from the ecliptic> over the <maximum> inclination of the apogee of the epicycle. The angle *XZH* is the maximum inclination of the perigee from the inclined orb in the <direction> opposite to the inclination of the apogee <of the epicycle>. The angle *OZX* is the excess of the inclination of the inclined orb <from the ecliptic over the maximum inclination of the epicycle>. These angles are known by observation. *IT* is the "adjusted anomaly" and *TB* is its complement. *TN* is the Sine of *TB*, and *HN* is equal to the Sine of *IT*. Both *TN* and *HN* are known in terms of *HT*, and *ZH* is <taken equal to> 60 parts. Then *ZN* is known. Its square plus the square of *NT* are equal to the square of *ZT*. So, *ZT* is known. Then *TN* is known based on <taking> *TZ* <equal to> 60 parts. It is the Sine of the angle *TZN*. So, the angle *TZN* is known. Then the angle *TZI* is known. Therefore, the whole angle *TZF* (i.e., the required latitude) is known. For Venus and Mercury, the resulting angle *TZN* is subtracted from the angle *HZI* being the maximum inclination angle of the apogee of the epicycle at one of the ascending or descending nodes. Also, the arc *XK* is the excess over the adjusted anomaly towards 90°. <Similarly,> *KL* is <equal to> the "adjusted anomaly". *KM* is the Sine of *KL* and *MH* is equal to the Sine of its complement *XK*. So, both *KM* and *MH* are known in terms of *HK*. *ZH* is <equal to> 60 parts. Then *ZM* is known. Its square plus the square of *MK* are equal to the square of *KZ*. Then *KZ* is known. So, *MK* is known based on <taking> *KZ* <equal to> 60 parts. It is the Sine of the angle *MZK*. So, the angle *MZK* and, therefore, the angle *KZX* are known. Then the whole angle *KZO* (i.e., the required latitude) is known. For Venus and Mercury the resulting angle *MZK* is subtracted from the angle *HZX* as we said before. That is what we wanted to demonstrate.

Description of its calculation: Minutes of the argument of latitude are <a certain number of> minutes whose ratio to 60 minutes is equal to the ratio of partial inclination of (i.e., the inclination of a point somewhere on) the inclined orb to its total (i.e., maximal) <inclination>, and equal to the ratio of partial latitude of the moon to its total magnitude (i.e., the "minutes of latitude" are proportional to the lunar latitude). We divide the partial latitude of the moon by its total magnitude, lowered. The result is the partial <latitude> in terms of the minutes of the argument of latitude. The inclination of Venus and Mercury at mean distance is called 'slant'. The ratio of its partial magnitude to its total (i.e., maximal) magnitude is equal to the ratio of the partial magnitude of the second equation to its total magnitude (i.e., the slant is proportional to the second equation). We multiply the partial magnitude of the second equation by the total slant, which is equal to 2 degrees and half, and divide <the product> by the total magnitude of the second equation. The result is the partial value of the slant. The inclination of the apogee and perigee of the epicycle is also computed from the adjusted anomaly, as indicated by the figure and proof that have been mentioned above.

Description of its tables: On the first <rows> of the tables <of planetary latitudes of the superior planets> is written "north" and "south", where the excess of the inclination of the inclined orb over the inclination of the apogee of the epicycle is <tabulated>. The "north" <column> is for the case when the center of the epicycle is in the northern half of the inclined orb. The "south" <column> is for the case when the center of the epicycle is in the southern half of the inclined orb. The inclinations <tabulated> for Venus and Mercury are their maximum inclination at one of the two nodes: for Venus at the ascending node and for Mercury at the descending node. In both cases the inclination of the apogee of the epicycle is southward.

Description of the operation by <using> tables: We take the minutes of the argument of the latitude from the adjusted centrum, to which (adjusted centrum) we add 50 degrees for Saturn, subtract 20 degrees for Jupiter, and <we take it> as it is for Mars. <We do so,> because the apogee of Saturn is shifted 50 degrees from the point *H* towards *G* which is the descending node, and the apogee of Jupiter <is shifted> 20 degrees from *H* towards *A*, and the apogee of Mars is at *H*. It (i.e., *H*) is the <point of> maximum inclination of the inclined orb. As we have already said, the minutes of the argument of the latitude take account of the <variable> inclination of the inclined orb for <varying> distance of the center of epicycle from the node. Then we take the latitude <corresponding to> the adjusted anomaly <from the "north" or "south" column>. If the adjusted center is in the semicircle *AHG*, the latitude is "north", because the inclination of the epicycle is towards the north in this semicircle. But if the adjusted center is in the semicircle *ASG*, the latitude is "south", because the inclination of epicycle in this semicircle is towards the south. Then we multiply the latitude by the minutes of the argument of latitude to obtain <the latitude> for <arbitrary> distance of the center of the epicycle from <one of> the two nodes.

<The cases with> Venus and Mercury <are as follows>. The apogee of Venus is at *H*, which is the northern extreme, and the apogee of Mercury is at *S*, which is the southern extreme. We take the inclination and slant for the <known> adjusted anomaly. The slant of Mercury at <its> apogee is 2; 15°, and at <the position> opposite to the apogee, 2; 45°. It was difficult to compose two tables for that. So, one table is composed for 2; 30°. Then a tenth of it is subtracted in <the region of> the apogee and a tenth of it is added in <the region> opposite to the apogee. This is sufficient for us. Then we add to the adjusted centrum, 3 <zodiacal> signs for Venus and 9 <zodiacal> signs for Mercury. The result is the distance from the ascending or descending node. If the result is less than 90° or greater than 270°, the distance is <regarded> from the ascending node. If the result is greater than 90° and less than 270°, the distance is <regarded> from the descending node. We use it (i.e., the distance) to find the minutes of the argument of latitude. We multiply it (i.e., the minutes of the argument of latitude) by the inclination for finding it (the latitude component) corrected for the distance from the node (i.e., for a position of the epicycle not coinciding with the node), because the extreme <magnitude> of this inclination is <achieved> at the two nodes. If the augmented center and the true anomaly are in the same half of the inclined orb, this latitude is southward. If their positions are <in> different <halves>, the latitude is northward. <They are so,> because the inclination of the apogee of epicycle is southward, and the inclination of <its> perigee is northward, between *SA* and *AH*. Conversely, if the result is between <the endpoints of the arc>, the center is between *SA* and *AH*. Then, if the true anomaly is also in the upper half, the inclination is southward. If the result is between <the endpoints of the arc> *GSA*, which is the lower half, the center is between <the endpoints of the arc> *HGS*.

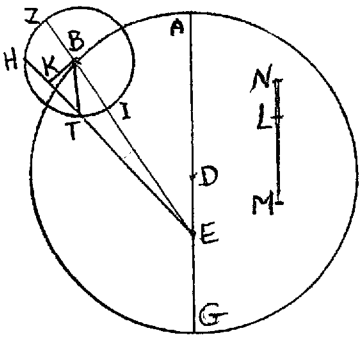
<Now> if the true anomaly is also in the lower half, the inclination is southward. From what we said, it is apparent that if the position of the result and the position of true anomaly are different, the latitude is northward. Then, we take the adjusted centrum of Venus as it is, and that of Mercury by adding 6 <zodiacal> signs. We obtain thereby the minutes of the argument of latitude, and we multiply these by the slant <at the highest point of the deferent> to obtain it (the slant) corrected for the distance of the center <of the epicycle> from the apogee for Venus, and for the distance to the point opposite to the apogee for Mercury. <We do so, > because the extreme of the slant is reached <when the epicycle center is> at the <point of> extreme inclination of the inclined orb. If the center is between <the endpoints of the arc> *AHG*, the upper half, and the true anomaly is between <the endpoints of the arc> *BIL* of the epicycle, then the latitude is northward. If the true anomaly is in the other half, the latitude is southward. <It is so,> because the endpoint *I* of the diameter *HI* is between <the endpoints of the arc> *AHG* <which lies> towards north and its other endpoint <lies> towards south. If the center is between <the endpoints of the arc> *GSA*, and the true anomaly is less than 180°, the latitude is southward. If the true anomaly is more <than 180°>, the latitude is northward. <It is so,> because the endpoint *I* of the diameter *HI* is between <the endpoints of the arc> *GSA* <which lies> towards south and the other endpoint <lies> towards north. Al-Battānī has neglected these directions in the text in his *zīj*, if it is not a scribal mistake. Then we multiply the <number of the> minutes of the argument of latitude which we have found lastly, by 1/6 degrees for Venus, and by a half plus a quarter of a degree for Mercury for finding the inclination of the inclined orb <at the center of the epicycle> from the distance of the center <of the epicycle> from the node. This inclination is always northward for Venus and southward for Mercury. Adding 6 <zodiacal> signs to the center of Mercury in the first and second <steps> is for shifting from the apogee region to its opposite <region>. <In this way,> the assertions about its latitudes and directions <become> like those for Venus. Then the single words (i.e., the rule) for it (i.e., for Venus) will be <valid> in general (i.e., also for Mercury). That is what we wanted to demonstrate.

Chapter 10: On the retrogradation of the planets.

<Let> *ABG* <be> the circle of the deferent with *D* as its center and *AG* as its diameter; *E* the center of the inclined orb; *ZHTI* the circle of the epicycle centered at *B*; and the line <segment> *BE* the distance of the center of the epicycle from the center *E*, <the method for> knowing which was already provided in the chapter on the first equation. We draw *ETH* passing through the first station. We join *BK* perpendicular to *HT*.

Based on what Ptolemy and his predecessors have demonstrated, the ratio of *KT* to *TE* is equal to the ratio of the motion (i.e., the motion during a particular day) of the center of the epicycle to the motion (i.e., the motion during the same day) of the planet on the epicycle.

BZ is the radius of the epicycle adjusted according to the distance of its center from the <position corresponding to the> mean distance (i.e., Kūshyār chooses *EB*=60), and it is known. We join *BT*. Then the arc *IT* is half the arc of retrogradation on the epicycle. The angle *BEK* is half the angle of retrogradation. *BE* and *BI* are known, so the remainder *IE* and the sum *ZE* are known. The product of *ZE* by *EI* is known. According to what has been demonstrated in the *Elements* (III.36), it is equal to the product of *HE* by *ET*. Then the product of *HE* by *ET* is known. The ratio of *KT* to *TE* is known, and *HT* is twice *KT*. Then the ratio of *HT* to *TE* is known. Let it be equal to the ratio of <two given segments> *NL* and *LM*, then the rectangle on *HE* and *ET* is similar to the rectangle on *NM* and *LM*, because their angles are equal and their sides are proportional. According to what has been demonstrated in the *Elements* (VI.23), the ratio of <the area of> the rectangle on *HE* and *ET* to



the <area of the> rectangle on *NM* and *ML* is equal to the ratio of the square of *HE* to the square of *NM*. The <areas of the> rectangles on *HE* and *ET*, and on *NM* and *ML* are known, and so is the square of *NM*. Then the square of *HE* is known; so *HE* is known. The ratio of *ZE* to *ET* is equal to the ratio of *HE* to *EI*, because the product of *ZE* by *EI* is equal to the product of *HE* by *ET*. But *ZE*, *HE*, and *EI* are known. Then *ET* is known. So, both *ET* and *TH* are known. Then *TK* and *KE* are known. So *KE* is known based on <taking> *BE* <equal to> 60 parts. Then its arc <Sine> is known, and it is <corresponding to> the angle *EBK*. Then the angle *EBK* is known. Also, *KT* is known based on <taking> *BT* <equal to> 60 parts. Then its arc <Sine> is known, and it is <corresponding to> the angle *TBK*. Then the angle *TBK* is known. If we subtract it from the angle *EBK*, the remainder is the angle *IBT*. It is the angle <corresponding to> the arc *TI*. Then the arc *TI* is known. It is half the arc of retrogradation on the epicycle. If we subtract the angle *EBK* from the right angle *BKE*, the remainder is the angle *BEK*. It is the angle of half the arc of retrogradation on the ecliptic. If the center of the epicycle had no movement towards east (i.e., if the distance of *B* from *E* was constant), the angle *BEK* and the arc *IT* would be adjusted (i.e., correct). But since it has a movement, we rely on finding a number (for adjustment of the retrogradation arc) whose ratio to the arc *IT* is equal to the ratio of the motion (i.e., the angular velocity) of the center of the epicycle to the motion (i.e., the angular velocity) of the planet on the epicycle. We subtract the obtained number from the angle *BEK* and the arc *IT*. The remainders are adjusted <magnitudes of> the angle *BEK* and the arc *IT* .If we divide the <magnitude of> the adjusted angle *BEK* in degrees by the daily mean motion of the planet, the result is half the <number of the> days of its retrogradation and its double is the whole <number> of the days of retrogradation. That is what we wanted to demonstrate.

Commentary

IV.4.1 In this chapter Kūshyār provides the definition of the equation of time and mentions its approximate bounds. Without going into details, he then shows that it is possible to calculate its magnitude. He presents the method for calculating the equation of time in I.4 and describes this subject in more detail in III.13. What Kūshyār says about the solar apogee reaching the sign of Leo may happen after around 20,000 years from his time. Kūshyār takes the mean solar longitude as the independent variable in his method, whereas al-Kāshī, for example, takes the true solar longitude as the independent variable. Benno van Dalen [1994a, 104] has shown that "as far as the computation of a table for the equation of time or its application is concerned, it makes little difference whether the independent variable is the true or mean solar longitude"; see also [Kennedy, 1988].
Kūshyār's text is confused. The equation of time is actually the sum of the differences between the successive mean and true solar days. The equation of time $E(t)$ at day t can be found by adding the solar equation (true minus mean longitude) at t to the mean solar longitude minus its right ascension at t. The result can be positive or negative. In order to avoid negative values, Kūshyār modifies his definition of equation of time. First he chooses u such that $E(u)$ is minimal. This turns out to be when the sun is in the middle of the sign Aquarius. Then Kūshyār defines his equation of time as $E(t) - E(u)$. This explains why he speaks about "twice" the differences in what is actually the determination of an upper bound of $E(t) - E(u)$. See also the commentary to I.4.5 where I have discussed Kūshyār's displacement method to avoid negative values of the equation of time. Ptolemy deals with the equation of time and its calculation in [1984, 169-72], and al-Battānī discusses this subject in [1907, 73-75]. See also [Pedersen 1974, 157-58; Neugebauer 1975, 61-68; van Dalen 1996, 211-18].

IV.4.2 Kūshyār's model for the solar motion is similar to that of Ptolemy. In this model, the sun B moves uniformly on a circle (the deferent) whose center E does not coincide with the earth D. Line DE points to point A on the ecliptic, which is taken as Gemini 18;31 for the beginning of the Yazdigird era (632 A.D.) and whose motion is very slow (1 revolution in 24,000 years). The radius EA is taken as 60 "parts". Kūshyār supposes that the uniform motion of B around E is known. The eccentricity is found to be 2; 29, 30° by Ptolemy [1984, 155], if the radius of the deferent is taken as 60. Al-Battānī, following Ptolemy's method but using his own observational data, found the eccentricity equal to 2 parts and 4 minutes plus half and a quarter of a minute (i.e., 2+4/60+1/120+1/240= 2;4,45°) [1988-1907, III, 66]. Kūshyār accepted the amount found by al-Battānī. The angle *EDB* in the figure is called "the angle of the true longitude". In the mss. A and M there is the justification that "if we add the true longitude of the apogee to it, the result will be the true longitude of the sun". Kūshyār provides the values of the equation of the sun in the table II.16, and describes its application in I.4.6. For avoiding negative values of the equation, Kūshyār adds 2 degrees to all of the tabular entries in II.16. Then in order to cancel this shift, he subtracts 2 degrees from all the tabular values of the table II.13 for the mean longitude of the sun. See also [Kashino 1998, 7-8; Ptolemy 1984, 157-166; Pedersen, 149-151].

IV.4.3 In this chapter, Kūshyār computes the "equation of center" of the moon, which is the difference between the mean epicyclic apogee *M* and the true epicyclic apogee *K* (which is on *ZB* extended). See [Pedersen 1974, 194]. Kūshyār's lunar model is similar to that of Ptolemy [1984, 226-33]. In this model, the earth is at *Z*. The moon *L* moves on an epicycle with center *B*. This center moves on a deferent circle whose center *E* does not coincide with the earth *Z*. The motion of *L* on its epicycle is contrary to the direction of the motion of *B* on the deferent. To further define the motion of *A* and *B* it is useful to bisect angle *AZB* by line *ZS* (not shown in the figure). Then *ZS* always points toward the mean sun. Points *A* and *B* move uniformly in opposite directions so that *ZS* always bisects the angle *AZB*, and angle *BZS* is the mean elongation, that is the difference between the mean lunar and the mean solar positions. We note that *B* moves on the circle with center *E*, but the motion is uniform with respect to *Z* (so not with respect to *E*). Finally define point *H* on *EZ* extended such that *ZH*=*EZ*, and extend *HB* to meet the epicycle at point *M*. Then the motion of the moon *L* on the epicycle is uniform with respect to *M*. All uniform motions are supposed to be known. See also pp. xxxii-xxxiv above.
However, Kūshyār deviates from Ptolemy and al-Battānī by taking the radius of the deferent *AE* (and not the inclined orb *AZ*) equal to 60 parts. So, his value for the distance between the center of the deferent and the center of the inclined orb is different from that of Ptolemy [1984, 226] and al-Battānī [1907, 82]:

$$10;19\times60/(60-10;19)=(619/60)\times(60-619/60)\approx 12.4589$$

Kūshyār uses the approximate value 12.5. He increases the values of the first equation of the moon tabulated in II.20 by 14 (the smallest integer greater than the maximum negative magnitude of the first equation) so that the first equation is always additive. The application of II.20 is described in I.4.7. In order to compensate the added 14 degrees, Kūshyār subtracts 14 degrees from the entries of the table II.18 for the lunar anomaly [cf. Kashino 1998, 12].

IV.4.4 For the moon, compare [Ptolemy 1984, 233-38; Pedersen 1974, 192-98]. Point *Z* is the earth, and *E* the center of the deferent. Kūshyār supposes that the epicycle is at maximum distance, so its center is at the apogee of the deferent. He computes the second (anomalistic) equation of the moon for any position of the moon on this epicycle. In the notation of Pedersen, Kūshyār computes the function in p. 196 (6.53). In this case, the mean and true anomaly are the same because the points *K* and *M* in the figure for IV.4.3 coincide.

For the planets, compare [Ptolemy 1984, 456-67; Pedersen 1974, 287-88]. Kūshyār's model for the planetary motion is similar to that of Ptolemy. In what follows, we exclude Mercury, for which see Section IV.4.6. The planet *T* moves on an epicycle whose center moves on a deferent, of which the center *E* does not coincide with the earth *Z*. The motion of the planet on its epicycle is in the same direction as the direction of the motion of its center on the deferent. In chapter IV.4.4, Kūshyār assumes that the center of the epicycle coincides with the apogee *A* of the deferent. The motion of the center of the epicycle on the deferent is uniform with respect to the equant point, that is a point *P* on *ZE* extended such that *EP*=*ZE*.

Again the values of the second equation of the moon are tabulated in II.20 and their application is described in I.4.7. Here Kūshyār's description is different from that of al-Battānī who follows Ptolemy exactly. Kūshyār adds 8 degrees to all values of the second equation of the moon to avoid negative values, and subtracts 8 degrees from all entries in the table II.17 for the mean longitudes of the moon. However, for compensating this shift and the 2 degrees shift in the entries for the mean longitudes of the sun, he subtracts 12 degrees from the entries of the table II.19 for the double elongation. See [Kashino 1998, 13].

IV.4.5 This chapter is a preparation for the computation of the "second equation", which is the angle between the center of the epicycle and the moon or planet, as seen from the earth. In the figure for IV.4.5, *E* is the earth, and *Z* the center of the deferent. Kūshyār discusses the case where the center of the epicycle is at *T*, not at the apogee of the deferent, and he only computes the maximum anomalistic equation. First he describes the computation and then he gives a proof. He assumes that the distance *ET* to the center of the epicycle is known. This distance can be computed from the angle *AZT* (mean motion from apogee) by the method of Chapter IV.4.2. Unlike Ptolemy, Kūshyār does not provide an exact computation of the second equation for an arbitrary point on the epicycle

whose center is an arbitrary position on the deferent. However, he presents a computation based on an interpolation method which is somewhat different from the method used by Ptolemy, for which see [Ptolemy 1984, 237-39; Pedersen 1974, 197-8]. Kūshyār briefly describes his interpolation method at the end of Chapter IV.4.5. Chapters IV.4.4 and IV.4.5 are used for the construction of interpolation tables in Book II. G. Van Brummelen [1998] studied Kūshyār's planetary tables and he reconstructed Kūshyār's interpolation method from the tables. Van Brummelen's mathematical reconstruction of Kūshyār's method is confirmed by the text at the end of Chapter IV.4.5 which Van Brummelen apparently did not consult. (Compare [Van Brummelen 1998, 273] "Kūshyār's second function gives the difference" with the text at the end of Chapter IV.4.5.)
Some of Kūshyār's parameter values are different from those of Ptolemy. Kūshyār mentions that he found the difference for the radius of the epicycle of Mars at its maximum distance one and a fifth degree less than the value found by Ptolemy, and that at its minimum distance two and a fifth parts less than that found by Ptolemy. Compare [Van Brummelen, 1998, 268]. For the moon, Kūshyār prescribes an interpolation procedure which is the same as his interpolation procedure for the planets, and different from the procedure in Ptolemy, for which see [Pedersen 1974, 196, (6.58)]. The "equation of center" of the planets is discussed in IV.4.6 for Mercury and in IV.4.7 for the other planets. Similar discussions are found in [al-Battānī 1899-1907,III, 80-81].

IV.4.6 See [Ptolemy 1984, 443-67] and especially [Pedersen 1974, 315-20] for a clear description of the very complicated Ptolemaic model for the motion of Mercury. Kūshyār uses the same model as Ptolemy. Z is the earth, and line ZA points towards a point on the ecliptic which moves so slowly that its motion is only noticeable after centuries. Points E, N and M are on ZA such that $ZE=EN=NM$. A small circle is drawn with center N and radius ND. On this circle, point D moves in the opposite direction of the sun, and with a velocity equal to the solar velocity; the exact position will be defined below. D is the center of the deferent with radius DT. Point T moves on the deferent such that the line ET is parallel to the direction of the mean sun (the direction of the line from the center of the solar deferent to the sun in the figure for IV.4.2). This also defines the position of D.
The values of the first equation of Mercury are tabulated in II.36 of Kūshyār's *zīj*. The magnitudes provided by Kūshyār for line segment ZT corresponding to the center being equal to 0°, 66°, 90°, 120° and 180° are correct and very close to the results of my recalculation. However, at the end of this chapter Kūshyār erroneously mentions the angle AZT – and

not *AET* – as the angle corresponding to the center. See also the commentary to I.4.8 and [Kashino 1998, 14-19; Pedersen, 315-28].

IV.4.7 Compare [Ptolemy 1984, 545-54] and [Pedersen 1974, 279-80, 283-85], and see the commentary to section IV.4.5 for a description of the model for the motion of the planets except Mercury. *H* is the earth, *Z* the center of the deferent, *E* the equant point, and *B* the center of the epicycle (which is not drawn). The values of the first equation of Saturn, Jupiter, Mars and Venus are provided in the relevant tables of Book II.

IV.4.8 In table II.37 are provided the values of the latitude of the moon for different values of the argument of latitude. Kūshyār presents two methods for calculating the latitude: one using both Sine and Shadow functions, and another using merely the Sine function. In the figure, *HZ* is the required latitude, but Kūshyār erroneously calculates *HK* as the latitude of the moon. See also the commentary on I.4.9.

IV.4.9 Kūshyār's theory for the latitudes of the planets is essentially Ptolemy's theory of latitudes in the *Almagest* for which see [Ptolemy 1984, 597-647] and [Pedersen 1974, 355-86]. Ptolemy's theory for the latitude of the superior planets (Saturn, Jupiter and Mars) is basically as follows. He supposes that the planes of the deferent and the ecliptic intersect in the nodal line, that is a straight line through the center of the earth, and that the plane of the deferent (Kūshyār's "inclined orb") makes a (small) angle with the plane of the ecliptic. The line through the earth *Z* perpendicular to the nodal line will intersect the deferent in a point *H*, the highest point, where the center of the epicycle has maximal northern distance from the ecliptic. For Mars, the highest point coincides with the apogee of the deferent, as in Kūshyār's figure. For Jupiter and Saturn, if *E* is the center of the deferent, angle *HZE* is equal to 20 or 50 degrees.
The epicycle makes a variable angle with the deferent which is maximal when the center of the epicycle is at the highest point *H* and the lowest point *S* of the deferent, and zero when the center of the epicycle is at the nodes, i.e., the two points of intersection of the deferent and the plane of the ecliptic. The apogee of the epicycle is between the deferent and the ecliptic, but the perigee on the other side of the deferent, so that the latitude of the planet is maximal when it is at the perigee of the epicycle.
As usual, Kūshyār only considers a special situation, where the center of the epicycle is in the highest point *H* of the deferent. He wants to show how the latitude of the planet can be computed from its true anomaly in this situation. He assumes that the angle between deferent and the ecliptic is known, and also the latitudes of the apogee and the perigee of the

epicycle are known. He approximate computation is similar to, but easier than Ptolemy's (which is also an approximation).

Kūshyār's figure is confusing for the modern reader because it is a superposition of two different figures, which are in two different planes. Some of the points in the paper play different and inconsistent roles in the two figures. The procedure to superpose different figures in two or sometimes even three planes in plane of the paper was sometimes used in ancient Greek and medieval Islamic geometry in a type of constructions which are called analemma-construction by modern historians of science.

The first figure (which we call the "horizontal figure") consists of the earth *E*, the ecliptic *ABGD*, the deferent *ASGH* with center *Z*, the nodes *A* and *G*, and the epicycle *BILX* with apogee *B* and perigee *L*. The center *H* of the epicycle is supposed to be at the highest point of the deferent.

The second figure (which we call the "vertical figure") describes the situation in the plane through *HZ* perpendicular to the ecliptic. In this second figure, *ZF* is the intersection with the ecliptic, and one considers a circle with center *H* and radius the "sine of the maximum inclination" of the apogee and perigee of the epicycle from the plane of the deferent. This has to be understood in the sense that the radius is equal to the (equal) distances of the apogee and perigee of the epicycle to the deferent. This small circle is essentially an interpolation device. Kūshyār also indicates this new circle by the same letters *BILX*. However, in the "vertical" figure, *I* is the position of apogee (which has approximately the minimum latitude in this model), and *X* the position of the perigee (the point on the epicycle with approximately the maximal latitude). Therefore, *I* in the vertical figure corresponds to *B* in the horizontal figure and vice versa. In the vertical figure, the angle *FZH* is the inclination of the deferent.

If *T* in the vertical figure is the position of the planet, *IT* is its "adjusted anomaly", and *TB* is its complement (we have emended the manuscript texts which say that *BT* is the adjusted anomaly and *TI* is its complement-this is true in the horizontal figure). Kūshyār can now easily compute the angle *TZF*.

It is instructive to express his result in the notation in [Pedersen 1974, 365-67], so that it can be compared to Ptolemy's much more complicated computation. In the notation of Pedersen, the quantity to be computed is $\beta(90^\circ, a_v)$= angle *FZT*. The radius of the epicycle is $r \sin j_m$, where j_m is the angle between the epicycle and the deferent. In Kūshyār's vertical figure we have $TI = a_v$ (the "adjusted anomaly"), $HN = r \sin j_m \sin a_v$,

$TN = r \sin j_m \cos a_v$, $ZH = \rho$, $ZT^2 = (\rho + r \sin j_m \sin a_v)^2 + (r \sin j_m \cos a_v)^2$,

and finally $\beta(90^\circ, a_v)$= (angle) *FZH* – (angle) *TZH*

$= i - \text{arc sin } [r \sin j_m \cos a_v / ((\rho + r \sin j_m \sin a_v)^2 + (r \sin j_m \cos a_v)^2)^{1/2}]$, where i is the angle between the deferent and the ecliptic.

Then Kūshyār wants to compute the latitude when the adjusted anomaly of the planet is between 90 and 180 degrees. He now constructs a new vertical figure, in which angle *XZO* is the latitude of the center of the epicycle (and also of the point *X*). Kūshyār now considers the position of the planet at *K* between *X* and *O*, in the horizontal and at the same time in the vertical figure. He computes the angle *KZO* and considers this to be the latitude. However, the resulted latitude function is strange, increasing slowly when *K* moves away from *X*, and sharply when *K* approaches the perigee *L*. This is in contrast with Ptolemy's more sensible method of interpolation. It is likely that Kūshyār copied the latitude tables of Ptolemy and did not use his own method of computation.

For reasons of space, I will not describe the complicated geometric theory of the latitude of the inferior planets here. However, I will provide some formulas. To compute the latitude of any (superior or inferior) planet, one needs the adjusted anomaly (of the planet on its epicycle, reckoned from the true apogee), and the difference x between the ecliptical longitude of the center of the epicycle minus the longitude of the ascending node (for all planets except Mercury this can be defined as the node where the epicycle center passes from southern to northern latitude). For the superior planets, Ptolemy's tabular computation of latitudes boils down to

$\sin(x)\, f(a)$ for x between 0 and 180 degrees, and

$\sin(x)\, g(a)$ for x between 180 and 360 degrees.

Kūshyār calls x-90 the "argument of latitude", and the function $|\sin(x-90)| = \cos(x)$ "minutes for the argument of latitude"; the function is expressed in minutes (1/60) of unity. Kūshyār tabulates $f(a)$, $g(a)$ and $|\sin(x-90)|$ for a and x between 0 and 360 degrees. For x between 0 and 180 degrees, the latitude is northern, and $f(a)$ is tabulated in the column "north". For x between 180 and 360 degrees, the latitude is southern, and $g(a)$ is tabulated in the column "south".

For Venus and Mercury, the latitude is the sum of three components: $c\sin^2 x + \sin x\, f(a) + k \cos x\, g(a)$ for a and x between 0 and 360 degrees. Here c =6 for Venus and c =0.75 for Mercury. The "inclination" $f(a)$ and "slant" $g(a)$ are tabulated in the third and fourth columns. They are the differences between the latitude of the planet and the latitude of the center of the epicycle, at x =90 (inclination) or x = 0 (slant of Venus). For Mercury $g(a)$ is a hypothetical slant with maximum value 2.5; the real slant is computed by multiplication by the constant k, which is 0.9 for x between 0 and 180 degrees, and 1.1 for x between 180 and 360 degrees. The constant k is 1 for Venus. The fifth column displays again values of $\sin x$ in "minutes" for the "argument of latitude" $x-90$.

The last part called "description of the operation by tables" closely resembles *Almagest* [Ptolemy 1984, 635-36]. Kūshyār reproaches

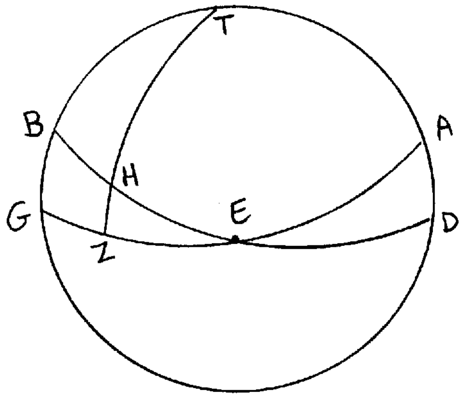
al-Battānī of not giving the correct rules for the determination of the direction of the latitude components (northern or western). However, these rules are found in the text of al-Battānī [1899, 174]. To find the third latitude component, Kūshyār multiplies the minutes of latitude by 1/6 degrees for Venus and by 0.75 degrees for Mercury (so he obtains $c\sin x$). Kūshyār then forgets to multiply the result again by the "minutes of latitude". The same mistake is found in al-Battānī's *zīj* [1899, 175, lines 14-16]. Ptolemy explains the computation correctly in *Almagest* [Ptolemy 1984, 636]: "Then we take these same sixties which were found by the second entry with the longitude, calculate the amount which is the same fraction of them as they are of 60, and for Venus, take 1/6th of this and set it out …."
The tables for the latitudes of the five planets are presented in II.38 to II.42.

IV.4.10 Kūshyār starts this chapter with the Apollonius Theorem provided also in the *Almagest* [Ptolemy 1984, 555-62; Pedersen 1974, 329-51], where he says that this preliminary lemma was demonstrated by a number of mathematicians, notably Apollonius of Perga [ibid, 555]. Kūshyār's wording corresponds to the generalized theorem of Apollonius [Pedersen 1974, 341-43]. Ptolemy [1984, 562-81] also provides the numerical calculation of the retrogradation arc for each planet at mean, greatest and least distances. Then he uses his usual interpolation method for other distances. See also [Pedersen 1974, 329-54], where the author also quotes a beautiful and interesting proof of Apollonius Theorem devised by B. L. van der Waerden [ibid, pp. 331-32]. The correction to the retrograde arc at the end of this chapter (to account for the eastward motion of the epicycle) corresponds to formula (11.40) in [Pedersen 1974, 347].

Section 5: On the operations relating to the ascendants of the day and night, <in> 16 chapters

Chapter 1: On the first declination.

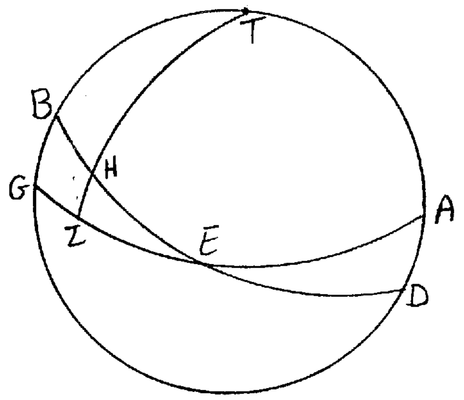
Let *ABGD* be the circle passing through the two poles of the <celestial> equator and the ecliptic, *AEG* the celestial equator with its pole at *T*, *BD* the ecliptic, and *E* one of the two equinoxes. We take *EH* as <the arc> from the ecliptic, whose first declination we want. We draw the arc *THZ*. Then *HZ* is the first declination of the arc *EH*. I say that it (i.e., *HZ*) is known.



Proof: The angle *Z* of the triangle *EHZ* is right and the angle *E* is <equal to> the greatest declination. So, the ratio of the Sine of *EH* to the Sine of *HZ* is equal to the ratio of the greatest Sine to the Sine of the angle *E*. But *EH* is known, and the greatest Sine is known through observation, so *HZ* is known. This is what we wanted to demonstrate.

Chapter 2: On the right ascensions of the <zodiacal> signs.

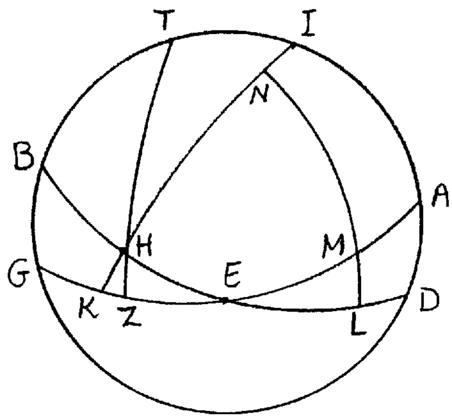
Let *ABGD* be the circle passing through the poles <of the celestial equator and the ecliptic>, *AEG* the celestial equator with its pole at *T*, *BED* the ecliptic, and *E* one of the two equinoxes. We take *EH* as <the arc> from the ecliptic, whose right ascension we want. We draw the arc *THZ*. Then *EZ* is the <right> ascension of the arc *EH*. I say that it is known.



Proof: The angle *Z* of the triangle *EZH* is right. *E* is the angle of the greatest declination, and *HZ* the declination of *HE*. According to what was demonstrated in the fourth premise, the ratio of the Sine of *EZ* to the Tangent of *ZH* is equal to the ratio of the greatest Sine to the Tangent of the angle *E*. But *ZH* and the greatest Sine are known, so *EZ* is known. This is what we wanted to demonstrate.
Another method: Again, the angle *Z* of the triangle *EZH* is right. Then, according to what was demonstrated in the second premise, the ratio of the Sine of the complement (i.e., Cosine) of *EZ* to the Sine of the complement (i.e., the Cosine) of *EH* is equal to the ratio of the greatest Sine to the Sine of the complement (i.e., the Cosine) of *ZH*. The complements of *EH* and *ZH* are known. Then the complement of *EZ* is known. So, *EZ* <itself> is known. This is what we wanted.

Chapter 3: On the second declination.

ABGD centered at *E*, is the circle passing through the poles <of the celestial equator and the ecliptic>. *AEG* is the celestial equator with pole at *T*, *BED* the ecliptic with pole at *I*, and *E* one of the two equinoxes. We take <the arc> *EH* from the ecliptic whose second declination we want. We draw the arc *IHK*. Then *KH* is the second declination of the arc *EH*. I say that it is known.



Proof: The angle *H* of the triangle *EHK* is right and the angle *E* is <equal to> the angle of the greatest declination. Then the ratio of the Sine of *EH* to the Tangent of *HK* is equal to the ratio of the greatest Sine to the Tangent of the angle *E*. But *EH* is known. Therefore the Tangent of *HK* is known. So, <*HK*> itself is known. This is what we wanted.
Another method: Again, we draw the arc *THZ*. We <also> draw the arc *NML*, taking its pole at *K* and its radius equal to the side of the <inscribed> square. Then *L* is the pole of the circle *IHK*, and both *KN* and *NL* are quadrants. *HZ* is the first declination of the arc *EH*, and *EL* is the complement of *EH*. *ML* is the first declination of the arc *EL*, and *MN* is

the complement of *ML*, <equal to the> magnitude of the angle *NKM*. So the angle *Z* of the triangle *HKZ* is right and the angle *K* is known. So the ratio of the Sine of *KH* to the Sine of *HZ* is equal to the ratio of the greatest Sine to the Sine of the angle *K*. But *HZ* is known and the angle *K* is known, therefore, *HK* is known. This is what we wanted.

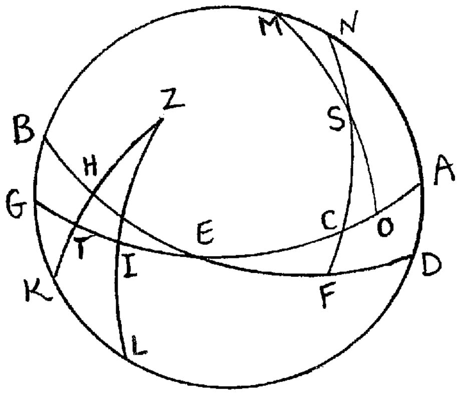
Another method: If we take the known <arc> *EH* <as a part> of the celestial equator, and <the arc> *EK* <as a part of> the ecliptic, <then> *KH* will be the first declination of the arc *EK*. If we find the arc corresponding to *EH* in <the table for> right ascensions, *EK* will become known, and it is called 'the inverse ascension'. If we take its first declination, it will be <equal to> *HK*, being the second declination of the arc *EH*. Then *HK* is known. This is what we wanted to demonstrate.

<Finding right> ascensions from the two declinations: *EH* is <an arc> of the ecliptic and *EZ* is <an arc> of the celestial equator. *HZ* is the first declination of the arc *EH* and the second declination of the arc *EZ*. If we find the arc corresponding to *HZ* in the table of the second declinations, *EZ* will become known. It is the right ascension of *EH*.

So the right ascension is known from the two declinations. This is what we wanted to demonstrate.

Chapter 4: On the distance of the stars from the celestial equator.

ABGD is the circle passing through the poles <of the celestial equator and the ecliptic>. *AEG* is the celestial equator with poles at *L* and *M*. *BED* is the ecliptic with poles at *K* and *N*. First, we suppose the star <to be at> the point *Z*, so that the latitude and the second declination are in the <same> direction. We draw the arcs *KTZ* and *LIZ*. Then *HZ* is the latitude of the star, *HT* is its second declination, and *ZI* is its distance from the celestial equator. I say that it (i.e., *ZI*) is known.

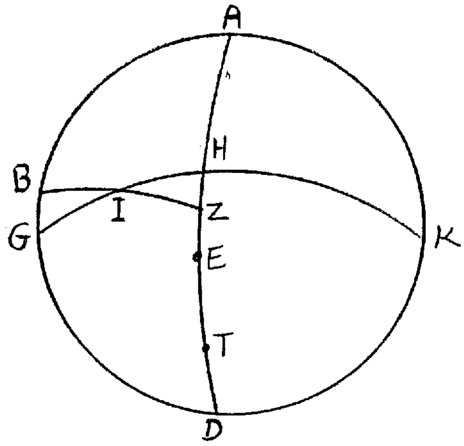


Proof: The two triangles *ZTI* and *KTG* are similar, because their angles *T* are equal and the angles *I* and *G* are right. Then the ratio of the Sine of *TZ* to the Sine of *ZI* is equal to the ratio of the Sine of *TK* to the Sine of *KG*. *TZ* is known, being <equal to> the latitude plus the second declination

<of *Z*>. *TK* is the complement of the second declination. *KG* is the complement of the greatest declination, because *KL* is <equal to> the greatest declination. So, *ZI* is known.
Again, we suppose the star <to be at> the point *S*, so that the latitude and the second declination are in two <opposite> directions. We draw the arcs *MSO* and *NSF*. Then *FS* is its latitude, *FC* its second declination, and *SO* its distance from the celestial equator. I say that it (i.e., *SO*) is known.
Proof: The triangles *CSO* and *CNA* are similar, because the angle *C* is common <to them> and the angles *A* and *O* are right. Then the ratio of the Sine of *CS* to the Sine of *SO* is equal to the ratio of the Sine of *CN* to the Sine of *NA*. *CS* is known and *CN* is the complement of the second declination. *NA* is the complement of the greatest declination because *NM* is <equal to> the greatest declination. Therefore *SO* is known. This is what we wanted to demonstrate.

Chapter 5: On the latitude of a <given> locality.

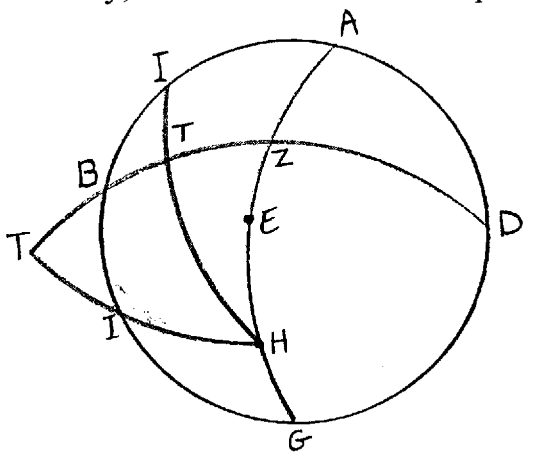
ABGD is the horizon circle, *E* the zenith, *AED* the meridian, *GIK* the celestial equator with *T* as pole, and *BIZ* the ecliptic. Then *EH* is <equal to> the latitude of the locality. I say that it is known.



Proof: *AZ* is the maximum altitude of the sun, found by some altitude <measuring> instrument. *ZH* is the declination of the sun. Therefore *AH* is known, but it is the complement of *EH*. Therefore, *EH* is known and it is <equal to> the latitude of the locality. If the point *H* is on the ecliptic, <the point> *Z* on the celestial equator, and *EZ* the latitude of the locality, then *AH* is the maximum altitude of the sun and *ZH* the declination of the sun. So, the sum *AZ* is known, but it is the complement of *EZ*. Thus *EZ*, the latitude of the locality, is known. This is what we wanted to demonstrate.

Chapter 6: On the ortive amplitude of the sun and the stars.

ABGD is the horizon circle, *E* the zenith, *AEG* the meridian, and *BZD* the celestial equator with its pole at *H*. Let *I* be the rising point of the sun or the star on that day, then *BI* is the ortive amplitude. I say that it is known.



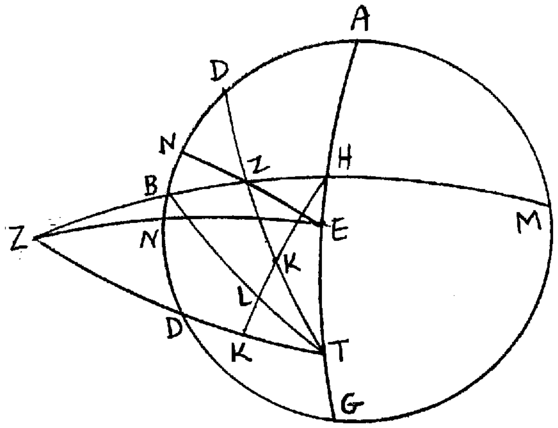
Proof: We draw the arc *HTI*. Arcs *BZ* and *BA* are both quadrants. *AZ* is the complement of the latitude of the locality. It is equal to the magnitude of the angle *ZBA*. In the triangle *BIT*, *T* is a right angle, the angle *B* is known, and *IT* is the declination of the sun or the star from the celestial equator. Then the ratio of the Sine of *BI* to the Sine of *IT* is equal to the ratio of the greatest Sine to the Sine of the angle *B*. So *BI* is known. This is what we wanted.

Another method: Based on what we will demonstrate in Chapter 10, *TZ* is half the day arc. *Z* and *A* in the two triangles *HTZ* and *HIA* are right angles. *TZ* is <equal to> the magnitude of the angle *H*, because *TH* is a quadrant. The ratio of the Sine of *HI* to the Sine of *IA* is equal to the ratio of the greatest Sine, being the Sine of *HT*, to the Sine of the angle *H*. The Sine of *HI* is equal to the Cosine of *IT*, and *IT* is the declination of the sun or the distance of the star from the celestial equator. The angle *H* is known, since it is <equal to the magnitude of> the arc *TZ*. Then *IA* is known, but it is the complement of *IB*, and *IB* is the ortive amplitude. Therefore the ortive amplitude is known. If we substitute the point *G* for the point *A*, the proof for finding the complement of the ortive amplitude in the northern and southern directions will be the same. This is what we wanted to demonstrate.

Chapter 7: On the equation of daylight of the sun and the star<s>.

ABGD is the horizon circle, *E* the zenith, *AEG* the meridian, and *BHM* the celestial equator with its pole at *T*. Let the point *D* be the degree whose equation of daylight is desired. We draw the arc *TZD* through it. Then *DZ*

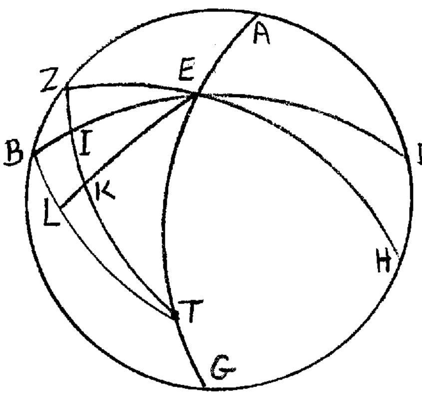
is the declination of the point *D*, *BZ* its equation of daylight, and *BD* its ortive amplitude. I say that *BZ* is known.
Proof: *BZD* is a right triangle, and <its right angle> is *Z*. Then the ratio of the Cosine of *DZ* to the Cosine of *DB* is equal to the ratio of the greatest Sine to the Cosine of *BZ*. But *DZ* and *DB* are known. Then *BZ* is known. This is what we wanted to demonstrate.



Another method: We draw the arc *EZN*. The angles *D* in the two triangles *DZN* and *DTG* are equal, and *N* and *G* are right angles. Therefore, according to what was demonstrated in the first premise, the ratio of the Sine of *DZ* to the Sine of *ZN* is equal to the ratio of the Sine of *DT* to the Sine of *TG*. *DZ* is the declination <of the sun> or the distance <of the star from the celestial equator>. <For northern *D*,> *DT* is the complement of the declination or of the distance. *TG* is the latitude of the locality. Therefore *ZN* is known. Again, the ratio of Sine of *BZ* to the Sine of *ZN* is equal to the ratio of the Sine of *BH* to the Sine of *HA*. But *ZN* is known, *BH* is a quadrant, and *HA* is the complement of the latitude of the locality. Therefore *BZ* is known. This is what we wanted.
Another method: In the triangle *BZD*, *Z* is a right angle. The angle *B* is equal to the complement of the latitude of the locality, being <equal to the magnitude of the arc> *HA*, because the arcs *BH* and *BA* are both quadrants. In view of what was demonstrated in the fourth premise, the ratio of the Sine of *BZ* to the Tangent of *ZD* is equal to the ratio of the greatest Sine to the Tangent of the angle *B*. So, we have to divide the Tangent of the declination, i.e. *ZD*, by the Tangent of the complement of the latitude of the locality, lowered. According to what was demonstrated in the third notice, being Chapter 7 of Section 3 <of Book IV in this *zīj*> on the premises, that <quotient> is equal to the product of the Tangent of the declination by the Tangent of the latitude of the locality, lowered. The result is the Sine of *BZ*. So *BZ* is known. This is what we wanted to demonstrate.

Another method <applicable> if the equation of daylight for the solstices is known: We draw the arc *BT*, being a quadrant. The circle *DZT* takes the place of the celestial equator, because if we fix the point *Z* and we rotate the arc *TZD*, it coincides with the arc *HBZ* of the celestial equator. We take on it the <right> ascension of the arc whose equation of daylight is desired. Let it be *TK*. We pass the arc *HKL* through it. It (i.e., *HKL*) is a quadrant, because the arc *BT* is drawn with *H* as its pole. Then *L* is a right angle. In the triangle *TKL*, *L* is a right angle and the angle *T* is equal to the total (i.e., maximal) equation <of the daylight>. On the basis of what was demonstrated in the first premise, the ratio of the Sine of *TK* to the Sine of *KL* is equal to the ratio of the greatest Sine to the Sine of the angle *T*. But *TK* is the ascension that was assumed, and the angle *T* is known. Then *KL* is known, and it is the partial equation of daylight. This is what we wanted.

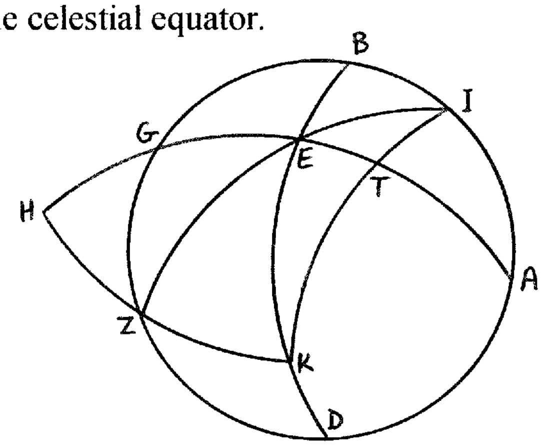
Another method for <finding> the equation of daylight <applicable> when the total equation <of daylight> is known: *ABGD* is the horizon circle, *AEG* the meridian, *BED* the celestial equator with its pole at *T*, and *ZEH* the ecliptic. Let the point *Z* be the first of Capricorn, and let us draw the arc *TIZ*. Then *BI* is the total (maximal) equation of daylight. We draw the arc *TB*. Both of the arcs *TI* and *TB* may take the place of the celestial equator, because if the intersection point *I* is fixed and the arc is rotated, it will coincide with the celestial equator. We take from the arc *TI* a magnitude <equal to the arc> whose equation of daylight is desired. Let



it be *TK*. We draw E*KL* <which> intersects the arc *TB* at right angles, because *TB* is drawn with *E* as the pole. So, *EL* is a quadrant. Then the ratio of the Sine of *TK* to the Sine of *KL* is equal to the Sine of *TI*, being the greatest Sine, to the ratio of the Sine of *IB*. But *TK* is taken from the celestial equator, *TI* a quadrant, and *IB* the total equation. So *KL* is known. This is what we wanted to demonstrate.

Chapter 8: On ascensions for a locality (i.e., oblique ascensions).

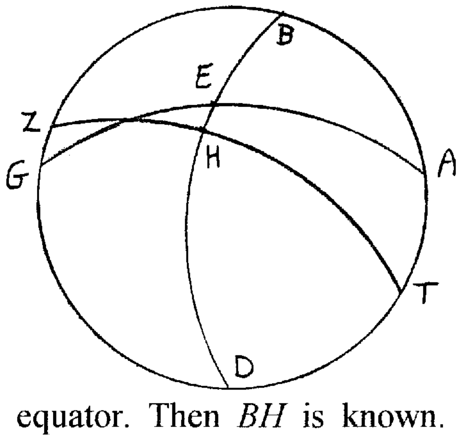
Let *ABGD* be the horizon circle, *BED* the meridian (this is not necessary; see commentary), *AEG* the celestial equator, *ZEI* the ecliptic, and *K* the pole of the celestial equator.



We draw the two arcs *KTI* and *KZH*. Then the arcs *TA* and *GH* are the equation of daylight for the points *I* and *Z*. The arcs *ET* and *EH* are the right ascensions of the two arcs *EI* and *EZ*. Let *EZ* be northern and *EI* southern. If we add *TA* to *ET*, the result is *EA*, the oblique ascension of *EI*. If we subtract *GH* from *EH*, the result is *EG*, the oblique ascension of *EZ*. This is what we wanted to demonstrate.

Chapter 9: On the maximum altitude of the sun and the star<s>.

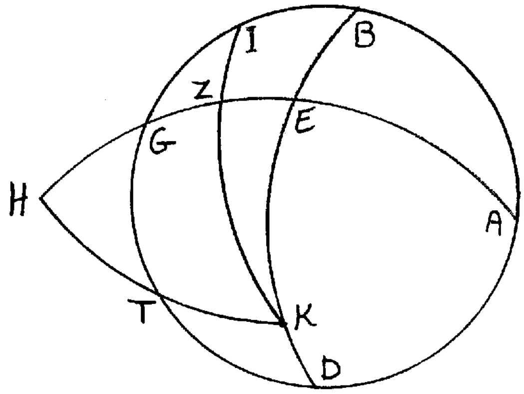
ABGD is the horizon circle, *BED* the meridian, *AEG* the celestial equator, and *ZHT* the ecliptic. We suppose the sun or the star <to be at> the point *H*. Then the arc *BH* is its maximum altitude, *BE* the complement of the latitude of the locality, and *EH* the declination of the sun or the star



from the celestial equator. Then *BH* is known. Again, let *ZHT* be the celestial equator, *AEG* the ecliptic, and *E* the position of the sun or the star. Then *BH* is the complement of the latitude of the locality, and *EH* the declination or the distance. So, *BE* is known. This is what we wanted to demonstrate.

Chapter 10: On half the day arc of the sun and the star<s>.

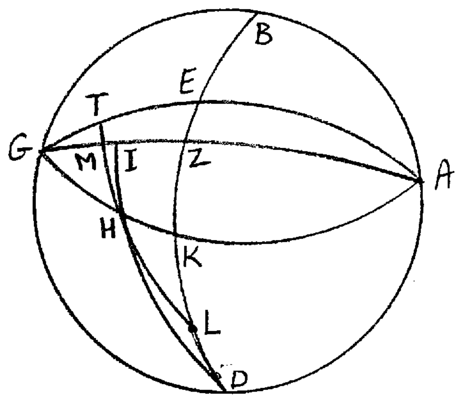
ABGD is the horizon circle, *BED* the meridian, and *AEG* the celestial equator. We assume the two points *I* and *T* as <two> rising positions of



the ecliptic, and *K* as the pole of the celestial equator. We draw the two arcs *KZI* and *KTH*. Then *GZ* is the equation of daylight for the point *I*, which is southern, and *ZE* is half its day arc. But *EG* is a quadrant, so *ZE* is known. *GH* is the equation of daylight for the point *T*, which is northern. *EH* is half its day arc and *EG* is a quadrant. So *EH* is known. This is what we wanted to demonstrate.

Chapter 11: On the <ecliptical> degree of the transit of a star through the meridian.

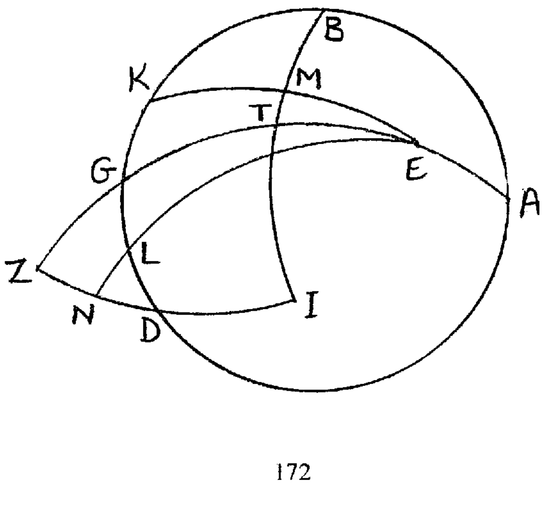
ABGD is the horizon circle (not necessary; see commentary), *BED* <the circle> passing through the poles <of the ecliptic and the celestial equator>, *AEG* the celestial equator with its pole at *L*, *AZG* the ecliptic with its pole at *D*, and *H* the body of the star. We draw <the arcs> *LHT*, *DHI*, and *AHG*. Then *I* is the <ecliptical> degree of the star, *IH* its latitude, *HT* its distance from the celestial equator, *M* the <ecliptical> degree of its transit, and *Z* the point of one of the two solstices. In the triangle *KDH*, *K* is a right angle, and the angle *D* is known, being <equal to the magnitude of> the arc *ZI*, because *DHI* is a quadrant.



ZI is the <longitude> distance of the <ecliptical> degree of the star from the solstice, and *DH* is the complement of the latitude <of the star>. The ratio of the Sine of *DH* to the Sine of *HK* is equal to the ratio of the greatest Sine to the Sine of the angle *D*. So, *HK* is known. Again, in the triangle *LHK*, *K* is a right angle, *LH* is the complement of the distance of the star from the celestial equator, and *HK* is known. The ratio of the Sine of *LH* to the Sine of *HK* is equal to the ratio of the greatest Sine to the Sine of the angle *L*. It (i.e., the Sine of *L*) is <equal to the magnitude of> the Sine of the arc *TE*. The arc *TE* is the right ascension of *ZM* <starting> from the first <degree> of the solstitial <sign> (i.e., Cancer or Capricorn). In view of what was demonstrated in the first notice (i.e., VI.3. 3), <since> the two middle terms in the <ratio of the> first <four> magnitudes are equal to the two middle terms in the <ratio of the> other <four> magnitudes, by ex aequali <of ratios>, the ratio of the Sine of *DH*, the complement of the latitude, to the Sine of *TE*, the right ascension of the <ecliptical> degree of the transit <taken> from the first <degree> of the solstitial <sign>, is equal to the ratio of the Sine of *LH*, the complement of the distance from the celestial equator, to the Sine of *IZ*, the distance of the <ecliptical> degree of the star from the solstice. *TE* is known, so *ZM* is known. Therefore the point *M* is known. This is what we wanted to demonstrate.

Chapter 12: On the <ecliptical> degree of the rising and setting of a star.

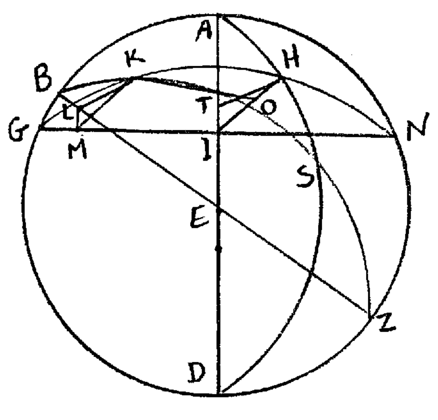
ABGD is the horizon circle, *AEG* is the celestial equator with pole at *I*, *E* is one of the two equinoxes, *EK* a southern <part> of the ecliptic, and *EL* a northern <part> of it. We assume *B* as the body of the star in the south, and *D* as its body in the north. *M* and *N* are the <ecliptical> degree of the transit of the star <in each of these two cases>. We draw *IBT* and *IDZ*. Then *GT* is the equation of daylight for *B*. If it (i.e., *GT*) is added to *ET*, the right ascension of the <ecliptical> degree of the transit of the star *B*, the result is *EG*, the oblique ascension of *EK*, and *K* is the <ecliptical> degree that rises <simultaneously> with the star.



Again, *EZ* is the right ascension of the <ecliptical> degree of the transit of the star *D*, and *GZ* is its equation of daylight. If it (i.e., *GZ*) is subtracted from *EZ*, *EG* remains, the oblique ascension of *EL*. But *L* is the <ecliptical> degree that rises <simultaneously> with the star. So *EG* is the <oblique> ascension of the <ecliptical> degree that rises <simultaneously> with the star *B*, and <with> the star *D*. If we imagine that the point *G* moves according to the general motion <of the celestial sphere> and arrives at the western horizon, i.e., at the point *A*, <then> it would have moved by the amount of the day arc of the star. <Then a certain> point of the celestial equator will have arrived at the eastern horizon, which <point> is (i.e., defines) the <oblique> ascension of the opposite to the <ecliptical> degree which sets <simultaneously> with the star. This is what we wanted to demonstrate.

Chapter 13: On <finding> the arc of revolution <of the celestial equator> since the rising of the sun and the star<s> from the altitude of the <sun or the star at a given> time, and <finding> the altitude from the arc of revolution.

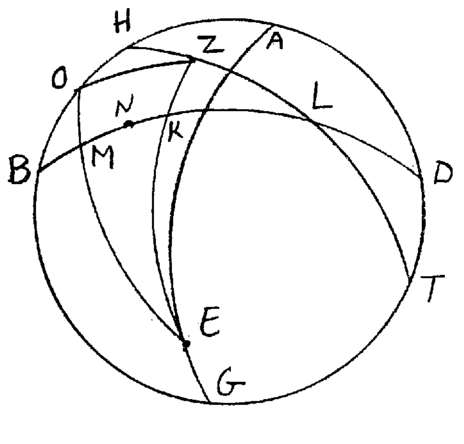
ABGD is the horizon circle, *ASD* the meridian and *AED* its diameter, *BSZ* the altitude circle, and *BEZ* its diameter. Then *S* is the zenith. Arc *GHN* is <a part> of the parallel circle <to the equator> above the earth, and *GN* its chord. Then *H* is the intersection <point> of the parallel circle and the meridian, and *K* the intersection <point> of the parallel circle and the altitude <circle>. We draw *HT* perpendicular to *AE*. Then it (i.e., *HT*) is the Sine of the arc *AH*, the meridian altitude for the point *H* of the parallel circle. We join *HI*; it is the Sagitta of the arc *GKH*, and *GKH* is half the day arc. We draw *KL* perpendicular to *BE*; it is the Sine of the arc *BK*, and *BK* is the altitude of the <sun or the star at a given> time. We draw *KM* perpendicular to the chord *GN*. It (i.e., *KM*) is the 'arrangement Sine' of the arc of revolution. If we imagine *ASD* and *BSZ* to be vertical <semicircles> in the <celestial> sphere, with diameters *AD* and *BZ*, it is clear that, as we said, *HT* is the Sine of the arc *AH*, *HI* the Sagitta of the arc *GKH*, *KL* the Sine of the altitude, and that *KM* is parallel to *HI*.



Then the two triangles *IHT* and *MKL* are similar, because *T* and *L* are right angles, and the two angles *H* and *K* are equal because the two line <segment>s *IH* and *HT* are parallel to the line <segment>s *MK* and *KL*, <respectively>. Then the ratio of *HT*, the Sine of the meridian altitude, to *KL*, the Sine of the altitude of the <sun or the star at a given> time, is equal to the ratio of *HI*, the Sagitta of half the day arc, to *KM*, the 'arrangement Sine' of the arc of revolution. So *KM* is known. We join *KO* perpendicular to *HI*. *KM* is equal to *OI* in the <celestial> sphere. The rest <from *HI*> is *HO*, the Sagitta of the arc *HK* <so *HK* is known>. *HK* is the excess of the arc of revolution, <and *HS*, half the day arc is known>, so *KG*, the arc of revolution, is known. This is what we wanted to demonstrate.
<Finding> the altitude from the arc of revolution: Again, if *KG*, the arc of revolution, is known, <then> the altitude *BK* is known. That is <based on the following proof:> The arc *GKH*, being half the day arc is known. The arc of revolution *GK* is also known. So *KH*, the excess of the arc of revolution, is known. Thus its Sagitta is known, and it is the excess of *IH* over *MK*. <*IH* is known> so *MK* is known. But the ratio of *MK*, the 'arrangement Sine' of the arc of revolution, to *KL*, the Sine of the altitude, is equal to the ratio of *IH*, the Sagitta of half the day arc, to *HT*, the Sine of the meridian altitude. So *KL* is known. So the altitude is known. This is what we wanted.

Chapter 14: On <finding> the ascendant from the arc of revolution <of e.g., the sun> and <finding> the arc of revolution from the ascendant.

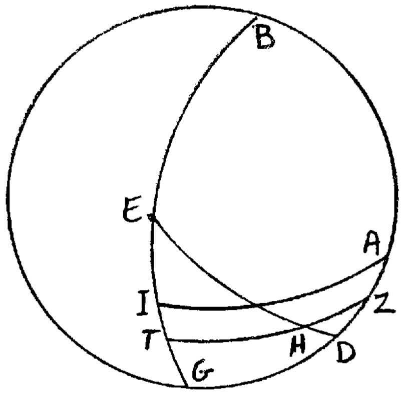
ABGD is the horizon circle, *AEG* the meridian, *BLD* the celestial equator with *E* as its pole. *HLT* is the ecliptic, its point *H* is <situated> on the horizon, and it (i.e., *H*) is required. I say that it is known. So we assume *Z* as the <known> position of the sun or the star on the ecliptic, and the arc *ZO* is its <known celestial> parallel. We draw two arcs passing through the pole of the celestial equator and the points *Z* and *O*. They intersect the celestial equator at *K* and *M*.



Then the arc *MK* is similar to the arc *ZO*, and *ZO* is the <known> arc of revolution, so *MK* is <also> the arc of revolution. *BM* is the equation of daylight for the point *Z*, and *LK* is the right ascension of *LZ*. We cut off *KN* equal to *BM*. Then *LN* is the oblique ascension of *LZ*, *BM* is equal to *KN*, and *MN* is a common <part>. So *MK* is equal to *BN*. But *MK* is the arc of revolution, so *BN* is equal to the arc of revolution. If *BN* is added to *LN*, the result is *LB*, which is known, because it is the oblique ascension of *LH*. Then the point *H*, which is the <ecliptical> degree of the ascendant, is known. This is what we wanted to demonstrate.
<Finding> the arc of revolution from the ascendant. Again, if the point *H* is known, and *Z* is the <ecliptical> degree of the sun or the star, <then> both <oblique> ascensions of *LH* and *LZ* are known, and they are *LB* and *LN* <respectively>. Then *BN*, equal to the arc of revolution, is known. This is what we wanted.

Chapter 15: On the proof <using> a base generally applicable to the arc of revolution and to what is related to it.

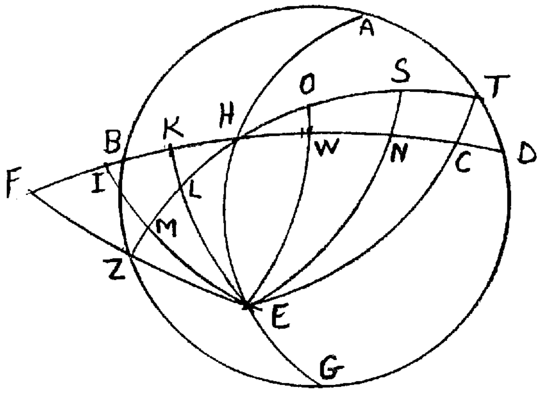
It is known from the forty-first figure (i.e., IV.5.13, see commentary), on the proof of the <method for finding the> arc of revolution from the altitude, that the ratio of the Sine of any altitude to the 'arrangement Sine' of its <corresponding> arc of revolution is equal to the ratio of the Sine of any other altitude to the 'arrangement Sine' of its <corresponding> arc of revolution. It is known that through any point from the ecliptic which is assumed on the horizon, a circle can be drawn passing through the two poles of the celestial equator. <The arc> between the assumed point and the celestial equator, of the circle passing through the two poles of the celestial equator, is the declination of the assumed point. The line drawn from the assumed point perpendicular to the diameter of the celestial equator is the Sine of the declination of the point. The diameter is <the line> drawn from the intersection of the celestial equator and the circle passing through its poles. <The line segment> between the foot of the perpendicular on this diameter and the complement of the radius is the Cosine of the declination of the <assumed> point, and it is equal to the radius of the parallel circle passing through the assumed point. The diameter <of the parallel circle> is drawn from that point, so the radius of any parallel circle is equal to the Cosine of its declination.
After this premise, let *ABGD* be the horizon circle, *BEG* the meridian, *EHD* <a part> of the altitude circle, *AI* the celestial equator, and *ZHT* <a part> of the parallel <circle>. Then the ratio of the Sine of *DH*, the altitude, to the 'arrangement Sine' of *HZ*, is equal to the Sine of *IG*, i.e. the altitude of the point *I*, to the 'arrangement Sine' of *AI*. But *AI* is a quadrant, and *IG* is the complement of the latitude of the locality.



So the product of the Sine of the altitude and the greatest Sine is equal to the product of the Cosine of the latitude of the locality and the 'arrangement Sine' of *HZ*. So the 'arrangement Sine' is known in terms of the magnitude (i.e., unit) for which the radius of the circle *AI* is 60 parts. We want to know this in terms of the radius of the circle *ZT*. The radius of the circle *ZT* is equal to the Cosine of the declination. So the ratio of the 'arrangement Sine' of *HZ* to the Cosine of the declination is equal to the ratio of the desired base (i.e., the 'arrangement Sine' *HZ* in terms of the radius of *ZT*) to the greatest Sine. So, the product of the known 'arrangement Sine' of *HZ* and the greatest Sine is equal to the product of the base (i.e., the value of the 'arrangement Sine' of *HZ*) in terms of the desired magnitude (i.e., the radius of *ZT*) and the Cosine of the declination of the degree (i.e., the point *H* on the parallel circle). Then the 'arrangement Sine' of *HZ* in terms of the radius of the circle *ZHT* is known. So, <to carry out> the multiplication, we may <actually> multiply the Sine of the altitude by the greatest Sine and <then> divide it by the Cosine of the latitude of the locality. Then we multiply the result by the greatest Sine and we divide it by the Cosine of the declination of the <ecliptical> degree. This is as if we multiply the Sine of the altitude by the greatest Sine twice and divide <the result> by the Cosine of the latitude of the locality, and then <we divide the final result> by the Cosine of the declination. That is <also> equal to multiplying it (i.e. the Sine of the altitude) by the greatest Sine twice and dividing it by the product of the Cosine of the latitude of the locality and the Cosine of the declination of the <ecliptical> degree. If we multiply the Cosine of the latitude of the locality by the Cosine of the declination of the <ecliptical> degree, lowered twice, because it should be multiplied by the greatest Sine twice, the result is the base from which the arc of revolution and what relates to it may be derived (i.e., computed). This is what we wanted to demonstrate.

Chapter 16: On the equalization of the houses.

ABGD is the horizon circle, *AEG* the meridian, *BHD* the celestial equator and *E* its pole, *ZHT* the ecliptic, the point *Z* the ascendant, *H* the midheaven, and *T* the descendant. We draw <the arcs> *EZF* and *ECT*.



Then *HF* is half the day arc of the ascending degree, and *HC* is half its night arc. If we divide *HF* into the three divisions *FI*, *IK* and *KH*, each division thereof will be equal to twice <the number of> the parts (i.e., degrees) <corresponding to> the hours of the ascendant. If we divide *HC* into three <equal> parts *HW*, *WN* and *NC*, each part will be equal to twice the <number of the> degrees <corresponding to> the hours of the descendant. <This is> because the time <-degrees corresponding to> each of <the arcs> *HF* and *HC* are 6 seasonal hours. If we draw circles from the pole of the celestial equator passing through these division <point>s, they cut the ecliptic at the division <point>s being the equal <i.e., ecliptical> degrees for the first division <point>s of the celestial equator. They are the divisions *HL*, *LM* and *MZ*, and the divisions *HO*, *OS* and *ST*. If we subtract 90° from the <right> ascension of the ascendant, i.e. *HB*, the right ascension of the tenth <house> remains. If we write down the right ascension of the tenth <house> in two positions, and add to it twice the <number of the> parts <corresponding to> the hours of the ascendant repeatedly, and if we subtract <in the other position> from it twice the <number of the> parts <corresponding to> the hours of the descendant repeatedly, the result of additions will be the right ascensions of the eleventh <house>, the twelfth <house>, and the ascendant. The result of the subtractions will be the right ascensions of the ninth <house>, the eighth <house>, and the descendant. If we write down the right ascension of the ascendant in two positions, and subtract from it twice the <number of the> parts <corresponding to> the hours of the ascendant repeatedly, and if we add <in the other position> to it twice the <number of the> parts <corresponding to> the hours of the descendant repeatedly, the result of these subtractions will be the right ascensions of the twelfth <house>, the eleventh <house>, and the tenth <house>. The result of the additions will be the right ascensions of the second <house>, the third <house>, and the fourth <house>. This is what we wanted to demonstrate.

Commentary

IV.5.1 The subject of first declination (the distance of any ecliptical degree from the celestial equator) is also discussed by Ptolemy [1984, 69-70]. Kūshyār's method based on the Sine theorem is general and simple, but Ptolemy's proof is longer and he performs the calculation for the specific arcs 30° and 60°. See also the commentary on I.5.1. The distance of a star to the equator is nowadays also called "declination".

IV.5.2 Kūshyār applies what he has proved in the premises IV.3.5 and IV.3.2 in the first and the second proofs, respectively. They are the Tangent Theorem and the Cosine Theorem for the right spherical triangles. See also the commentary on I.5.2.

IV.5.3 Here Kūshyār first proves his second method in I.5.3. See the commentary on I.5.3.

IV.5.4 In this chapter, Kūshyār supposes that the (ecliptical) latitude *HZ* and second declination *HT* are known. The application of the concept "similarity" for spherical triangles by Kūshyār is rather strange. He applies the so called "Rule of Four" for spherical right triangles that have an acute angle in common or equal acute angles. The same concept of "similarity" of spherical triangles is used in the solution of the problem in al-Kāshī's *Khāqānī Zīj*, see [Kennedy 1985, 9]. See also the commentary on I.5.4.

IV.5.5 Kūshyār uses the same figure for the two cases in which the declination of the sun is towards north and south, thereby interchanging the circles representing the ecliptic and the celestial equator. See also the commentary on I.5.5.

IV.5.6 See the commentary on I.5.6.

IV.5.7 In the first method for finding the equation of daylight for the sun and the stars, it is assumed that the declination and the ortive amplitude of the sun or the star are known. In the second and the third methods, it is assumed that the latitude of the city and the declination of the sun or the star are known. In the fourth method, it is assumed that the right ascension of the arc whose equation of daylight is requested and the equation of daylight for the solstices, *BZ*, are known. It is interesting that Kūshyār applies a rotation in this method. The last method is not mentioned in I.5.7 where the first four methods are provided. However, this additional method is similar to the fourth one, except that *TK* is taken equal to the arc whose equation of daylight is desired, and not its right ascension as taken in the fourth method. In both fourth and fifth methods, the values found for the equinoxes and the solstices are accurate, and the intermediate values vary uniformly. The results can be good approximations for the desired equation of daylight. Kūshyār's proofs for these two methods show that *KL* can be found from the existing data. But he does not prove that *KL* is equal to the desired equation of daylight. See also the commentary on I.5.7. The fourth and the fifth methods are mathematically correct.

Proof: since $\text{Sin}\Delta = \text{Tg}\delta \times \text{Tg}\varphi / R$ and $\text{Sin}M = \text{Tg}\varepsilon \times \text{Tg}\varphi / R$, we have $\text{Sin}\Delta \,/\, \text{Sin}M = \text{Tg}\delta \,/\, \text{Tg}\varepsilon = \text{Sin}\alpha / R$, where *M* is the known equation of daylight for solstices. Kūshyār's proof seems to imply some sort of geometrical transformation, but the idea escapes us.

IV.5.8 In this chapter, if we take *BED* as the meridian, then the problem will lose its generality. So this is a superfluous phrase possibly inserted into the text because it exists in the next chapters. *BED* can be any great circle through the celestial pole *K*. See also the commentary on I.5.8.

IV.5.9 Again, for using the same figure for the cases in which the declination of the sun or a star from the celestial equator is towards the north or south, Kūshyār interchanges the circles representing the ecliptic and the celestial equator for the second case. See also the commentary on I.5.9.

IV.5.10 See also the commentary on I.5.10.

IV.5.11 In this chapter, the starting phrase "*ABGD* is the horizon circle" is superfluous. It has possibly been inserted to the text by mistake, because it appears at the beginning of the former and the next chapters. *ABGD* is a great circle through the intersections *A* and *B* of the ecliptic and the equator. Kūshyār assumes that the ecliptical latitude *HI* and the ecliptical distance along the ecliptic to the solstice *IZ* are known (this distance can be derived from the ecliptical longitude).

Kūshyār concludes the desired proportion $\text{Sin}DH : \text{Sin}TE = \text{Sin}LH : \text{Sin}IZ$ by IV.3.3 from $\text{Sin}DH : \text{Sin}HK = R / \text{Sin}D$, $\text{Sin}LH : \text{Sin}HK = R / \text{Sin}L$ and from the fact that the magnitudes of the angles *D* and *L* are measured by arcs *IZ* and *TE* respectively. The technical term "ex aequali" is defined in definition 17 of Book V of Euclid's *Elements*. On the problem of finding *TE* (the right ascension of the star at *H*), see also al-Kāshī's *Khāqānī Zīj* [Kennedy 1985, 10-14]. See also the commentary on I.5.12.

IV.5.12 In the last part: "If we imagine...", the description is confusing. Apparently, Kūshyār means that "If we imagine that the point G moves according to the general motion <of the celestial sphere> and B arrives at the western horizon, then the point G would have moved by the amount of the day arc of the star. Then a certain point of the celestial equator will have arrived at the eastern horizon, which is the oblique ascension of the <ecliptical degree> opposite to the <ecliptical> degree which sets <simultaneously> with the star." See also the commentary on I.5.13.

IV.5.13 For a modern mathematical formula for finding the arc of revolution from the altitude of the time, and a definition of the 'arrangement Sine', see the commentary on I.5.14.

IV.5.14 Here Kūshyār provides the proof for the method described in I.5.16 and for its inverse described in I.5.17.

IV.5.15 The chapter on the proof of the validity of the method for finding the arc of revolution from the altitude of the sun or the star, IV.5.13, corresponds to the 41st figure (*shakl*) in Book IV. However, in the ms. A, it is referred to as the '44th figure'. IV.5.13 is Chapter 44 in the ms. A (which presents the chapters of Book IV as numbered consecutively) and apparently the word *shakl* in the reference is used in its other meaning "theorem".
In this chapter Kūshyār demonstrates that the 'arrangement Sine' can be found by dividing the Sine of the altitude by $\text{Cos}\varphi\,\text{Cos}\delta\,/R.R$. He calls this expression the "base". In I.5.21 this base is used in short efficient methods for finding different astronomical quantities. For a modern mathematical formulation of these methods see the commentary on I.5.21.
In finding the value of the arrangement Sine of HZ in terms of the radius of the parallel ZT, all manuscripts erroneously put '60' instead of 'the Cosine of the declination of the degree (i.e., the point H)', and the correct version is only mentioned in a marginal note of the ms. M.
The concept of "base" was widespread in medieval Islamic astronomy, see for example [King 2004, 114,909].

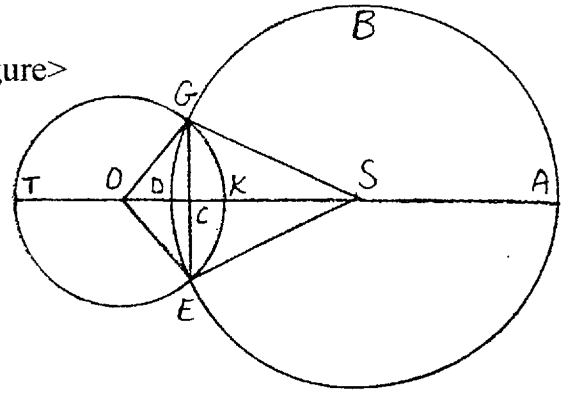
IV.5.16 Here Kūshyār actually explains the geometrical basis of the method for the equalization of the houses which was the most popular one among the medieval Islamic authors of astrological works. See also the commentary to I.5.22.

Section 6: On eclipses and what pertains to them, <in> 14 chapters

Chapter 1: On the absolute and adjusted magnitudes of a lunar eclipse in digits.

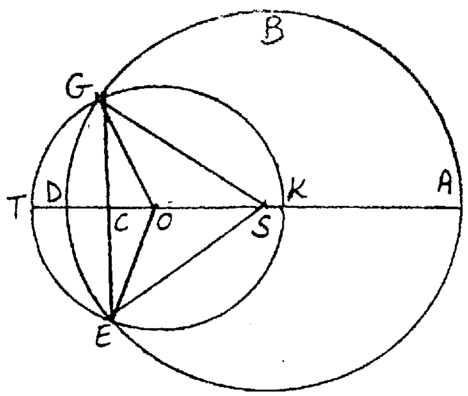
ABGD is the disk of the <earth's> shadow in the transition position <of maximum obscuration> of the moon, and *GTEK* is the disk of the moon's surface. They are perceived as being in the same plane. *AD* is the diameter of the disk of the shadow, and *KT* is the diameter of the disk of the moon. *SD* <plus> *KO* is the sum of the two radii. *SO* is the latitude of the moon. *KD* is the excess of *SD* <plus> *KO* over *SO*. So, *KD* the magnitude of the lunar eclipse in minutes, is known. *KT* is <also> known. So *KD* is known by taking *KT* <equal to> 12 digits. It (i.e., the magnitude of 12*KD*/*KT*) is <called> 'the absolute magnitude of the lunar eclipse in digits'. The area of <the segment> *DGEK*, the surface of the moon's disk is <called> the 'adjusted magnitude of the lunar eclipse in minutes'. It is <called> the 'adjusted magnitude of the lunar eclipse in digits', by assuming the area of the moon's disk <equal to> 12 digits. It (i.e., the adjusted magnitude in digits) is desired <if the absolute magnitude is known>. <To this end>, we join *EG* and we draw the line <segments> *SG*, *SE*, *OG* and *OE*. Since *AD* and *GE* intersect in a circle, the product of *AC* by *CD* is equal to the product of *GC* by *CE* (*Elements*, III.36), and the product of *TC* by *CK* is equal to the product of *GC* by *CE*. Thus the product of *AC* by *CD* is also equal to that of *TC* by *CK*. So, the ratio of *AC* to *CT* is equal to that of *KC* to *CD*. If we subtract *KD* from both diameters *AD* and *KT*, the ratio of the remainder, *AK*, to *DT* is equal to that of *KC* to *CD* (*Elements*, V.17). By composition of ratios, the ratio of the sum of *AK* and *DT* to *TD* is equal to that of *KD* to *DC*. The sum of *AK* and *DT*, *DT*, and *KD* are known. So, *DC* is known, and it is the Sagitta of the <arc *GE* of the> shadow's disk. *KC* is <also> known, and it is the Sagitta of the <arc *GE* of the> the moon's disk. So both *TC* and *CK*

The first <figure>



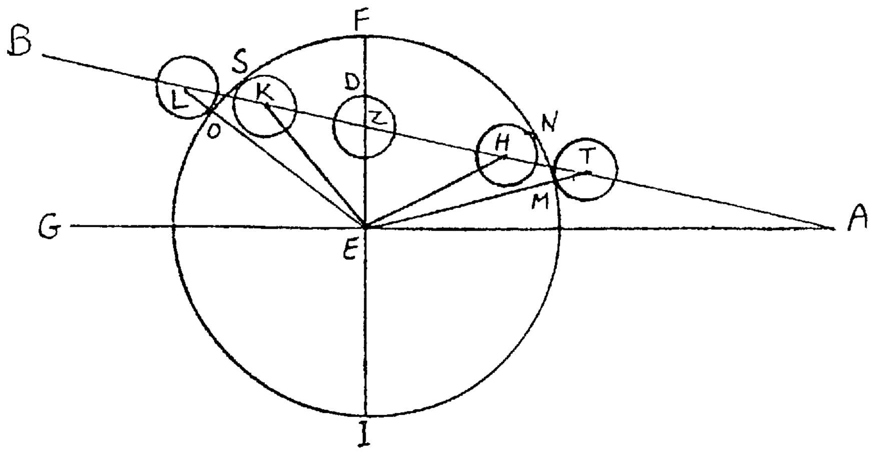
are known. The product of *TC* by *CK* is equal to the square of *GC*, because *GC* is equal to *CE*. So, *GC* is known, and it is the Sine of the arc *GK* by assuming *OG* as the radius of the moon. Thus it (i.e., *GC*) is known by assuming *OG* <equal to> 60 parts. Hence the arc *GK* of the great circle is known, and it is the complement of the arc *GT* towards 180 <degrees> in the second figure. Its ratio to 360 <degrees> is equal to the ratio of the arc *GK* on the circumference of the moon's disk to the whole circumference of the moon's disk. Thus <the ratio of the length of> the arc *GK* to <the circumference of> the moon's disk is known. *OK* is <also> known. So the area of the sector *OGKE* is known. *OC* and *GC* are known, so, the area of the triangle *OGE* is known. So the area of the segment *GKEC* of the moon's disk is known. Again, *GC* is the Sine of the arc *GD* <based> on <taking> *SG* the radius of the disk of the shadow. So, it (i.e., *GC*) is known <based> on <taking> *SG* <equal to> 60 parts. So the arc *GD* of the great circle is known. Its ratio to 360 <degrees> is equal to the ratio of the arc *GD* of the disk of the shadow to the whole circumference of the disk <of the shadow>. Therefore the arc *GD* of the disk of the shadow is known and *SD* is known. Thus the area of the sector *SGDE* is known. <Also> *SC* and *GC* are known. So, the area of the triangle *SGE* is known. Then the area of the segment *GDEC* of the disk of the shadow is known. <Now> the sum of *GKEC* and *GDEC* is known, and it is the 'adjusted magnitude of the lunar eclipse in minutes'. Its ratio to the area of the surface of the moon's disk is equal the 'adjusted magnitude of the lunar eclipse in digits' to 12. This is what we wanted.

The second <figure>



Chapter 2: On the absolute times of a lunar eclipse.

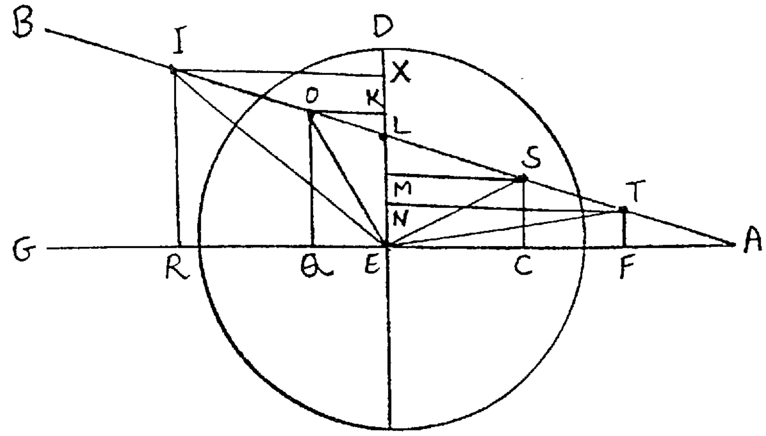
Let *AB* be a segment of the inclined <lunar> orb, *AG* a segment of the ecliptic, and the point *E* the center of the <earth's> shadow disk. *ED* is a segment of the circle *IEF* passing through the two poles of the ecliptic, <the part> *EZ* on it being the latitude of the moon in the middle of the eclipse. *T* is the center of the moon's disk at the beginning of the eclipse when it is tangent to the shadow's disk at the point *M*. <Then the moon> proceeds to enter the eclipse. *H* is the center of the moon's disk when it is totally eclipsed and the beginning of its total immersion when it is tangent to the shadow's disk at *N*. <Then the moon> proceeds to enter the <total> eclipse, and *Z* is the center of the moon's disk in the middle of the eclipse, being nearest to the center of the shadow <disk>. *K* is the center of the moon's disk at the end of the total immersion and at the beginning of emersion, and it is tangent to the shadow's disk at *S*. <Then the moon> proceeds to go out of it (i.e., out of the shadow) and *L* is the center of the moon's disk at the end of emersion, and it is tangent to the shadow's disk at *O*. <Then the moon> proceeds to separate from the shadow. Both the line segments *ET* and *EL* are the sum of the two radii, and both *EH* and *EK* are <equal to> the radius of the shadow <disk> minus the radius of the moon <disk>. *ZT* <corresponds to the duration of> immersion in minutes, from the beginning of the eclipse up to its middle. *ZH* <corresponds to> the duration <of totality> in minutes, from the beginning of the total immersion up to the middle of the eclipse. *ZK* <corresponds to> the duration <of totality> in minutes, from the middle of the eclipse up to the beginning of emersion. *ZL* <corresponds to the duration of> immersion in minutes, from the middle of the eclipse up to the end of emersion. <The lengths of> these line <segments> are desired, because each of them being divided by the lunar gain, provides the hours (i.e., time interval) corresponding to these minutes <of arc>. We assume all <the arcs> *AB*, *AG* and *ED* to be straight line <segments>, because they are small, and there is no <noticeable> difference between taking them as arcs or as straight line <segments> in the eclipses. *ET* is the sum of the two radii, *EZ* the latitude of the moon at the middle of the eclipse, and *Z* is approximately a right angle. If the square of *EZ* is subtracted from the square of *ET*, the result is the square of *TZ*. So *TZ* is known, and it is <corresponding to the duration of> the immersion. If we subtract its <time in> hours from the <time in> hours of the middle of the lunar eclipse, the <time in> hours of the beginning of the lunar eclipse will result.



If it is added <to the time of the middle of the eclipse>, the <time in> hours of the end of emersion will be obtained, because *ET* is equal to *EL*. Again, *EH* is <equal to> the radius of the shadow <disk> minus the radius of the moon <disk>. If we subtract the square of *EZ* from its square (i.e., from *EH* squared), the result is the square of *HZ*. Thus *HZ* is known. If its <time in> hours are subtracted from those of the middle of the lunar eclipse, the result is the <time in> hours of the beginning of its total immersion. If it is added <to the time of the middle of the eclipse>, the result is the <time in> hours of the beginning of emersion, because *EH* is equal to *EK*. These are the five <desired> times. If there is no total immersion in the eclipse, the <time in> hours of the beginning of the total immersion and of the beginning of emersion will be deleted. This is what we wanted.

Chapter 3: On the correction of the times.

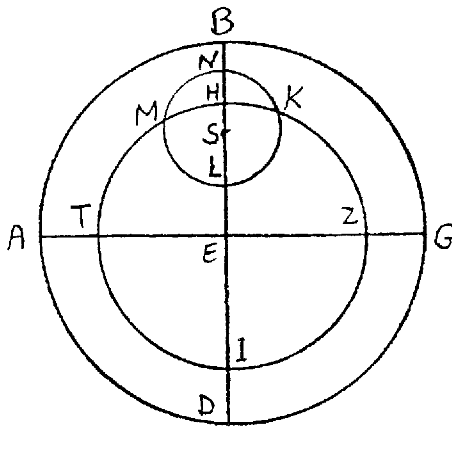
Let *AB* be a segment of the inclined <lunar> orb, *EG* a segment of the ecliptic, and *E* the center of the shadow's disk. *ED* passes through the two poles of the ecliptic, and <the part> *EL* of it is the latitude of the moon in the middle of the eclipse. The points *T*, *S*, *L*, *O* and *I* are the centers of the moon at the beginning of the lunar eclipse, the beginning of total immersion, the middle of the lunar eclipse, the beginning of the emersion, and the end of emersion, <respectively>. From these points, we draw the line <segments> *TF*, *SC*, *OQ* and *IR* parallel to the line <segment> *LE*. Each of them is the latitude of the moon corresponding to these centers. We draw the line <segments> *TN*, *SM*, *OK* and *IX* parallel to the line <segment> *AG*. *LEG* is a right angle. We join the line <segments> *TE*, *SE*, *OE* and *IE*. Since *TN* is parallel to *FE*, *EN* is equal to *TF*, both being the latitude of the moon at the beginning of the lunar eclipse (whose time is known from Chapter 2). *TE* is the sum of the two radii, and *ENT* is a right angle. Then *TN* is known, and *NL* is <the difference> between the latitude <of the moon> at the beginning <of the eclipse> and its latitude in the middle <of the eclipse>. Then *LT* is known, and it is <corresponding to> the adjusted <duration of> immersion in minutes.



Again, *ES* is the remainder of subtracting the radius of the moon from that of the shadow. *SC* is the latitude <of the moon> at the beginning of total immersion. *ECS* is a right angle. Then *EC* is known, being equal to *SM*. So, *SM* is known. *LM* is <the difference> between the latitude <of the moon> at the beginning of total immersion and <the latitude in> the middle of the eclipse. *LMS* is a right angle. Then *LS* is known, and it is <corresponding to the adjusted> duration <of totality> in minutes. Also, *EO* is the remainder of subtracting the radius of the moon <disk> from that of the shadow <disk>. *OQ* is the latitude of the moon at the beginning of emersion. Then *QE* is known, being equal to *OK*. *KL* is <the difference> between the latitude <of the moon> in the middle <of the eclipse> and <the latitude at> the beginning of emersion. *OKL* is a right angle. Then *LO* is known, being <corresponding to the adjusted> duration <of totality> up to the beginning of emersion in minutes. Again, *EI* is the sum of the two radii, and *IR* the latitude <of the moon> at the end of emersion. Then *RE* is known, being equal to *IX*. *LX* is <the difference> between the latitude <of the moon> at the middle <of the eclipse> and <its latitude at> the end of emersion. Then *LI* is known, and it is <corresponding to the adjusted duration of> immersion from the middle <of the eclipse> to the end of emersion. So, the five adjusted times are known. This is what we wanted.

Chapter 4: On drawing the figure of a lunar eclipse.

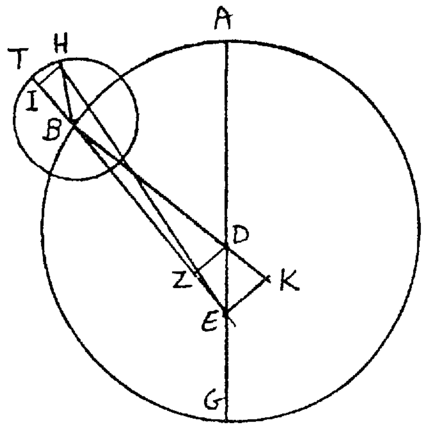
Let *ABGD* be a circle whose radius is equal to the sum of the two radii in minutes, centered at *E*. <Also let> *ZHTI* <be> a circle centered at this <point *E*>, its radius being equal to that of the shadow. The two line <segments> *AG* and *BD* intersect at *E* at right angles. Let *EB* be the south line, *ED* the north line, *EA* the east line, and *EG* the west line, and let *ES* be the latitude of the moon in the middle of the lunar eclipse.



KLMN is the moon circle centered at *S*. Its arc *KLM* is situated in the shadow circle and it (i.e., the lunule *KLM*) is the magnitude of the eclipsed part of the surface of the moon, by taking its whole surface <equal to> 12 digits. *LH* is the non-adjusted (i.e., absolute) <magnitude of the> lunar eclipse in digits (based on taking *NL* equal to 12 digits). *EB* is <equal to> the radius of the shadow plus the radius of the moon. Its <part> *EH* is the radius of the shadow. The remaining <part> *HB* is the radius of the moon, and *LH* is the <absolute magnitude of the> lunar eclipse in digits. This is what we wanted to demonstrate.

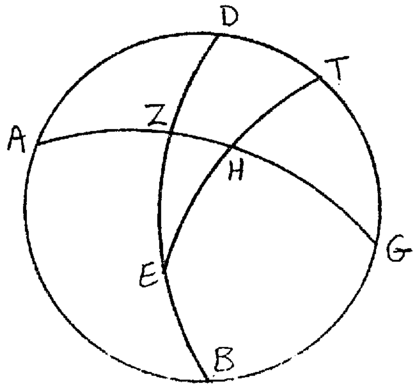
Chapter 5: On the distance of the moon from the earth.

Let the circle *ABG* be the eccentric orb centered at *D*, *AG* its diameter, *E* the center of the ecliptic, *B* the center of the epicycle of the moon, *T* the apogee of the epicycle, and *H* the body of the moon. We join the lines. *EH* is, as desired, the distance of the moon from the earth. The two lines *DZ* and *HI* are perpendicular to *ET*. The angle *AEB* is known, being the double elongation. *Z* is a right angle. So the angle *EDZ* is known. *DE* is <equal to> 10 parts and a third, if we take *AE* <equal to> 60 parts. So both *DZ* and *ZE* are known. *DB* is <equal to> 49 parts and two thirds, and its square is equal to <the sum of> the squares of *BZ* and *ZD*. Then *BZ* and *ZE* are known. So is *EB*, being the distance of the center of the epicycle of the moon from the earth. Also, the angle *TBH* is the adjusted mean anomaly (i.e., true anomaly) of the moon. *I* is a right angle. So the angle *BHI* is known. *BH* is the radius of the epicycle in terms of the distance of its center from the point *A*. Both *HI* and *IB* as well as *EB* are known. So, *EI* is known, its square plus the square of *HI* being equal to the square of *EH*. So, *EH* is known, being the distance of the moon from the earth. This is what we wanted to demonstrate.



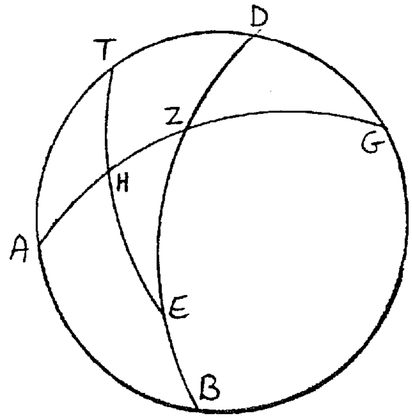
Chapter 6: On the altitude of the pole of the ecliptic.

<Let> *ABGD* <be> the horizon circle, *BED* the meridian, *AHG* the ecliptic, and *ET* <a part> of the altitude circle drawn with its pole at *A* and its radius equal to the side of an <inscribed> square. Then *HT* is <equal to> the magnitude of the angle *HAT*, for *AT* and *AH* are both quadrants. <Now> *EH* is desired, for it is equal to the altitude of the pole of the ecliptic. In the triangle *ADZ*, *D* is a right angle, the side *AZ* <the distance> between the ascendant and the midheaven along the ecliptic, and *ZD* the altitude of the <ecliptical> degree of the midheaven. The ratio of the Sine of *AZ* to that of *ZD* is equal to the ratio of the greatest Sine, being the Sine of *AH*, to the Sine of *HT*. So *HT* is known. Therefore its complement *EH* is known. This is what we wanted to demonstrate.



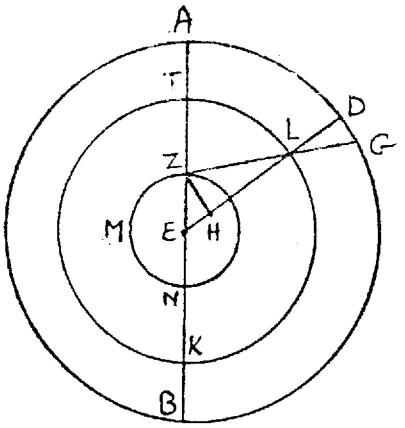
Chapter 7: On the altitude of any desired degree of the ecliptic.

<Let> *ABGD* <be> the horizon circle, *BED* the meridian, *AZG* the ecliptic, the two points *A* and *Z* the ascendant and the tenth <house, *E* the zenith>, *ET* <a part> of the altitude circle, and *H* the <ecliptical> degree whose altitude is desired. <So,> the arc *HT* is desired. In the two triangles *AHT* and *AZD*, *D* and *T* are right angles. *AH* is <the distance> between the ascending <ecliptical> degree and the <ecliptical> degree whose altitude is desired. *ZD* is the magnitude of the altitude of the midheaven degree. The ratio of the Sine of *AH* to that of *HT* is equal to the ratio of the Sine of *AZ* to that of *ZD*. So, *HT* is known. This is what we wanted to demonstrate.



Chapter 8: On the parallax of the two luminaries in the altitude circle.

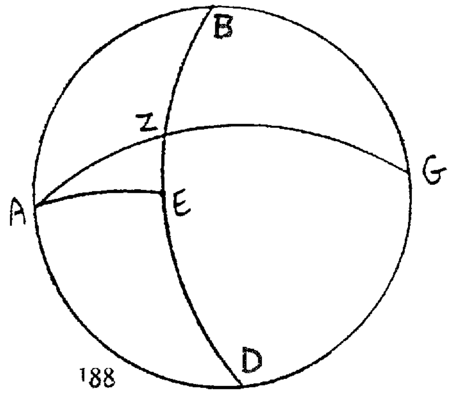
<Let> *ABGD* <be> the altitude circle on the surface of the Sphere of the Whole (i.e., the Universe), *TKL* the altitude circle on the surface of the sphere of the moon (i.e., a sphere through the moon with center the center of the earth) with the point *L* as the moon on it, and the circle *ZMN* the surface of the earth. The three circles are in the same plane and centered at *E*. The line <segments> *AE*, *TE*, and *ZE* are the radii of these circles passing through the zenith. From the two points *E* and *Z*, we draw two lines intersecting at the point *L* and ending at the two points *G* and *D*. The angle *ELZ* is the parallax, because it is the excess of the angle *LZT*, i.e. the arc *AG* visible on the surface of the earth, over the angle *LET*, i.e. the arc *AD* visible from the center of the earth. From the point *Z*, we draw *ZH* perpendicular to *EL*. The arc *AD* is the complement of the altitude visible from the center of the earth. So, the angle *ZEH* is known <from the computation of the lunar position>. *ZHE* is a right angle, and *ZE* is the radius of the earth, being <equal to> one degree (i.e. the unit of length). So, both *EH* and *HZ* are known. *EL* is the distance of the moon from the earth. Then *HL* is known, and so is *LZ*. Then *ZH* is known by taking *LZ* <equal to> 60 parts, and its arc is known. Then the angle *ELZ* is known and its magnitude is <equal to> the arc *GD*. This is what we wanted to demonstrate.



Chapter 9: On the six angles needed in <the calculation of> solar eclipses.

First <case>, when the position of the moon is at the first <degree> of Aries or Libra and it is at the ascending <ecliptical> degree of the time:

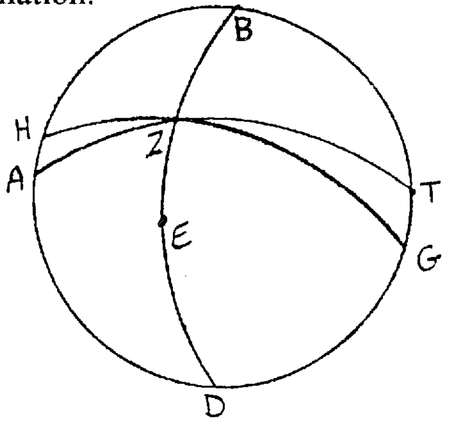
<The first figure>



Let *ABGD* in the first figure be the horizon circle, *BED* the meridian, *E* the zenith, *AZG* the ecliptic, *EA* <a part> of the altitude circle, and the point *A* the rising position of the equinox. The angle *EAZ* is desired, its magnitude being <equal to> the arc *EZ*. Both *AZ* and *EA* are quadrants, and *EZ* is known, because the point *Z* is the first of Cancer or the first of Capricorn. Therefore the angle *EAZ* is known (see commentary).

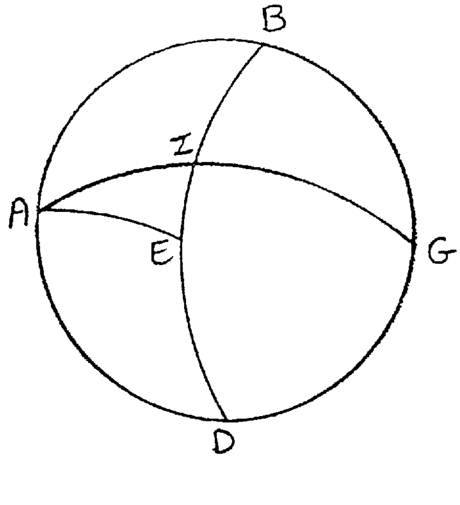
Second <case>, when the position of the moon is at the first <degree> of Aries or Libra and it is at the <ecliptical> degree of the tenth <house> of the time: It (i.e., the desired angle) is the angle *GZD* in the second figure, taking *AZT* as the celestial equator, *HZG* as the ecliptic, the point *Z* as the equinox, and *BED* as the meridian. *TZD* is a right angle, and the angle *TZG* is the total declination. So the remaining angle *GZD* is the complement of the <total> declination.

<The second figure>



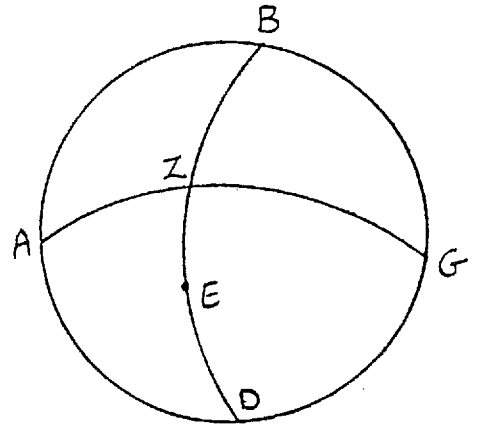
Third <case>, when the position of the moon is other than the first <degree> of Aries or Libra, and it is at the ascending <ecliptical> degree of the time: It is the angle *ZAE* in the third figure, assuming *A* to be other than the rising position of the equinox, and the circle *BED* drawn with its pole at *A* and its radius <equal to> the chord of the quadrant of <a great> circle. Then the arc *EZ* is equal to the altitude of the pole of the ecliptic (computed in Chapter 6), and its magnitude equal to the angle *EAZ*.

<The third figure>



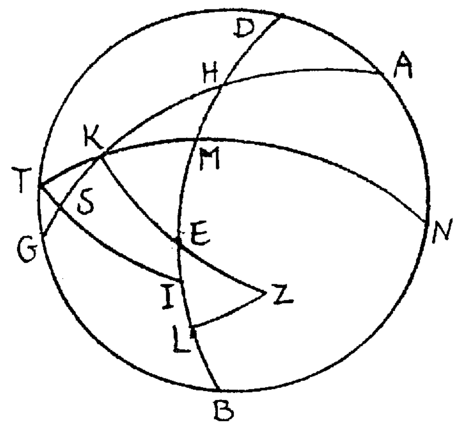
Fourth <case>, when the position of the moon is the first <degree> of Cancer or the first <degree> of Capricorn, and it is the <ecliptical> degree of the tenth <house> of the time: It is the angle *AZD* in the fourth figure, *AZG* being the ecliptic and *BED* the meridian. It (i.e., the desired angle) is a right angle, for *AZ* is a quadrant (*A* is an equinox).

<The fourth figure>



Fifth <case>, when the position of the moon (*H* in the fifth figure) is other than an equinoctial or solstitial point, and it is the <ecliptical> degree of the tenth <house> of the time: Let *ABGD* in the fifth figure be the horizon circle, *BED* the meridian, *L* the pole of the celestial equator, *GKA* the ecliptic, and *Z* its pole. *KHE* is the desired angle. In the triangle *KHE*, *K* is a right angle. The side *EH* is <the distance> between the zenith and the ecliptic along the meridian. *EK* is equal to the altitude of the pole of the ecliptic. The ratio of the Sine of *HE* to that of *EK* is equal to the ratio of the greatest Sine to that of the angle *H*. So the angle *H* is known. (Some manuscripts contain an alternative method which uses the points *I* and *S*, see the commentary to this chapter.)

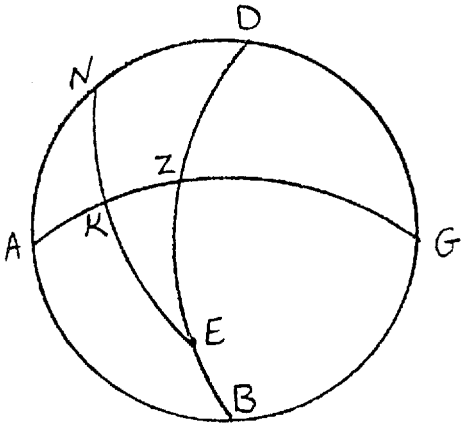
<The fifth figure>



Sixth <case>, when the position of the moon is any <arbitrary ecliptical> degree between the ascendant and the descendant: Let *ABGD* in the sixth figure be the horizon circle, *AZG* the ecliptic, its point *K* the <ecliptical> degree of the moon, *BED* passing through its (i.e., ecliptic's) poles <and

the zenith *E*>. *EKN* is <a part> of the altitude circle. *EKZ* is the desired angle. In the triangle *EKZ*, *Z* is a right angle. The side *KE* is the complement of the altitude of the <ecliptical> degree of the moon (computed in Chapter 7). The side *EZ* is equal to the altitude of the pole of the ecliptic (computed in Chapter 6).

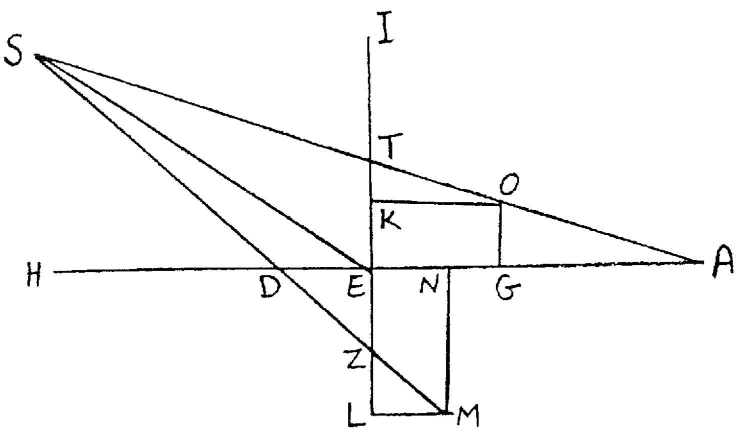
<The sixth figure>



The ratio of the Sine of *KE* to that of *EZ* is equal to the ratio of the greatest Sine to that of the angle *K*. Then the angle *K* is known. This is what we wanted.

Chapter 10: On <finding> the longitudinal and latitudinal parallax of the moon from these angles.

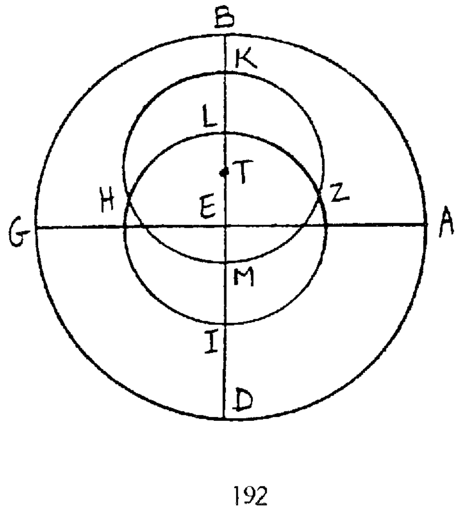
Let *AH* be an arc of the ecliptic, *IL* an arc of the latitude circle, *ET* the northern latitude of the moon, *E* the <ecliptical> degree of the moon, *T* the body of the moon, and *S* the zenith. We draw two arcs of the <two> altitude circle<s> passing through the two points *T* and *E*. They are the arcs *SA* and *SE*. Let *TO* be the parallax in the altitude circle. We draw *OK* parallel to *AH*, and *OG* parallel to *IE*. The lines in this figure are arcs. However, there is no <noticeable> difference between taking them as arcs or as straight lines, because they are small at the time of eclipses (because the lunar latitude is almost zero).



The two line <segments> *EG* and *OG* are desired. *EG* is the difference in longitude, and *OG* the apparent latitude. The angle *SEH* is the latitude angle (computed in Chapter 9). There is no noticeable <difference> between it and the angle *SAH*. The angle *SAH* is equal to the angle *TOK*, because *OK* is parallel to *AE*. Both the angles *SAH* and *TOK* are equal to the angle *SEH*; so they are known. *OKT* is a right angle; so the angle *OTK* is known. Since *OT*, the hypotenuse of the right angle is known, both *OK* and *KT* are known. *KT*, the difference in latitude, <is known>, so *KE* is known, and it is equal to *OG*. Then *OG* is known, and it is the apparent latitude. *OK* is equal to *GE*. So *GE* is known, and it is the difference in longitude. So because of its latitude *ET*, the moon is seen at the point *G* of the ecliptic. Again, let *EZ* be the latitude in the south <direction>, and *ZM* the parallax in the altitude circle. We join *ML* parallel to *AH* and *MN* parallel to *EL*. The two line <segments> *MN* and *NE* are desired. The angle *SDH* is approximately equal to the angle *SEH*. The angle *ZML* is equal to the angle *SDH*, because *ML* is parallel to *EH*. So the angle *ZML* is equal to the angle *SEH*. *L* is a right angle, and *ZM* is known. Then the angle *MZL* is known; so the sides of the triangle *MLZ* are known. *EZ* is known; so *EL* is known, and it is equal to *MN*. So *MN* is known, and it is the apparent latitude. *ML* is known, so *EN* is known, and it is the difference in longitude. So because of its latitude *EZ*, the moon is seen at the point *N* of the ecliptic. This is what we wanted to demonstrate.

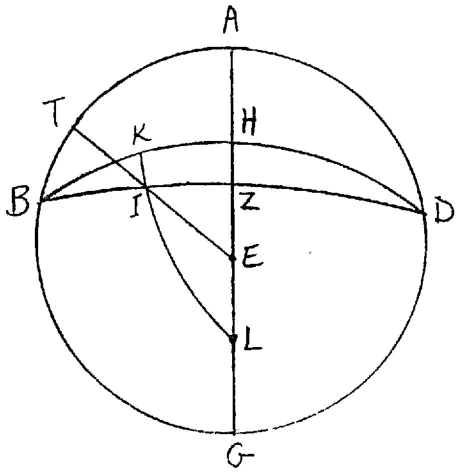
Chapter 11: On drawing the figure of a solar eclipse.

<Let> *ABGD* <be> a circle <with its radius equal to> the sum of the two radii, centered at *E*, *EB* equal to the radius of the moon plus the radius of the sun, *EL* the radius of the sun and *ZLHI* the circle of its surface. *ET* is the <apparent> latitude of the moon, *TK* its radius, and *ZKHM* the circle of its surface. Then <the segment> *ML* of the diameter of the sun is <the magnitude of> the solar eclipse in digits. The line <segment> *AG* is the east-west line, and the line <segment> *BD* is the north-south line. This is what we wanted.



Chapter 12: On <finding> the altitude of the moon, taking account of its latitude.

<Let> *ABGD* <be> the horizon circle, *BHD* the ecliptic with its pole at *L*, *AEG* passing through its poles <and the zenith>, and *I* the body of the moon. We draw through it (i.e., the point *I*) <the arcs> *LIK*, *BID* <,and> *EIT*. The arc *IT* is desired. <The arc> *IK* is the latitude of the moon. In the two triangles *LIZ* and *LKH*, the angle *L* is common, and *Z* and *H* are right angles.



So the ratio of the Sine of *LI* to the Sine of *IZ* is equal to that of the Sine of *LK* to the Sine of *KH*. But *LI* is the complement of the latitude of the moon, *LK* is a quadrant, and *HK* is the complement of the distance of the <ecliptical> degree of the moon from the ascendant. So *IZ* is known, therefore, its complement *IB* is known. Again, in the two triangles *BIK* and *BZH* the angle *B* is common, and *K* and *H* are right angles. So the ratio of the Sine of *BI* to that of *IK* is equal to the ratio of the Sine of *BZ* to the Sine of *ZH*. But *BI* is known, *IK* is the latitude of the moon, and *BZ* is a quadrant. So *ZH* is known. But *AH* is the complement of the altitude of the pole of the ecliptic <, so it is> known. Thus the whole *AZ* is known. Also, in the two triangles *BIT* and *BZA*, the angle *IBT* is common, and *T* and *A* are right angles. So the ratio of the Sine of *BI* to that of *IT* is equal to the ratio of the Sine of *BZ* to the Sine of *ZA*. But *BI* is known, *BZ* is a quadrant, and *ZA* is known. So *IT* is known. This is what we wanted to demonstrate.

Chapter 13: On <finding> the longitudinal and latitudinal parallax of the moon by a method <the validity of> which can be proved.

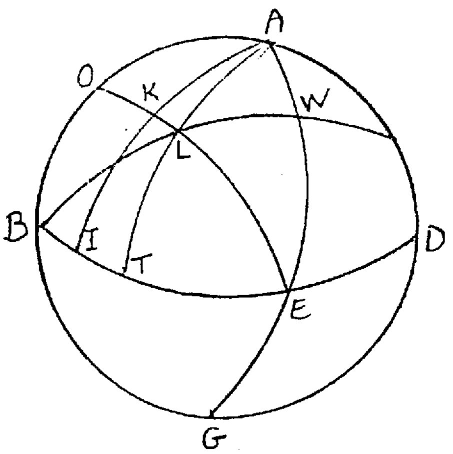
It was said in Chapter 11 of Section 6 in the First Book that the altitude obtained from the calculation is true altitude. <If> the parallax is subtracted from it, <the remainder> is the apparent altitude. Having said that, <I add that the subject of> this chapter occurs in five cases.

First <case>, when the altitude of tenth house at the time is 90 degrees and the moon has no <non-zero> latitude: The parallax in the altitude circle is longitudinal parallax alone.

Second <case>, when the distance of the <ecliptical> degree of the moon from the ascendant of the time is 90 degrees, whether the moon has or does not have a <non-zero> latitude: The parallax in the altitude circle is the apparent latitude alone (there is no longitudinal parallax).

Third <case>, when the altitude of the tenth <house> at the time is 90 degrees and the moon has a <non-zero> latitude. Let *ABGD* in the first figure be the horizon circle, *BED* the ecliptic and the two points *A* and *G* its poles (*E* is the zenith). *AEG* passes through these two poles. *EO* is <a part> of the altitude circle, *L* the body of the moon, and *LK* the parallax in the altitude circle.

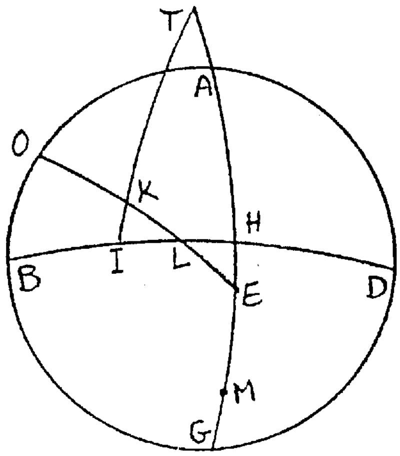
The first <figure>



We draw the arcs *ALT*, *AKI* and *BLW* through the points *L* and *K*. Then *LT* is the southern latitude of the moon (the method is also valid for northern latitudes), *KI* the apparent latitude, and *TI* the difference in longitude. In the two triangles *ELT* and *EKI* the angle *E* is common, and *T* and *I* are right angles. So the ratio of the Sine of *EL* to the Sine of *LT* is equal to the ratio of the Sine of *EK* to the Sine of *KI*. *EL* is the complement of the true altitude (computed in Chapter 12), *LT* the latitude of the moon, and *EK* the complement of the apparent altitude. So *KI* is known, and it is the apparent latitude. Also the angle *A* is common to the two triangles *AKO* and *AIB*, and *O* and *B* are right angles. So the ratio of the Sine of *AK*, the complement of the apparent latitude, to the Sine of *KO*, the apparent altitude, is equal to the ratio of the Sine of *AI*, the greatest Sine, to the Sine of *IB*. So *IB* is known, and *TB* is the distance of the <ecliptical> degree of the moon from the ascendant. So *TI* is known, being the difference in longitude.

Fourth <case>, when the altitude of the tenth <house> of the time is less than 90 degrees and the moon has no <i.e., zero> latitude. Let *ABGD* in the second figure be the horizon circle, *BHD* the ecliptic and the two points *T* and *M* its poles through which passes the circle *AEG* (*E* is the zenith). *EO* is <a part> of the altitude circle, *L* the body of the moon, and *LK* the parallax in the altitude

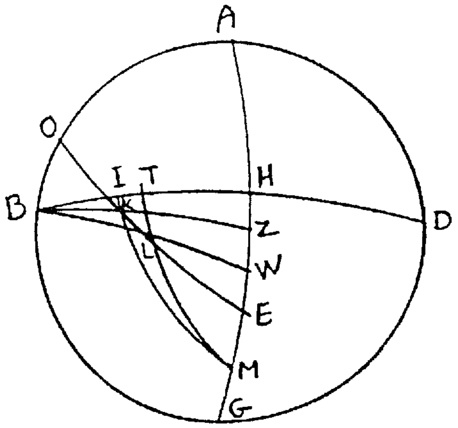
The second <figure>



circle. We draw the arc *TKI* and through the two points *L* and *K*. So *KI* is the apparent latitude, and *LI* is the longitudinal parallax. In the two triangles *KLI* and *HLE* their two angles *L* are equal, and *I* and *H* are right angles. So the ratio of the Sine of *LE* to the Sine of *EH* is equal to the ratio of the Sine of *LK* to the Sine of *KI*. *LK* is the parallax in the altitude circle, *LE* is the complement of the true altitude, and *EH* is the altitude of the pole of the ecliptic. So *KI* is known, and it is the apparent latitude.
Difference in longitude: In the triangle *LKI*, *I* is a right angle. So the ratio of the Cosine of *IK*, the apparent latitude, to the Cosine of *KL*, the parallax in the altitude circle, is equal to the ratio of the greatest Sine to the Cosine of *LI*, the longitudinal parallax. So *LI* is known, <and it is the difference in longitude>.

Fifth <case>, when the altitude of the tenth <house> of the time is less than 90 degrees and the moon has a <non-zero> latitude. Let *ABGD* in the third figure be the horizon circle, *BHD* the ecliptic and *M* its pole. *AEG* is the circle passing through it (i.e., *M*) <and the zenith *E*>. *EO* is <a part> of the altitude circle, *L* the body of the moon, and *LK* the parallax in the altitude circle. We draw the arcs *BKZ*, *BLW*, *MLT*, and *MKI* through the points *L* and *K*. Then *TL* is the northern latitude of the moon, *KI* the apparent latitude, and *TI* the longitudinal parallax.

The third <figure>

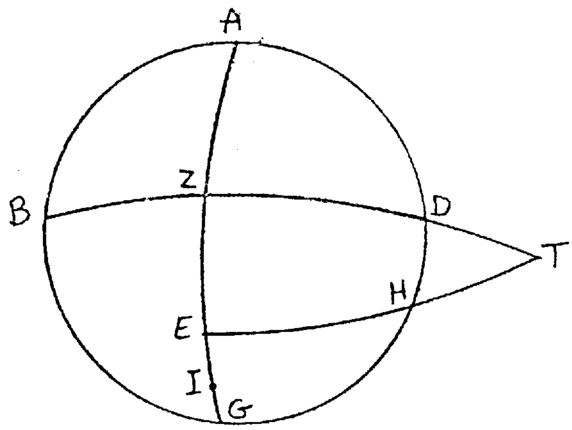


In the two triangles *MLW* and *MTH* the angle *M* is common, and *W* and *H* are right angles. So the ratio of the Sine of *ML* to the Sine of *LW* is equal to the ratio of the Sine of *MT* to the Sine of *TH*. *ML* is the complement of the latitude of the moon, *MT* is a quadrant, and *TH* is the complement of the distance of the <ecliptical> degree of the moon from the ascendant. So *LW* is known. Also, in the triangles *ELW* and *EKZ* the angle *E* is common, and *W* and *Z* are right angles. So the ratio of the Sine of *EL* to the Sine of *LW* is equal to the ratio of the Sine of *EK* to the Sine of *KZ*. *EL* is the complement of the true altitude, *LW* is known, and *EK* is the complement of the apparent altitude. So *KZ* is known, and its complement *KB* is known. Also, in the triangles *BKO* and *BZA* the angle *B* is common, and *O* and *A* are right angles. So the ratio of the Sine of *BK* to the Sine of *KO* is equal to the ratio of the Sine of *BZ* to the Sine of *ZA*. *BK* is known, *KO* is the apparent altitude, and *BZ* is a quadrant. So *ZA* is known. *AH* is the complement of the altitude of the pole of the ecliptic. So *HZ* is known. Also, in the triangles *BKI* and *BZH* the angle *B* is common, and *H* and *I* are right angles. So the ratio of the Sine of *BK* to the Sine of *KI* is equal to the ratio of the Sine of *BZ* to the Sine of *ZH*. *BK* is known, *BZ* is a quadrant, and *ZH* is known. So *KI* is known, being the apparent latitude.

Difference in longitude: In the triangles *MKZ* and *MIH* the angle *M* is common, and *Z* and *H* are right angles. So the ratio of the Sine of *MK* to the Sine of *KZ* is equal to the ratio of the Sine of *MI* to the Sine of *IH*. *MK* is the complement of the apparent latitude, *KZ* is known, and *MI* is a quadrant. So *IH* is known. *TH* is known, and so is *TI*, which is the longitudinal parallax. So the apparent latitude and the longitudinal parallax in <all> different situations are known. That is what we wanted to demonstrate.

Chapter 14: On the visibility arc<s>.

<Let> *ABGD* <be> the horizon circle, *BZD* the ecliptic and *I* its pole. *AEG* passes through the two poles of the ecliptic <and the zenith *E*>. *EHT* is the altitude circle <through the sun *T*>, *D* the <ecliptical> degree with which the moon sets <simultaneously>, *ZA* is the complement of the altitude of the pole of the ecliptic, and it is <equal to> the magnitude of the angle *ADZ* which is equal to the angle *TDH*. The arc *TH* is desired. In the triangles *DHT* and *DZA* the two angles *D* are equal, and *H* and *A* are right angles. So the ratio of the Sine of *DT* to the Sine of *TH* is equal to the ratio of the Sine of *DZ* to the Sine of *ZA*. *DT* is the distance between the sun, being the point *T*, and the <ecliptical> degree with which the moon sets <simultaneously>, being the point *D*. *DZ* is a quadrant and *ZA* is the complement of the altitude of the pole of the ecliptic. So *TH* is known, being the desired <arc>. As found up to now, its minimum value <for the visibility of the lunar crescent> is 6½ degrees to 7 degrees. That is what we wanted to demonstrate.



Commentary

IV.6.1 This is a proof of the validity of the method for converting 'absolute digits' (=linear digits) into 'adjusted digits' (=area digits) in the eclipses. Here, and in the subsequent "proofs", Kūshyār assumes that all arcs of great circles on the sphere can be represented as straight lines in a plane. See also I.6.4 and its commentary.

IV.6.2 In the figure, the angle *Z* is usually very close to 90 degrees. So to simplify the calculations, Kūshyār assumes it to be a right angle. This implies that *AB* and *AG* are approximately parallel, although they actually intersect in the figure. See also I.6.5 and its commentary.

IV.6.3 Here, *AB* is no longer assumed parallel to *AG*, and the small change of the moon's latitude during the eclipse has been taken into account. Since point *L* is the midpoint of the chord through *S* and *O*, which is not parallel to *AG*, the angle *LEA* is not exactly equal to 90 degrees. According to my computation, the maximum difference between the results from the methods of Chapter 2 and Chapter 3 is less than 0.08%.

IV.6.4 This is merely an illustration of what Kūshyār describes in I.6.6. The figure illustrates the situation in the middle of the eclipse. The references to the four cardinal directions are merely symbolic, since the moon can be on every side of the shadow cone of the earth (depending on the lunar latitude).

IV.6.5 The proof is correct. The determination of the distance*EB* of the epicycle center corresponds to the procedure which Kūshyār describes in I.6.7. Here Kūshyār deviates from the Ptolemaic lunar model by measuring the adjusted mean anomaly from the epicyclic (or true) apogee, while Ptolemy takes it from another point on the epicycle (mean apogee) which lies on the extension of the line connecting *B* and a new point *E*′, so that *E* is the midpoint of the line segment *DE*′ [Ptolemy 1984, 249-251; Kashino 1998, 9].
Kūshyār then determines an "adjusted radius" of the epicycle which does not seem to play a role in the proof of IV.6.5. In the light of the proof of IV.6.5, the "adjusted radius" in I.6.7 seems superfluous, and can be replaced by the normal value of the epicycle radius (which Kūshyār also used in the computation of the solar distance).

IV.6.6 In the figure, *E* represents the zenith and *EH* is drawn perpendicular to the ecliptic. Since *E* is the pole of the horizon, the pole *A* of circle *ET* is on the horizon. Since circle *EHT* is perpendicular to the ecliptic, the pole of ecliptic is on circle *EHT*, 90 degrees away from point *H*. Thus the altitude of the pole of the ecliptic is equal to the complement of *EH* of the zenith distance.

IV.6.7 This is a proof of the validity of the method provided in I.6.9

IV.6.8 This is a proof of the validity of the method provided in I.6.11. In a personal communication, Prof. E. S. Kennedy remarks: "'The Sphere of the Whole' (in Arabic *kurrat al-kull*) means 'the celestial sphere together with its contents': the immobile terrestrial sphere at the center (containing us), next the moon, next the five planets, comets etc. in between, and finally the fixed stars on the inside of the celestial sphere."
The figure is very similar to that used by Ptolemy [1984, 248] for finding the distance of the moon from a known lunar parallax. Since the computation of parallax is mostly used for the prediction of lunar eclipses, which happen when the center of the lunar epicycle is at maximal distance from the earth, it is reasonable to take the lunar distance equal to 60 earth radii. Kūshyār's assumption that the magnitude of the angle *L* is equal to arc *GD* is approximately true, since the radius *EA* of the universe is very large compared to the radius of the lunar sphere *ET*.

IV.6.9 The proofs of the validity of the methods provided in I. 6.12 for the six cases of the angle between the ecliptic and the altitude circle passing through the ecliptical degree of the moon are presented in this chapter. These angles are used in Chapter IV.6.10 for finding the longitudinal and the latitudinal parallax of the moon.
In the first case, the required angle is equal to $90^{\circ}-\varphi \pm \varepsilon$, where φ is the geographical latitude of the locality and ε is the obliquity of the ecliptic. In the second case, this angle is equal to $90^{\circ}-\varepsilon$ (note that we always take the smaller angle). The tenth house is the point of intersection of the ecliptic with the meridian above the horizon.
In the fifth case, the ms. A only contains an alternative proof which is found as marginal note in F, V and L. The points *I* and *S* in the figure are related to this alternative proof. The ms. Y only provides a proof almost similar to that of A, but using the triangle *KHM*. The ms. M only contains the proof of the main method in the text of F, V and L. As mentioned in the footnote of the Arabic text, the mss. F, V , L and M use the first figure for the cases one to four. But in A and Y one figure is drawn for each case, and we have followed them in this regard.

IV.6.10 This chapter presents the proofs of the validity of the methods for the two cases provided in I.6.13 in which the latitude of the moon is towards north or south, respectively. In the figure, *O* is the apparent lunar position, as seen by the observer, and *T* is the "real" lunar position, as seen from the center of the earth. See IV.6.12 for a more refined treatment (in which *SHE* and *SAH* are no longer considered as equal).

IV.6.11 This is an example of the drawing described in I.6.16. Also see the commentary to IV.6.4.

IV.6.12 This is a proof of the method provided in I.6.17. In that method, 'the first arc' is *IB*, 'the second arc' is *HZ*, and 'the result from the complement of the altitude of the pole' is *AZ*. Here we have provided the version found in A and Y. Other mss. contain proofs of the case in which *BHD* is the celestial equator. They correspond to 'another method' in I.6.17 which I have deleted, because that method and its proofs do not seem authentic. See I.6.17 and the footnote to the Arabic text of IV.6.12.

IV.6.13 In this chapter more accurate methods of computation of the longitudinal and latitudinal parallax are provided, in contrast to the approximate methods of Chapter 10. We have not seen this refined determination of the longitudinal and latitudinal parallax in other Arabic sources. Of course, since the arcs involved in this subject are very small, the results of the approximate methods are sufficient for practical purposes and the accurate methods merely have theoretical significance.
The first and the second cases are trivial. The proof of the third and the fourth cases are provided only for the first method of these cases in I.6.18. No proof is given for the alternative methods relating to the third and the fourth cases in I.6.18. In the proof of the validity of the method for finding the apparent latitude in the fifth case, the arcs *LK*, *KZ*, *ZA* and *ZH* are corresponding to the 'first', 'second', 'third' and 'fourth' arcs mentioned in the description of the method in I.6.18. For finding the difference in longitude in this case, *IH* represents 'the first arc'.

IV.6.14 This is a proof of the procedure of I.6.20 for finding the visibility arc. The minimum value of the visibility arc for the moon is mentioned to be 6 degrees in I.6.20 (less than 6½ to 7 degrees in this chapter). The limit values of the visibility arc for the planets are mentioned in I.6.20.
In the single chapter of Book III on the definition of the astronomical terms, Kūshyār defines the visibility arc (*qaus al-ru'ya*) as follows: "The arc <which is part> of the altitude circle, between the horizon on which lies the planet, and the sun <situated> under the earth (i.e., the horizon); it may also be regarded as the arc of the altitude circle between the planet

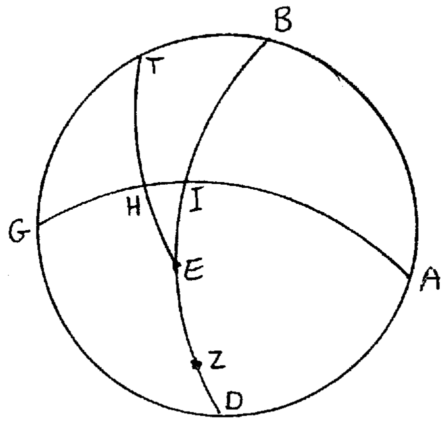
<situated> above the earth (i.e., the horizon) and the horizon on which lies the sun." Ptolemy gives the limit magnitudes of *arcus visionis* (visibility arc) for the stars and the planets as the sun's depression at the moment the celestial body is on the horizon [Ptolemy 1984, 413-415, 639-640]. In I.6.20, Kūshyār uses both cases of the visibility arc, below and above the horizon. Bīrjandī states that the arc of visibility was considered above the horizon because the depression is a calculated value, whereas an altitude could be observed [al-Ṭūsī 1993, II, 464]. For a description of the visibility arc (arc of vision, *arcus visionis*) see [Pedersen 1974, 388; Neugebauer 1975, I, 234-236].

Section 7: On what pertains to astrology, in one chapter

On the projection of the ray taking the latitude of the planet into account.

Those earlier astrologers who had some knowledge of astronomy said that when the planet has a <non-zero> latitude, its rays are not taken from the ecliptic, but from the <great> circle passing through the planet and cutting the ecliptic at <a distance equal to> the supposed distance. Al-Battānī (in Chapter 54 of his *al-Zīj al-Ṣābī* [1899, 196-197]), wishing to calculate this, has gone to great lengths to calculate and describe this. If this (i.e. the projection of the ray) is influential in astrology, and if it is needed in astrology, the way to <find> it is really short and its calculation is as I have demonstrated in Book I.

Proof: Let $ABGD$ be the ecliptic circle, E its pole, and the point H the body of the planet. EHT passes through the two poles of the ecliptic. So T is the <ecliptical> degree of the planet, TH is its latitude, and EH is the complement of the latitude. Let AIG pass through the planet, and Z <be> its pole. BED passes through the two poles (i.e., E and Z). GH is assumed to be 60 degrees, being the sextile arc on this circle, and its complement HI is 30 degrees. In the two triangles EHI and ETB, the angle E is common, and I and B are right angles. So the ratio of the Sine of EH to that of HI is equal to ratio of the Sine of ET, being the greatest Sine, to the Sine of BT. Then BT is known, being the desired magnitude on the ecliptic. If it is subtracted from BG (i.e., 90°), the remainder TG is the sextile arc. If it (i.e., BT) is added to BG, the result is the trine arc. If the arc GT is assumed <to be> 60 degrees, and the arc GH is desired, the ratio is the same as before. The arc HI will be known, its complement HG will be the sextile arc, and its addition to GI <will result> in the trine arc. GI and GB are the quartile arcs, any of which may be taken. That is what we wanted to demonstrate.



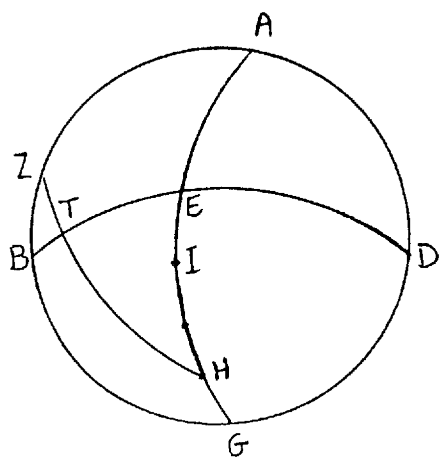
Commentary

IV. 7 This is a proof of the method presented in I.7.2 for the calculation of the projection of the rays when the planet has non-zero latitude. See I.7.2 and its commentary.

Section 8: On the operations which are less needed, <in> 8 chapters

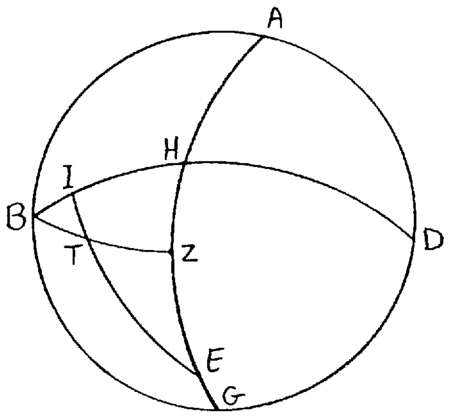
Chapter 1: On <finding> the latitude of a locality from the hours (i.e., the duration) of <its> longest and shortest days.

<Let> *ABG* <be> the horizon circle, *AEG* the meridian, *BED* the celestial equator, *H* its pole, and *Z* the rising position of <the beginning of> Capricorn or Cancer. We draw *HTZ*. *BZ* is the ortive amplitude and *ZT* is the total declination (i.e., the obliquity of the ecliptic). *TE* is half the day arc. It is known from the multiplication of half the <number of equinoctial> hours of the day by 15. *EA* is the complement of the latitude of the locality. *EI* is the desired latitude of the locality (*I* is the zenith). In the two triangles *THE* and *HZA* the angle *H* is common, and *E* and *A* are right angles. So the ratio of the Sine of *HZ* to that of *ZA* is equal to the ratio of the Sine of *TH* to that of *TE*. The Sine of *HZ* is equal to the Sine of the complement of the <total> declination. *HT* is a quadrant and *TE* is known. So *ZA* is known, and it is the complement of the ortive amplitude. Then *ZB*, the ortive amplitude, is known. In the two triangles *BTZ* and *BEA* the angle *B* is common, and *T* and *E* are right angles. So the ratio of the Sine of *BT*, the equation of the daylight, to the Tangent of *TZ*, the total declination, is equal to the ratio of the Sine of *BE*, the greatest Sine, to the Tangent of *EA*, the complement of the latitude of the locality (see the premise proved in IV.3.5).
<Proof of another method:> In the two triangles *BZT* and *BAE* the angle *B* is common, and *T* and *E* are right angles. So the ratio of the Sine of *BZ* to that of *ZT* is equal to the ratio of the Sine of *BA* to that of *AE*. *BZ* is the ortive amplitude, *ZT* is the <total> declination, and *BA* is a quadrant. Then *AE* is known, and it is the complement of the latitude of the locality. Then *IE* is known, and it is the latitude of the locality. This is what we wanted to demonstrate. Now it has become clear that this proof is generally valid for the hours of every day of the year, if the declination of the sun corresponding to its ecliptical degree is taken (instead of the total declination).



Chapter 2: On <finding> the altitude without (i.e., with zero) azimuth.

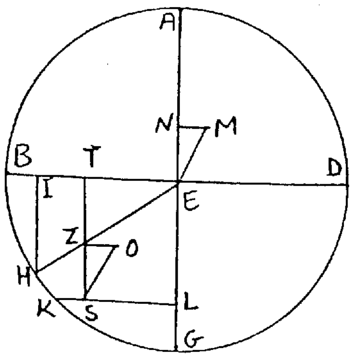
<Let> *ABGD* <be> the horizon circle, *Z* the zenith, *AZG* the meridian, and *BHD* the celestial equator. *ZTB* is <a part> of the altitude circle passing through the rising position of the equinox (i.e., the East point with zero azimuth), and *T* is the body of the sun or the planet. So *IT* is the declination of the sun or the distance of the planet from the celestial equator. In the two triangles *BTI* and *BZH*, the angle *B* is common, and *I* and *H* are right angles. So the ratio of the Sine of *BT* to that of *TI* is equal to the ratio of the Sine of *BZ* to that of *ZH*. *TI* is the declination or the distance, *BZ* is a quadrant, and *ZH* is the latitude of the locality. So *BT* is known, and it is the altitude without (i.e., with zero) azimuth. This is what we wanted to demonstrate.



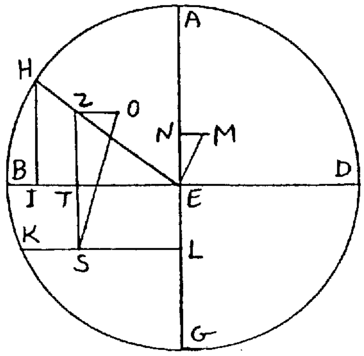
Chapter 3: On <finding> the azimuth of <a point of given declination and> any assumed altitude.

<Let> *ABGD* <be> the horizon circle, *AEG* the intersection of the meridian <plane> and the horizon, *BED* the intersection of the celestial equator <plane> and the horizon, *LK* the intersection of the parallel circle <plane> and the horizon, and *EH* the radius of the altitude circle. *OZ* is the perpendicular drawn from the intersection <point> of the altitude circle and the parallel circle to the horizon plane. So it is the Sine of the altitude. We draw *ZT* perpendicular to *EB*. It is also perpendicular to *LK*, because *BE* and *KL* are parallel. We draw *HI* perpendicular to *BE*. It is the Sine of the azimuth. *MN* is the perpendicular drawn from the intersection <point> of the meridian and the celestial equator to the horizon plane. It is the Cosine of the latitude of the locality. We join *ME* and *OS*. The <corresponding> sides of the two triangles *MNE* and *OZS* are parallel. So the ratio of *MN*, the Cosine of the latitude of the locality,

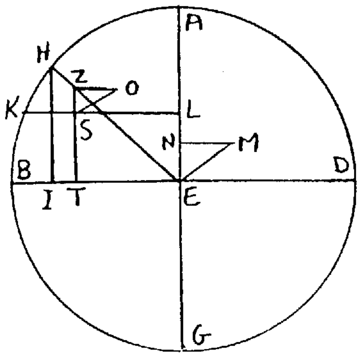
to *NE*, the Sine of the latitude of the locality, is equal to the ratio of *OZ*,



southern declination
and azimuth



northern declination
and southern azimuth

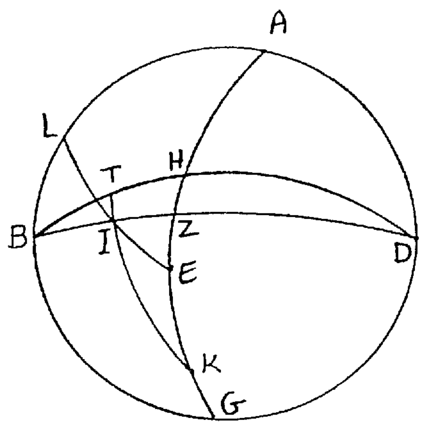


northern declination
and azimuth

the Sine of the altitude, to *ZS*, <which is called> 'the argument of the azimuth'. So *ZS* is known, and *ST* is the Sine of an arc equal to *BK*, the ortive amplitude. Then *ZT* is known, and it is <called> 'the equation of the azimuth'. Again, in the two triangles *EZT* and *EHI*, the two bases *ZT* and *HI* are parallel. So the ratio of *EZ*, the Cosine of the altitude, to *ZT*, the equation of the azimuth, is equal to the ratio of *EH*, the greatest Sine, to *HI*, the Sine of the azimuth. So *HI* is known, and it is the required azimuth. This is what we wanted to demonstrate.

In the second figure, *ZS*, the argument of the azimuth, is greater than *TS*, which is equal to the Sine of the ortive amplitude. If *ST* is subtracted from *SZ*, the remainder is the equation of the azimuth, and the azimuth *BH* is southern. In the third figure, *ZS*, the argument of the azimuth, is less than *TS*, which is equal to the Sine of the ortive amplitude. If *ZS* is subtracted from *ST*, the remainder is *ZT*, which is the equation of the azimuth, and the azimuth is northern. This is what we wanted to demonstrate.

Another method <for the case> when the ascendant and the tenth <house> are known: <Let> *ABGD* <be> the horizon circle, *AEG* the meridian, *K* the pole of the celestial equator, and *BHD* the celestial equator.

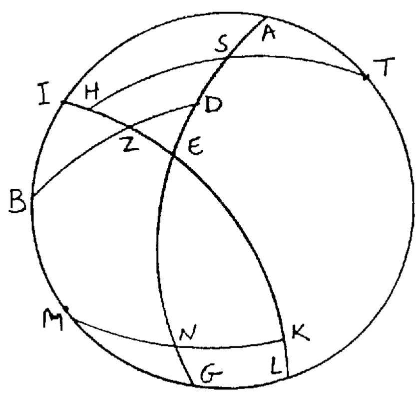


I is the <ecliptical> degree of the sun, and *BID* passes through it. *EIL* is <a part> of the altitude circle. *KIT* passes through the pole of the celestial equator and the <ecliptical> degree of the sun. The arc *BL* is required, and I say that it is known.

Proof: In the two triangles *KIZ* and *KTH*, the angle *HKT* is common, and *Z* and *H* are right angles. So the ratio of the Sine of *KI* to that of *IZ* is equal to the ratio of the Sine of *KT* to that of *TH*. So *IZ* is known, because *KI* is the complement of the declination and *TH* is the right ascension (in the ancient sense) of the distance of *I* from the meridian (*TH* is known because the ascendant and the tenth house are known). Again, in the two triangles *EIZ* and *ELA*, the angle *ZEI* is common, and *Z* and *A* are right angles. So the ratio of the Sine of *EI* to that of *IZ* is equal to the ratio of the Sine of *EL* to that of *LA*. Then *LA*, the complement of *BL*, is known. So *BL* is known. This is the case, because *EI* is the complement of the altitude, and *IZ* is known. Since the two middle terms in the first proportion are equal to the two middle terms in the other proportion, the ratio of the Sine of *KI* to that of *EI* is equal to the ratio of the Sine of *AL* to that of *HT*. So *AL*, the complement of the azimuth, is known. Therefore, *BL* is known. This is what we wanted.

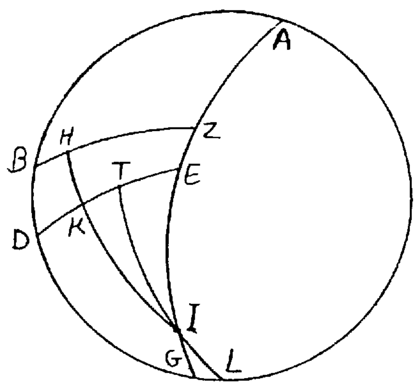
Chapter 4: On <finding> the altitude from the azimuth <and the declination>.

Premise: If the circle of the celestial equator and an altitude circle intersect, and we take an arc of the meridian <starting> from the horizon and equal to the latitude of the locality, then <the distance > between the zenith and the celestial equator on the altitude circle is equal to <the distance> between the horizon and the circle passing through the pole of the altitude circle and <the endpoint of an arc> equal to the latitude of the locality.

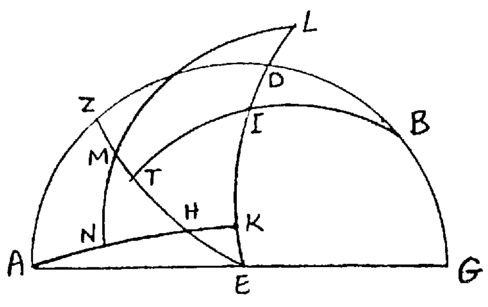


<Proof:> Let *ABG* be the horizon circle, *AEG* the meridian, *BZD* the celestial equator and *N* its pole. *IEL* is the altitude circle, and each one <of the arcs> *GN*, *ED* and *SA* is <equal to> the latitude of the locality. *T* and *M* are the two poles of the altitude circle. We draw <the arcs> *MNK* and *TSH*. I say that *EZ* is equal to *HI*. <Proof:> The point *Z* is the pole of the circle *MNK*; <so> *KZ* is a quadrant. *LE* is <also> a quadrant. We subtract the common <arc> *KE*; the remaining <arcs> *LK* and *EZ* are equal. The ratio of the Sine of *MN* to that of *NG* is equal to the ratio of <the Sine of> *MK* to that of *KL*. *MN* is equal to *TS*, *NG* is equal to *SA*, and *MK* is equal to *TH*. So the remaining arcs *HI* and *KL* are equal. Hence it is demonstrated that *KL* is equal to *EZ*. Then *HI* is equal to *EZ*. This is what we wanted to demonstrate.

When the azimuth is northern: <Let> *ABGD* <be> the horizon circle, *AEG* the meridian, *I* the pole of the celestial equator, *BHZ* <a part> of the celestial equator, *ED* <a part> of the altitude circle, <*E* is the zenith,> and *K* the <ecliptical> degree of the sun. Then *BD* is the given azimuth, and *DG* is its complement. *IG* is the latitude of the locality. We take *GL* equal to *BD*, and we draw *LIT* and *HKI*. The arc *KD*, the altitude <of the sun> at the <given> time, is desired. *KH*, the declination of the sun, is always northern, and *KI* is its complement. So the ratio of the Sine of *EI* to that of *IT* is equal to the Sine of *EG* to that of *GD*. *EI* is the complement of the latitude of the locality, and *DG* is the complement of the azimuth. So *IT* is known, and its complement, *LI*, is known. This is the case (i.e., the above proportion holds), because in the two triangles *EIT* and *EGD* the angle *E* is common, and *T* and *D* are right angles. Also, in the two triangles *LIG* and *LTD* the angle *L* is common, and *G* and *D* are right angles. So the ratio of the Sine of *LI* to that of *IG* is equal to the ratio of the Sine of *LT* to that of *TD*. *LI* is known, and *IG* is the latitude of the locality. So *TD* is known. Also, in the triangle *IKT*, *T* is a right angle. So the ratio of the Cosine of *TI* to the Cosine of *IK* is equal to the ratio of the greatest Sine to the Cosine of *KT*. *TI* is known, and *IK* is the complement of the declination of the sun. So the complement of *TK* is known. Then, *TK* is known. *TD* was already known, so the remainder *KD*, which is desired, is known.

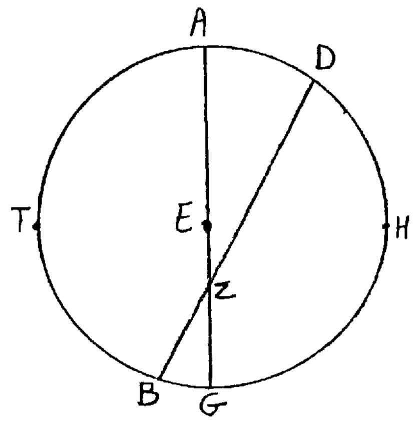


When the azimuth is southern: <Let> *ADG* <be> the southern horizon semicircle, *ED* the meridian, *AK* <a part> of the celestial equator and *L* its pole, and *EZ* <a part> of the altitude circle. Then *AZ* is the given azimuth. We take *DB* equal to *AZ*, and *DI* equal to *EK*, which is the latitude of the locality. We draw *BIT*. *M* is the body of the sun. We draw *MLN*. *HZ* is the argument of the altitude, *MH* the equation of the altitude, *AH* the argument of the arc of revolution, and *NH* the equation of the arc of revolution. Based on what was demonstrated in the premise, *EH* is equal to *TZ*. If *EH* and *ZD* are known, *HK* is <also> known, and so is its complement *HA*. So the circle of the celestial equator and the altitude circle intersect at a point <that can be> known either from the two triangles *EHK* and *EZD*, or from the two triangles *AHZ* and *AKD*. In the triangle *MNH*, *N* is a right angle. So the ratio of the Cosine of *MN* to the Cosine of *MH* is equal to the ratio of the greatest Sine to the Sine of *HN*. So *HN* is known. In this figure, the arc *MZ* is desired. In the two triangles *ETI* and *EZD*, the angle *E* is common, and *T* and *Z* are right angles. So the ratio of the Sine of *EI*, the complement of the latitude of the locality, to the Sine of *IT* is equal to the ratio of *ED*, the greatest Sine, to the Sine of *DZ*, the complement of the azimuth. Then *TI* is known, and its complement *BI* is known. Also, in the two triangles *BID* and *BTZ* the angle *B* is common, and *D* and *Z* are right angles. So the ratio of the Sine of *BI*, which is known, to that of *ID*, the latitude of the locality, is equal to the ratio of the Sine of *BT*, which is the greatest Sine, to the Sine of *TZ*, which is unknown. So *ZT* is known. It was already demonstrated to be equal to *EH*. So *EH* is known. Then *ZH*, the argument of the altitude, is known. Also, *L* is the pole of the celestial equator and *M* is the <ecliptical> degree of the sun. Then, *LMN* is the declination circle, and *MN*, the declination, is southern. In the two triangles *HMN* and *HEK*, their angles *H* are equal, and *K* and *N* are right angles. The ratio of the Sine of *HM*, which is the equation of the altitude, to the Sine of *MN*, the declination of the sun, is equal to the ratio of the Sine of *HE*, the complement of the argument of the altitude, to the Sine of *EK*, the latitude of the locality. So *HM* and *ZH* are known. Then *MZ*, which is the desired altitude, is known. This is what we wanted to demonstrate.



Chapter 5: On the distance between two stars, one of which has a <non-zero> latitude.

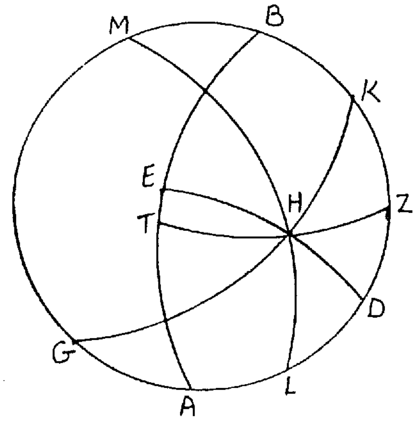
<Let> *ABGD* <be> the latitude circle centered at *E*, *AEG* the ecliptic with *H* and *T* as its poles and centered at *E*. We assume the point *B* as the star having a <non-zero> latitude. *DZB* is the circle passing the star and cutting the ecliptic at *Z*, where *Z* is a degree on the ecliptic, or the position of the star without a (i.e., with zero) latitude. The arc *BZ* is desired. *GB* is the latitude of the star, and *GZ* is <the distance> between the <ecliptical> degree of the star and the <ecliptical> degree from which the distance of the star is desired. In the triangle *ZGB*, *G* is a right angle. So the ratio of the Cosine of *GZ* to that of *ZB* is equal to the ratio of the greatest Sine to the Cosine of *GB*. So *BZ* is known. This is what we wanted to demonstrate.



Chapter 6: On the distance between two stars <both> having <non-zero> latitudes.

<Let> *ABG* <be> the latitude circle, *AEB* the ecliptic and *Z* its pole. First, we assume the two stars <to be> the points *G* and *H* <with latitudes> in opposite directions. We draw <the arcs> *GHK*, *ZHT*, and *EHD* (*E* is the pole of the circle *ABG*). The required arc is <the arc> *GH* which passes through the two stars. In the triangle *HET*, *T* is a right angle. So the ratio of the Cosine of *TE*, the complement of <the distance> between the two stars <measured> in terms of the ecliptical degrees, to the Cosine of *EH*, is equal to the ratio of the greatest Sine to the Cosine of *HT*, the latitude of one of the two stars. Thus *EH* is known. In the two triangles *EHT* and *EDA*, the angle *E* is common, and *T* and *A* are right angles. So the ratio of the Sine of *EH*, which is known, to that of *HT*, the latitude of the first star, is equal to the ratio of the Sine of *ED*, the greatest Sine, to that of *DA*. So *DA* is known. *AG* is the latitude of the other star. So the sum *GD* is known. In the triangle *GHD*, *D* is a right angle. So the ratio of the

Cosine of *GD*, which is known, to the Cosine of *HG*, which is desired, is equal to the ratio of the greatest Sine to the Cosine of *HD*, which is already known. So *HG* is known.

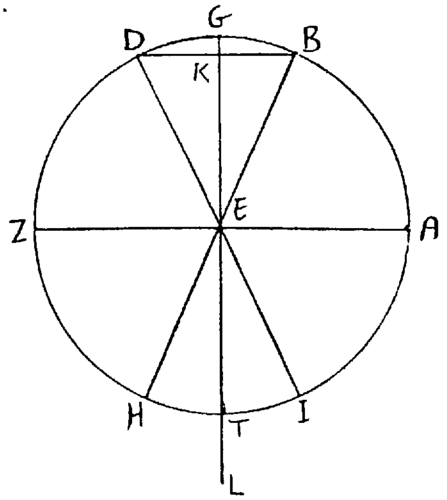


Again, we assume the two stars <to be> the points *H* and *L* <with latitudes> in the same direction. We draw *LHM*. Based on what has already been explained above, *EH*, *HD*, and *DA* are known. *AL* is the latitude of the other star. So *LD* is known. In the triangle *LHD*, *D* is a right angle. So the ratio of the Cosine of *LD*, which is known, to the Cosine of *LH*, which is desired, is equal to the ratio the greatest Sine to the Cosine of *DH*, which is known. So *LH* is known. It is <also> known that if *DA* is less than *AL*, and this is <possible> if the point *D* is situated between the points *A* and *L*, then <*DA*> must be subtracted from *AL*. Then the remainder *DL* is known. In the triangle *DHL*, *D* is a right angle, so *LH* is known. This is what we wanted.

Chapter 7: On the extraction of the meridian line.

Let *AGZ* be the horizon semicircle, and *ATZ* the semicircle of the celestial equator coinciding with the <plane of the> horizon circle or perpendicular to it; and let their <common> center <be> *E*. We suppose the sun rotating on it (i.e., the equator) or parallel to it in its universal rotation. *T* is the midheaven. The arcs *TH* and *TI* are equal. A gnomon should be mounted evenly and vertical at *E*. When the sun is at the point *H*, the shadow of the gnomon is *EB*. When it is at the point *I*, its shadow is *ED*. We join *BD*, and we bisect it at *K*. We draw *KE* and extend it to *L*. Then the line *KL* is the meridian line. This is because the shadow of the gnomon is always opposite to the body of the sun. So *ED* is in the direction of *EI*, and *EB* in the direction of *EH*. So the arc *BD* is equal to the arc *HI*. When we bisect it at *G* and join *GEL*, <it> will be the intersection of the plane of the meridian circle and the plane of the horizon circle. This is what we wanted to demonstrate.

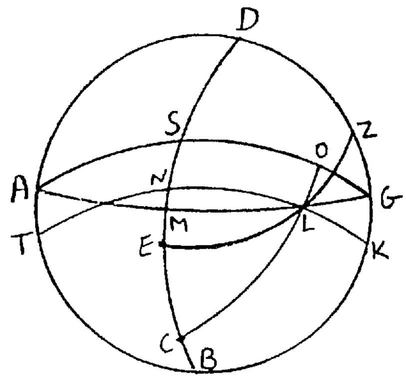
Another method <for the case> when the azimuth of the sun is known: Let *AGZT* be the horizon circle, *A* the equinoctial rising point <of the sun>, and *Z* its <equinoctial> setting point. If the azimuth of the sun is along the point *I*, the shadow of the gnomon will be *ED*. *AI* is the known azimuth, and it is equal to *ZD*. Then *ZD* is known, and so is its complement *DG*. If we draw the line *GET*, <it> will be the intersection of the plane of the meridian circle and the plane of the horizon circle. This is what we wanted to do.



Chapter 8: On the deviation of <the directions of> the <other> localities from the meridian of our locality.

<Let> *ABGD* <be> the horizon circle, *E* the zenith, *A* and *G* the equinoctial rising position and setting position, *BED* the meridian, *C* the pole of the celestial equator, *ASG* the celestial equator, and *ES* the latitude of our locality (then *E* is the zenith). We assume *SO* <to be the difference> between the longitude of our locality and that of Mecca. We draw *CO* as a quadrant of the meridian of Mecca. We take *C* as the pole, and we draw a circle with its distance (i.e., radius) <equal to> the chord of the colatitude of Mecca, and parallel to the celestial equator. It is <marked as> *TNK*. It intersects the arc *CO* at *L*. Then *L* is the zenith of the inhabitants of Mecca. The two arcs *SN* and *OL* are <equal to> the latitude of Mecca. So *EN* is <the difference> between the two latitudes (i.e., that of our locality and of Mecca). We draw *ELZ* <as a part> of the circle passing through <our> zenith. It is the circle of the distance between the two localities, because *EL* is the distance <on the sphere> between our locality and Mecca. Then the arc *ZD* is the deviation of <the direction of> Mecca <from the meridian of our locality>. We draw the semicircle passing through the equinoctial rising position and the point *L*. It is <marked by> *AMLG*. In the two triangles *CLM* and *COS*, the angle *C* is common, and *M* and *S* are right angles. <So> the ratio of the Sine of *CL*, the colatitude of Mecca, to the Sine of *ML*, <which is called> 'the

equation of longitude', is equal to the ratio of the Sine of *CO*, the greatest Sine, to the Sine of *OS*, <the difference> between the two longitudes. So *LM* and its complement *LG* are known. In the two triangles *GLO* and *GMS*, the angle *G* is common, and *O* and *S* are right angles. So the ratio of the Sine of the known *GL*, to that of *LO*, the latitude of Mecca, is equal to the ratio of the Sine of *GM*, the greatest Sine, to the Sine of *MS*, <which is called> 'the equation of the latitude'. Then *MS* is known, and *SE* is the latitude of our locality. Then, *ME* is known, and it is <called> 'the adjusted latitude of the locality'. Then *MD* is known. In the two triangles *GLZ* and *GMD*, the angle *G* is common, and *Z* and *D* are right angles.



So the ratio of the Sine of the known *GL* to that of *LZ*, the complement of the distance between the two localities, is equal to the Sine of *GM*, the greatest Sine, to the Sine of the known *MD*. Then *LZ* is known, and it is the complement of the distance between the two localities. So its complement *LE* is known, and it is the distance <between the two localities>. In the two triangles *ELM* and *EZD*, the angle *E* is common, and *M* and *D* are right angles. So the ratio of the Sine of *EL*, the distance between the two localities, to the Sine of the known *LM*, is equal to the ratio of the Sine of *EZ*, which is a quadrant, to the Sine of *ZD*, the deviation of Mecca. Then, the deviation of <the direction of> Mecca from the meridian of our locality is known. This is what we wanted to demonstrate. Based on this figure, it is possible that while the two latitudes are equal, <the direction of> Mecca be not along the east-west line. It is because <the direction of> the zenith of the inhabitants of Mecca will be situated on the side inclined towards the <north> pole (i.e., *C*) from the circle passing through the equinoctial rising position and our zenith. That will be demonstrated if we draw a parallel circle with the same pole as the celestial equator and with its distance (i.e., radius) equal to the chord of the colatitude of our locality.

Having kept our promise at the beginning of the Book on Chapters and Proofs, we finish the section in this chapter, the book in this section, and the treatise in this book. Praise be to Allāh alone, and that is enough, and His blessing be on the best of His creatures, Muḥammad the Chosen <prophet by Allah>!
[The copying was finished on 18 Ramażān of the year 545 of Hejira, by the hands of Maḥmūd b. Aḥmad b. al-Ḥusayn al-Muʿallimī.]

Commentary

IV.8.1 The first proof corresponds to the second method in I.8.1. Cf. the second proof in IV.5.6. The second proof corresponds to the first method in I.8.1. Cf. the first proof in IV.5.6. Both methods are applicable for the solstices and for any other day for which the length of the day and the declination of the sun is known. See also the commentary of I.8.1.

IV.8.2 This is a proof of the method provided in I.8.2 for the calculation of the altitude for zero azimuth. In the figure, *E* is the pole of the celestial equator.

IV.8.3 The first proof is for the second method provided in I.8.3, and the second proof is for the first method thereof. See I.8.3 and its commentary. In the first proof, Kūshyār applies the archaic "analemma" construction which was known in classical times and the Middle Ages. In this case, the plane of the horizon is the plane of the paper. Points may be projected perpendicularly on the horizon plane. For performing any operation upon an arc or line in a different and nonparallel plane, it is rotated into the plane of projection. Then arc or line will appear in its true shape and size. For more details about the "analemma" construction see [Neugebauer 1969, 214-20; Id 1969, 669-72]. The terms 'argument of azimuth' and 'equation of azimuth' used by Kūshyār are rather strange. Al-Battānī who uses chords instead of sines, applies the term *watar ikhtilāf al-ufuq* (i.e., "chord of the horizon difference") in his description of a similar method [al-Battānī 1989-1907, III, 33-34]. For a discussion of the terms *ḥiṣṣa* ("argument") and *ta'dīl* ("equation") as used by al-Sijzī, Abu'l-Wafā and Ḥabash al-Ḥāsib see [Kennedy-Kunitzsch-Lorch 1999, 101].

IV.8.4 In the premise, point *Z* is the pole of the great circle through *M* and *N* since *M* and *N* are the poles of the circles *TL* and *BD*, which intersect at *Z*. In the proof itself, arc *LT* is 90 degrees since point *L* is the pole of *DE* (because arc *LD* is assumed to be 90 degrees and arc *LE* is 90 degrees since *E* is the zenith) The proof of the validity of the methods provided in I.8.4 for finding the altitude of the sun when its northern or southern azimuth is known are presented here. For a modern formulation of these methods see the commentary of I.8.4. In the figure for the northern azimuth, *IT* is the first arc, *TD* is the second arc or the complement of the argument of the altitude, and $(90^\circ - TK)$ is the third arc or the equation of altitude, mentioned in I.8.4. In the figure for the southern azimuth, *IT* is the first arc, *ZT* is the second arc or the complement of the argument of the altitude, and *HM* is the third arc or the equation of the altitude. In the figure for the northern azimuth, *TD* is

greater than *TK*; so, the argument of the altitude is smaller than the equation of the altitude. Therefore, we should subtract the argument of the altitude from the equation of the altitude to obtain a positive value for the altitude. We are unaware of other occurrences of this proof in the medieval Arabic astronomical literature.
In a short fragment following the proof for the northern azimuth, only found in the manuscript A, we read:

و لايظن ان نقطة **ك** يقع بين نقطتى **ه ط** لان الشمس هما (لما؟) زاد ارتفاعها فرعه قل سمت **ب د**
و قرب نقطة **ط** من نقطة **ه** الى ان يجوز الارتفاع بنقطة **ب** فيصير قوس **د ط** تسعين

("It is not be expected that the point *K* lies between the two points *E* and *T*, because when the altitude of the sun increases, it leads to decrease of the azimuth *BD*, and to the approach of the point *T* towards the point *E*, until the altitude <circle> passes through the point *B*, and the arc *DT* becomes <equal to> 90°").

IV.8.5 See I.8.5 and its commentary. In the figure, *E* is the pole of the celestial sphere and the vertical projection of the ecliptic passes through its projection.

IV.8.6 See I.8.6 and its commentary. The arcs *DH* and *DA* are the first arc and second arc mentioned in I.8.6, respectively. The arcs *GD* and *LD* are the "third arc" for the cases in which the latitudes are in different or in the same directions, respectively.

IV.8.7 Here Kūshyār first shows the validity of the method of the Indian circle for finding the meridian line, but not the validity of the shortest shadow method provided in I.8.7. Then he demonstrates the validity of the second method in I.8.7, based on the known azimuth of the sun. This is of course trivial. Here Kūshyār applies the "analemma" method again (see the commentary to IV.8.3). The equinoctial rising/setting points are East/West points.

IV.8.8 A proof for the method of finding the direction of Mecca, essentially al-Bīrūnī's 'method of the *zīj*es', provided in I.8.8, is presented here. The distance between two localities is described as the arc of the terrestrial great circle passing through them. This is equal to the arc joining the zenith points of the two localities on the celestial sphere. It is supposed that the longitude difference and the two latitudes are known. In the figure, *ML*, *MS* and *ME* are the arcs of the "equation of longitude", the "equation of latitude", and the "adjusted latitude of the locality", respectively. The figure is drawn for a locality North-East of Mecca, of course, since Kūshyār lived in Iran. See also I.8.8 and its commentary. It is interesting that Kūshyār shows here that in a locality having the same latitude as Mecca, the direction of Mecca is not necessarily on the east-west line. For an account of different texts that use the terms "equation [or correction] of latitude" (*taʿdīl al-ʿarż*) and "adjusted [or corrected] latitude" (*al-ʿarż al-muʿaddal*) in the calculation of the direction of Mecca, see [Berggren 1985]. Abū Jaʿfar al-Khāzin stated in his *Zīj al-Ṣafā'iḥ* that if Mecca is at the same latitude as your locality, the *qibla* is towards the East or the West. This is why Kūshyār may have discussed the error at the end of this chapter [Abū Naṣr ibn ʿIrāq 1948, 34].

بسم اللّه الرحمن الرحيم[١]

و بك الاعانة يا كريم قال كوشياربن لبّان بن باشهري[٢] الجيلي انّي لمّا تصفّحت[٣] الزيجات المؤلّفة في صناعة التنجيم و تأمّلتها فكان في بعضها فساد[٤] يحتاج الى اصلاح و في بعضها تطويل و تبعيد يحتاج الى تقريب و في بعضها نقصان يحتاج الى اتمام و ما خلا المجسطي منها و كلّها حساب[٥] غفل لا يرجع الى بيان شاف و لايستند الى برهان كاف اردت[٦] ان اعمل زيجاً يجمع علماً و عملاً اصلح فيه الفاسد و اقرب البعيد و اتمم الناقص[٧] و اكشف عن معني كل لفظ فاشرحه و ابرهن علي كل حساب فيه فاقيده[٨] فما وجد[٩] من التفاوت بين هذا و غيره في ايّ شيء وجد فهو امّا لفاسد[١٠] اصلح و امّا لبعيد قرب[١١] و امّا لناقص تمّم[١٢] و اقدم العمل على العلم لسهولة وصول المبتدئ اليه و سرعة فايدته[١٣] له و اجعله اربع مقالات الاولى منها في حساب الابواب و[١٤] الثانية في جداولها و[١٥] الثالثة في الشرح والهيئة و[١٦] الرابعة في البرهان على صحّة حساب الابواب و لمّا صحّ عزمي على ذلك و تأكّدت نيّتي فيه سألت اللّه التوفيق والهداية[١٧]

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[١] The opening phrase in praise of Allāh and the Prophet Muḥammad, up to قال كوشيار, is not the same in different mss.; F is illegible here, so we quote from C.

[٢] C باشهريار instead of باشهري

[٣] C تصحّفت instead of تصفّحت

[٤] F فسادا instead of فساد

[٥] C om. حساب

[٦] F illegible from here to اصلح فيه

[٧] F illegible from here to معني

[٨] C فانقذه instead of فاقيده

[٩] C وجدت instead of وجد

[١٠] C الفاسد instead of لفاسد

[١١] C البعيد اقرب instead of لبعيد قرب

[١٢] C الناقص اتمم instead of لناقص تمّم

[١٣] C فايدة instead of فايدته

[١٤] C om. و

[١٥] C om. و

[١٦] C om. و

[١٧] C والرشد والهداية والعصمة والكفاية انه هوالمعين instead of والهداية

المقالة الاولى في حساب الابواب: ثمانية فصول و خمسة و ثمانون بابا

الفصل الاول في التواريخ: ستة ابواب

ا في ذكر مبادئ تواريخ قديمة و ما بين كل اثنين[18] منها من السنين والايام

ب في ذكر التواريخ الثلثة[19] المستعملة في زماننا

ج في نقل سني هذه التواريخ الىالايام[20] والايام الى سنيها بالحساب والجدول[21]

د في استخراج هذه التواريخ[22] بعضها من بعض

ه في مدخل هذه التواريخ في ايام الاسبوع

و في[23] الاعياد والتوقيعات التي في هذه التواريخ

الفصل الثاني في الجيوب والاوتار: ستة ابواب

ا في مقدمة لمعرفة الجيب

ب في تعديل ما بين سطري الجيب و ساير الجداول

ج في جيب القوس و قوس الجيب من الجدول[24]

د في سهم القوس و قوس السهم من جدوله و جدول الجيب

ه في وتر القوس و قوس الوتر من جدول الجيب

و في تصحيح الجيب اذا شككنا في شيء منه[25]

الفصل الثالث في الاظلال: ثلثة[26] ابواب

ا في حساب الظل الاول والثاني و قطريهما و قوسيهما

ب في ظل القوس و قوس الظل من الجدول

ج في نقل الاظلال الى مقاييس مختلفة

---

[18] C تاريخ instead of اثنين

[19] C الثالثة instead of الثلثة

[20] F الىالايام illegible

[21] C الجداول instead of الجدول

[22] F هذه التواريخ illegible

[23] F و في illegible

[24] C الجداول instead of الجدول

[25] C the last two sections are interchanged

[26] C ثلاثة instead of ثلثة

الفصل الرابع في تقويم الكواكب واحوالها: اثنا عشر بابا

ا في ذكر اصول و مقدمات لاوساط الكواكب

ب في استخراج الاوساط من جداولها

ج في نقل الاوساط من طول الى طول

د في مواضع الاوجات والجوزهرات و حركاتها

ه في تعديل الايام بلياليها

و في تقويم الشمس

ز في تقويم القمر و جوزهره

ح في تقويم الكواكب الخمسة

ط في عرض القمر

ى في عرض الكواكب الخمسة

يا في رجوع الكواكب و استقامتها[٢٧] و رؤيتها و خفائها

يب في صعود الكواكب و هبوطها[٢٨] في افلاكها

الفصل الخامس في اعمال طوالع النهار والليل: اثنان[٢٩] و عشرون بابا[٣٠]

ا في الميل الاول

ب في مطالع البروج بخط الاستواء

ج في الميل الثاني

د في بعد الكواكب عن معدل النهار

ه في عرض البلد

و في سعة مشرق[٣١] الشمس والكوكب[٣٢]

ز في تعديل نهار الشمس والكوكب[٣٣]

ح في مطالع البلد

ط في غاية ارتفاع الشمس والكوكب

---

٢٧ C استقاماتها instead of استقامتها

٢٨ C om. و هبوطها

٢٩ C اثنا instead of اثنان

٣٠ F باباillegible

٣١ C المشرق instead of مشرق

٣٢ C الكواكب instead of الكوكب

٣٣ C للشمس والكواكب instead of الشمس والكوكب

الفصل السادس في الكسوفات و ما يليق بها: عشرون بابا

---

[٣٤] C من المطالع instead of من الطالع

[٣٥] C فصل instead of اصل

[٣٦] C and F add والليل in the text but not in this list of contents

[٣٧] C ساعاتهما instead of ساعتهما

[٣٨] F om. from here to the middle of I.2.2; titles of the sections are taken from the sections in the text

[٣٩] C add. و يقال في ارتفاع اى درجة معلومة

يا في اختلاف منظر النيرين في[41] دايرة الارتفاع
يب في الزوايا الست التي يحتاج اليها في[42] الكسوفات الشمسية
يج في اختلاف منظر القمر طولاً و عرضاً[43] من هذه الزوايا
يد في اصابع كسوف الشمس مطلقه و معدله
يه في ازمان الكسوف[44] مطلقه و معدله
يو في تصوير الكسوف[45]
يز في ارتفاع القمر بحسب عرضه
يح في اختلاف منظر القمر طولاً و عرضاً بطريقة مبرهنة
يط في استخراج طول البلدان
ك في رؤية الهلال والكواكب من جهة قسي محدودة لها

الفصل السابع في اعمال[46] تتعلق بالاحكام: ستة ابواب
ا في ساعات بعد درجة الكوكب[47] من الاوتاد
ب في مطرح الشعاع بدرجة السواء
ج في مطرح الشعاع بدرجة المطالع
د في[48] التسييرات
ه في تحاويل[49] السنين و طوالعها
و في نقل طالع سنة العالم من بلد الى بلد

الفصل الثامن في اعمال يقل الاحتياج اليها: عشرة ابواب
ا في عرض البلد من ساعات النهار الاطول[50]

---

[40] C add. بين
[41] C من instead of في
[42] C من instead of في
[43] C add. بالتقريب
[44] C الخسوف instead of الكسوف
[45] C الخسوف instead of الكسوف
[46] C ابواب instead of اعمال
[47] C الكواكب instead of الكوكب
[48] C add. معرفة
[49] C تحويل instead of تحاويل
[50] C add. والاقصر

ب في الارتفاع الذي لا سمت له

ج في سمت اى ارتفاع نفرض[51]

د في الارتفاع من السمت

ه في البعد بين الكوكبين[52] لاحدهما عرض

و في البعد بين كوكبين ذوى عرض

ز في استخراج خط نصف النهار

ح في انحراف البلدان المعلومة الطول والعرض[53] عن نصف نهار[54] بلدنا

ط في ذكر[55] الكواكب الثابته و علامات بعضها[56] لتعرف بالعيان

ى في اسماء منازل القمر و ايام[57] طلوعها

[٥٨]فهذه ابواب هذه المقالة قدّمت الاهم فالاهم والاكثر احتياجاً اليه فـالاكثر واللّـه الموفـق للصواب و اليه المرجع والمآب

---

[51] C السمت من الارتفاع instead of سمت اى ارتفاع نفرض

[52] C كوكبين instead of الكوكبين

[53] C om. المعلومة الطول والعرض

[54] C نصف هار instead of نصف النهار

[55] C add. بعض

[56] C و علاماها instead of و علامات بعضها

[57] C ازمان instead of ايام

[58] The rest of the missing part in F up to the middle of I.2.2 is supplied here from C

الفصل الاول في التواريخ ستة ابواب

الباب الاول في ذكر مبادي تواريخ قديمة و ما بين كل اثنين منها من السنين والايام

التواريخ المشهورة المحفوظة عند القدماء تاريخ الطوفان و تاريخ بختنصر[١] و تاريخ فيلبس و تاريخ ذىالقرنين و تاريخ اغسطس و تاريخ دقلطيانوس و تاريخ الهجرة و تاريخ يزدجرد

الطوفان فتاريخ الطوفان تستعمله اصحاب الزيجات القديمة مثل السندهند والشاه و اوله يوم الجمعة قريب من ظهور الماء في ايام نوح عليه السلام الشمس عند طلوعها في ذلك اليوم كانت في الحمل والقمر معها مجتمعان في اول الحمل و ساير الكواكب حول اول الحمل و الى هذا التاريخ تنسب ساير التواريخ التي بعده

بختنصر[٢] و هو بختنصرالاول من ملوك بابل و اول يوم من تاريخه يوم الاربعاء و على هذا التاريخ وضع بطلميوس اوساط الكواكب في المجسطي و وضع مواضع الكواكب الثابتة لاول سنة ثمان مائة و ست و ثمانين منه[٣] و هو اول يوم من ملك انطينس و بين يوم الجمعة اول يوم من الطوفان و يوم الاربعاء اول يوم من هذا التاريخ ٨٦٠١٧٢ يوماً تكون من السنين الفارسية المصرية التي عدد ايامها ثلثمائة و خمسة وستين يوماً الفي و ثلثمائة و ستة و خمسين سنة ومائتي و اثنين و ثلثين يوماً تامة

فيلبس هو فيلبس المعروف بالبناء و هو والد ذىالقرنين و هو ملك من ملوك اتون و هو بعد ممات الاسكندرالماقدوني و علي تاريخه وضع ثاون[٤] الاسكندراني زيجه الملقب بالقانون و اول يوم من تاريخه يوم الاحد بينه و بين تاريخ الطوفان ١٠١٤٨٣٤ يوماً تكون هذه الايام الفى و سبعمائة و ثمانون سنة و مائة و اربعة و ثلاثون يوماً

ذوالقرنين هوالاسكندرالثاني المعروف بذي القرنين و اول يوم من تاريخه يوم الاثنين اول السنة السابعة من ملكه حين خرج من بلاد مقدونية فسار في الارض و بلغ من معمورها ما بلغ و بين يوم الاثنين من هذا التاريخ وبين تاريخ الطوفان ١٠١٩٢٧٣ يوماً تكون هذه الايام الفي و سبعمائة و اثنين و تسعين سنة و مائة و ثلثة و تسعين يوماً تامة

اغسطس هو ملك من ملوك الروم و في بعض سنيه ولد المسيح و اول يوم من تاريخه يوم الخميس بينه و بين تاريخ الطوفان من الايام ١١٢٢٣١٦ و من السنين ثلاثة آلاف و اربعة و سبعين سنة و ثلثمائة و ستة ايام

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[١] C بختصر instead of بختنصر

[٢] C بختصر instead of بختنصر

[٣] C سنة; B, L, and Y منه

[٤] C has an abundant word ثيون here, being an alternative Arabic form of Theon's name, as found in P

دقلطيانوس هو ملك من ملوك النصرانية و اول يوم من تاريخه يوم الاربعاء بينه و بين تاريخ الطوفان <من الايام> ١٢٣٦٦٣٩ و من السنين ثلاثة آلاف و ثلثمائة و ثمان و ثمانين سنة و تسعة عشر يوماً تامة

الهجرة هو هجرة النبي صلى اللّه عليه و سلم من مكة الى المدينة و كان دخوله اياها يوم الاثنين الثامن من شهر ربيع الاول والتاريخ مأخوذ من اول السنة و هو يوم الخميس اول يوم من المحرم فاذا بينه و بين ذلك سبعة و ستون يوماً فالسنة ثلثمائة و اربعة و خمسون يوماً و خمس وسدس فاذا صارت هذه الكسور اكثر من نصف يوم زيد في ايام ذي الحجة يوم واحد فتصير ايامه ثلثين يوماً و ايام تلك السنة ثلثمائة و خمسة و خمسون يوماً و ذلك في حساب كل ثلاثين سنة احدى عشر مرة لان الاحدى عشر خمس و سدس الثلاثين و بينه و بين تاريخ الطوفان من الايام ١٣٥٩٩٧٣ و من السنين ثلاثة آلاف و سبعمائة و خمس و عشرين سنة و ثلثمائة و ثمانية و اربعين يوماً و معرفة الكبيسة منها هي ان تلقي السنين مع السنة التي تريد ثلثين ثلثين و ما بقي تضربه في احد عشر و تلقيه ثلثين ثلثين فان كان الباقي اكثر من خمسة عشر فتلك السنة كبيسة و ان كان اقل فلا

يزدجرد هو يزدجرد بن شهريار بن[٥] كسرى آخر ملوك الفرس و اول يوم من السنة التي ملك فيها يوم الثلاث بينه و بين تاريخ الطوفان من الايام ١٣٦٣٥٩٧ و من السنين ثلاثة آلاف و سبعمائة و خمسة و ثلاثين سنة و ثلثمائة و اثنى و عشرين يوماً

فاذا اردنا ما بين كل تاريخين انقصنا سنى الاقرب الى الطوفان او ايامه من سنى الابعد منه او ايامه فما بقي فهو ما بينهما من السنين او الايام

الباب الثاني في ذكر التواريخ <الثلثة> المستعملة في زماننا

التواريخ المستعملة عندنا و في زماننا فهو تاريخ ذى القرنين و هو الرومي والسرياني لانه لاخلاف بينهما الا في اسامي الشهور و ان اول شهور السنة عند الروم كانون الثاني[٦] باسم رومي <ثم>[٧] على ترتيبها و تاريخ الهجرة و هو التاريخ العربي و تاريخ يزدجرد و هو التاريخ الفارسي و اما السرياني فاوله يوم الاثنين على ما تقدم ذكره و اسماء شهوره بالسريانيه و عدد[٨] ايامها مجملاً و مفصلاً على ما اقول تشرين الاول احد و ثلاثون يوماً **لا** تشرين الثاني ثلاثون يوماً **سا** كانون الاول احد و ثلاثون يوماً **صب** كانون الثاني احد و ثلاثون يوماً **فكج** شباط ثمانية و عشرون

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[٥] C ابن instead of بن

[٦] C تشرين الاول instead of كانون الثاني found in B, L, P, and Y

[٧] ثم added from B and Y

[٨] C تذكر instead of عدد found in B, L, P, and Y

يوماً و ربع يوم **قنا** آذار احد و ثلاثون يوماً **ققب** نيسان ثلاثون يوماً **ريــب** ايار احد و ثلاثون يوماً **رمج** حزيران ثلاثون يوماً **رعج** تموز احد و ثلاثون يوماً **شد** آب احد و ثلاثون يوماً **شله** ايلــول ثلاثون يوماً **شسه** فالسنة ثلثمائة و خمسة و ستون يوماً و ربع يوم فاذا صار الربع اكثر من نصف يوم زيد في ايام شباط يوم واحد فتصير ايامه تسعة و عشرين و ايام تلك السنة ثلثمائـــة و ســـتة و ستون و هي السنة الكبيسة و معرفتها ان تلقى السنين مع السنة التي تريد اربعة اربعة فان بقيـــت ثلاثة فتلك السنة كبيسة و ان بقي اقل فلا

و اما العربي فاوله يوم الخميس اول يوم من السنة التي هاجر فيها النبي صلى الله عليه و ســلم و هو الخامس عشر من تموز سنة ثلاث و ثلاثين و تسعمائة لذى القرنين و اسماء شـــهوره و عـــدد ايامها مجملاً و مفصلاً على ما اقول المحرم **ل** صفر **كط نط** ربيع الاول **ل فط** ربيع الاخر **كط قيح** جمادي الاول **ل قمح** جمادي الاخر **كط قعز** رجب **ل رز** شعبان **كط رلو** رمضان **ل رسو** شوال **كط رصه** ذي القعده **ل شكه** ذي الحجه **كط** و خمس و سدس يوم **شند** <**كب**>[9] فالسنة ثلثمائة و اربعة و خمسون يوماً و خمس و سدس يوم فاذا صارت هذه الكسور اكثر من نصف يوم فكمـــا تقـــدم [و] حسابه <و> استخرجت ايام هذه الشهور بان تنقص وسط مسير يوم الشمس من وسط مسير يـــوم القمر و قسم الدور علىالباقي يحصل تسعة عشرون يوماً و احدى و ثلاثون دقيقه و خمسون ثانيه بالتقريب فوضع شهر ثلثين يوماً وشهر تسعة و عشرين يوما و جمعنا الكسور الفاضله اى الزايـــدة على نصف يوم[10] في اخر السنة فاجتمع منها خمس و سدس يوم

و اما الفارسي فاوله يوم الثلثاء اول يوم من السنة التي ملك يزدجرد بن شهريار فيها و هو الثـــاني والعشرون من ربيع الاول سنة احدى عشر للهجرة والسادس عشر من حزيـــران ســـنة ثلاثـــة و اربعين و تسعمائة لذى القرنين و اسماء شهوره و عدد ايامها مفصلاً و مجملاً علـــى مـــا اقـــول فروردينماه **ل ل** ارديبهشتماه **ل س** خرداذ[11] ماه **ل ص** <تيرماه ل>[12] **قك** مرداذمـــاه **ل** <**قـــن**>[13] شهرير[14] ماه **ل قف** مهرماه **ل رى** آبان ماه **له رمه** آذر[15] ماه **ل رعه** دى ماه **ل شه** بهمن مـــاه **ل شله** اسفندارمذ[16] ماه **ل شسه** فالسنة ثلثمائة و خمسة و ستون يوماً والخمسة الزايدة في[17] آخر ابان ماه تسمى المسترقه و لان السنة الفارسية تنقص عن الشمسية بربع يوم تقريباً صار في كل اربـــع

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[9] كب added from Y

[10] C بيوم instead of يوم found in B, L, and Y

[11] L and P substitute ذ for final د in the names of the months

[12] C تيرماه ل illegible

[13] C قن illegible

[14] Y شهريور instead of شهرير , which conforms to the modern Persian name of this month

[15] C ازر instead of اذر found in other mss.

[16] C اسفندار instead of اسفندارمذ found in other mss.

[17] C هىinstead of في

سنين يوم واحد و في كل مائة و عشرين سنة شهر واحد و كانت الفرس في ايام دولتهم يكبسون في كل مائة و عشرين سنة شهراً واحداً فيكون تلك السنة ثلثة عشر شهراً يعدون اول شهر من شهور السنة مرتين مرة في اول السنة و مرة في آخرها و يجعلون الخمسة الزائدة في ايام الشهر المكبوس و اول شهور السنة الشهر الذي تحل فيه الشمس الحمل فكانت الخمسة و اول السنة تنتقل في كل مائة و عشرين <سنة>[18] من شهر الى شهر و كان في ايام كسرى بن قباد انوشروان[19] تحل الشمس الحمل في آذر[20] ماه والخمسة الموضوعة في آخر آبان ماه و لما اتت عليه مائة و عشرون سنة كان اواخر ايام ملك الفرس و اضطراب دولتهم و استيلاء العرب عليهم فاهمل ذلك الرسم و بقيت الخمسة في آخر آبان ماه الى سنة خمس و سبعين و ثلثمائة ليزدجرد و حلت الشمس الحمل في اليوم الاول من فروردينماه فنقلت الخمسة بفارس و تلك الديار علي ما بلغنا الى آخر اسفندارمذماه علىالرسم القديم فاما في ديارنا التي هى الرى و جرجان و طبرستان فهى في آخر آبان ماه فانهم يظنون ان ذلك دين و سنة للمجوس لايجوز ان يبدل و يغير و لكل يوم من ايام الشهر اسم مخصوص يسمى به و هو هرمزد[21]، بهمن، ارديبهشت، شهرير، اسفندارمذ، خرداذ، مرداذ[22]، ديباذر، آذر[23]، آبان، خور، ماه، تير، كوش[24]، ديبمهر، مهر، سروش[25]، رشن، فروردين، بهرام، رام، باد، ديبدين، دين، ارد، اشتاد، اسمان، زامياد، مارسفند[26]، انيران، والخمسة المسترقة : اهنود، اشتود، اسفندمد[27]، وهخشتر، وهشتوشت[28]

الباب الثالث في نقل سني هذه التواريخ الىالايام والايام الى سنيها بالحساب والجدول

اما الحساب السرياني فتضرب السريانية بالسنة التامه في احد و عشرين الفاً و تسعمائة و خمسة عشر و تقسم المبلغ على ستين[29] فتحصل ايام تلك السنين فان فضل من القسمة شئ اكثر من ثلثين جبرناه يوماً و تضرب الايام التي تفرض في ستين و تقسم المبلغ على احد و عشرين الفاً و

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[18] C om. سنة found in B, L, and Y

[19] C انواشروانinstead of انوشروان found in B, P, and Y

[20] C ازر instead of اذر found in other mss.

[21] L, P, and Y هرمز instead of هرمزد

[22] Y امرداد, a more ancient form of the name alternatively used in modern Persian, instead of مرداد

[23] C ازر instead of اذر found in other mss.

[24] L, P, Y جوش instead of كوش

[25] C شروس instead of سروش found in other mss.

[26] P and Y مهراسفد and L مهراسبند instead of مارسفند

[27] L and Y اسبتمد instead of اسفندمد

[28] B, L, and P substitute ذ for final د in the names of the days

[29] C السنين السريانية instead of ستين found in other mss.

تسعمائة و خمسة عشر فيحصل سنو تلك الايام و ما فضل من القسمة قسمناه على ستين فتحصل الايام من السنة الناقصة

العربي تضرب السنين العربية التامة في احد و عشرين الفاً و مائتين و اثنين و ستين و تقسم المبلغ على ستين[٣٠] فيحصل ايام تلك السنين و تضرب الايام التي تفرض في ستين و تقسم المبلغ على احد و عشرين الفاً و مائتين و اثنين و ستين فيحصل سنو تلك الايام و ما فضل من القسمة قسمناه على ستين[٣١] فيحصل ايام من السنة الناقصة

الفارسي تضرب السنين الفارسية التامة في ثلثمائة و خمس و ستين فيصير ايام تلك السنين تامة و تقسم الايام التي تفرض على ثلثمائة و خمس و ستين فيحصل سنون تامة و مابقي فايام من السنة الناقصة

الجدول ان وضعنا جداول اثبتنا فيها السنين المجموعة والمبسوطة والشهور و بازائها ايامها مرفوعة ستين ستين فالاول منها هو الايام المطلقه والثاني منها مرفوع مرة اى مقسوم علىالستين مرة والثالث مرفوع مرتين اى مقسوم علىالستين مرتين والرابع مرفوع ثلث مرات[٣٢] فاذا اردنا ايام سنين مفروضة و شهور دخلنا بالسنين التامة في جدول السنين المجموعة [ثم الباقي في المبسوطة] و نأخذ الايام التي بازاء اقرب عدد اليها مما هو اقل منها فنثبتها[٣٣] و ندخل بالباقي من السنين في جدول السنين المبسوطة ونأخذ الايام التي بازائها و نزيدها على ما اثبتناها كل جنس على[٣٤] جنسه ثم نأخذ الايام التي بازاء الشهر التام و نزيدها على ما اجتمع من قبل فتحصل ايام السنين و الشهور المفروضة

و اذا اردنا سني ايام و شهورها دخلنا بالايام في ايام المجموعة و نأخذ السنين التي بازاء اقرب عدد اليها ما هو اقل منها فنثبتها و ننقص الايام الموجودة في الجدول من الايام التي معنا كل جنس من جنسه ثم ندخل بالباقي من الايام في ايام المبسوطة و نأخذ السنين التي بازاء اقرب عدد اليها مما هو اقل منها فنزيدها على السنين التي اثبتناها و ننقص الايام الموجودة في الجدول المبسوط من الايام التي معنا كل جنس من جنسه و ما بقيت من الايام اخذنا الشهور التي بازاء اقرب عدد اليها مما هو اقل منها و ما بقي من الايام بعد ذلك فهي ايام من الشهر الناقص

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[٣٠] C السنين instead of ستين found in other mss.

[٣١] C السنين instead of ستين found in other mss.

[٣٢] C مراتب instead of مرات

[٣٣] B adds على التحت

[٣٤] C الى instead of على

الباب الرابع في استخراج هذه التواريخ بعضها من بعض

اذا كان احد هذه التواريخ الثلاثة معلوماً و اردنا ان نعرف منه احد الباقيين جعلنا المعلوم اياماً الى اليوم الذي انت فيه و حفظناها ثم ان كان المعلوم اقدم من المجهول نقصنا من الايام المحفوظة ايام ما بين التاريخين[٣٥] و ان كان المجهول اقدم من المعلوم زدنا ايام مابين التاريخين على الايام المحفوظة فما بقي او بلغ[٣٦] فهو التاريخ المجهول اياماً فنجعلها سنين كما تقدم القول فيه والتاريخ السرياني اقدم من العربي بايام عددها ٣٤٠٧٠٠ و هو اقدم من الفارسي بايام عددها ٣٤٤٣٢٤ والعربي اقدم من الفارسي بايام عددها ٣٦٢٤ و يمتحن الحاصل من التاريخ بان يعرف مدخل اليوم المفروض من التاريخ المعلوم في ايام الاسبوع و مدخل اليوم المجهول فان اتفقا فصحيح و ان اختلفا بيوم او يومين الحقنا المجهول بالمعلوم

الباب الخامس في مدخل هذه التواريخ في ايام الاسبوع

السرياني نجعل تاريخه اياماً الى اليوم الذي نريد مع ذلك اليوم و نلقيها سبعة سبعة و ما بقي نعده من يوم الاثنين فاليوم الذي ينتهى اليه هو مدخل ذلك اليوم المفروض و ان شئنا القينا من السنين مع السنة التي نريد ثمانية و عشرين ثمانية و عشرين و ما بقي دخلنا به في جدول المدخل و نأخذ ما بازائه من مدخل اى [سنة نريدها ثم نزيد عليه مدخل اى] شهر نريده

العربي نجعل تاريخه اياماً كما تقدم في السرياني و نلقيها سبعة سبعة و ما بقي نعده من يوم الخميس فاليوم الذي ينتهي اليه العدد هو مدخل اليوم و ان شئنا القينا من السنين مع السنة التي نريد مائتين و عشرة مائتين و عشرة و ما بقي دخلنا به في جدول المدخل و نأخذ ما بازائه

الفارسي تلقي سنيه مع السنة التي تريد سبعة سبعة و ما بقي تعده من يوم الثلاثاء فاليوم الذي ينتهي اليه هو مدخل تلك السنة و تزيد عليه لكل شهر بعد فروردين ماه يومين يومين و لا من مدخل اى سنة نريده ثم نزيده على مدخل الشهر الذي نريد تزيد لمدخل آذرماه شيئاً لان مدخل آبان ماه و آذرماه في يوم واحد لوقوع المسترقة

[٣٥] C om. from here to المحفوظة , recovered from B, L, and Y

[٣٦] C بلغ او بقى instead of بقى او بلغ found in B

الباب السادس في الاعياد والتوقيعات التي في هذه التواريخ

السرياني

ماعلثا ان كان اليوم التاسع والعشرون من تشرين الاول يوم الاحد فهو ماعلثا والا فالاحد الذي بعده **السبار** ان كان اليوم الثامن والعشرون من تشرين الثاني يوم الاحد فهو السبار والا فالاحد الذي بعده **الميلاد** الليلة التي صبيحتها الخامس والعشرون من كانون الاول **الدنح**[37] السادس من كانون الثاني **صوم العذاري** هو **عيد الغيطاس** الاثنين الذي بعد الدنح **صوم نينوى** ثلثة ايام اولها الاثنين الذي قبل الصوم الكبير باثني و عشرين يوماً **عيد الهيكل** الثاني من شباط **الصوم الكبير** حسابه ان نأخذ سني ذي القرنين مع السنة التي نريد و نزيد عليها خمسة و نلقيها تسعة عشر تسعة عشر و ما بقي[38] ضربناه في تسعة عشر فان كان المبلغ اكثر من مأتي و خمسين نقصنا منه واحداً ابداً[39] و ان كان اقل لم ننقص منه شيئاً فما كان نلقيه ثلاثين ثلاثين و[40] ما بقي نظرنا و ان كان مثل ايام شباط او دونه فالصوم في ذلك اليوم من شباط ان كان يوم الاثنين و الا فالاثنين الذي بعده و ان كان اكثر من ايام شباط القينا منه ايام شباط و ما بقي فهو اول الصوم من آذار ان كان يوم الاثنين و الا فالاثنين الذي بعده[41] و قد وضعنا لذلك جدولاً و العمل به ان نأخذ سني ذي القرنين مع السنة التي نريد و نضعها في موضعين و نقسم احد الموضعين على ثمانية و عشرين و نزيد على موضع الآخر خمسة ابداً و نقسمه على تسعة عشر ثم ندخل بما بقي من القسمة على ثمانية و عشرين في طول الجدول و ما بقي من القسمة على تسعة عشر من عرض الجدول فموقع الالتقاء العددين هو اول الصوم فان كان بالسواد فهو من شباط و ان كان بالحمرة فهو من آذار[42] **وجه آخر** الاقرب الاثنين الى الاجتماع الكاين فيمابين اليوم الثاني[43] من شباط الىاليوم الثامن[44] من آذار[45] فان شككنا في الاثنين الاقرب فهو الذي يقع بين الشعانين والفطر استقبال[46] **الشعانين**[47] يوم الاحد الثاني والاربعون من الصوم **الفطر** يوم الاحد الذي بعد الشعانين **الشعانين الصغيرة**[48]

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[37] C الذبح instead of الدنح

[38] B add. ان كان تسعة عشر او دونه

[39] B om. ابداً

[40] B ان كان ثلاثين او دونه فان كان اقل من ايام شباط تلك السنة و كان يوم الاثنين فهو صوم instead of here up to ان كان يوم الاثنين

[41] This calculation method is found only in L and B.

[42] This method based on table 7 of Book II is found only in L.

[43] C الثامن instead of الثاني found in B, P, and Y

[44] C الثاني instead of الثامن found in B, P, and Y

[45] This alternative method is found in C, B, Y, and P. Y and P mention that there is also a calculation for this fast that accords with this method.

[46] The sentence regarding the doubtful case found in C, Y, and P is ambiguous, because the beginning of the Lent cannot be in its last week.

[47] B السعانين الكبير instead of الشعانين

[48] B, L, and Y الصغير instead of الصغيرة

الجمعة التي بعد الفطر **السلاق** يوم الخميس بعد الفطر باربعين يوماً **فنطيقسطي** يوم الاحد بعد السلاق بعشرة ايام **صوم السليحين** الاثنين الذي بعد فنطيقسطي **صوم مارت**[٤٩] **مريم** اول يوم من آب[٥٠] **ظهور المسيح** السادس من آب **فطرمريم** الخامس عشر من آب **عيدالصليب** الرابع عشر من ايلول و عند نسطور الثالث عشر من ايلول و عند الروم و يعقوب الرابع عشر منه **سقوط الجمار**[٥١] اليوم السابع والرابع عشر والحادي والعشرون من شباط **ايام العجوز** سبعة اولها السادس والعشرون من شباط **نيروزالمعتضد**[٥٢] الحادي عشر من حزيران **ايام الباحور** ثمانية اولها التاسع عشر من تموز و يستدل بما يكون في هذه الايام من اختلاف الهواء[٥٣] على ما في السنة من ذلك

العربي

العاشورا هو مقتل الحسين بن علي كرم اللّه وجهه و رضي عنه العاشر من محرم **مولد النبي** صلى اللّه تعالى عليه و سلم الثاني عشر من ربيع الاول **يوم الجمل** الخامس عشر من جمادي الاول **مبعث النبي** صلّى اللّه عليه و سلم السادس والعشرون من رجب **المعراج** ليلة السابع والعشرون من رجب **ليلة الصك** ليلة خامس عشر من شعبان **الصوم** ايام رمضان **فتح مكة** العشرون من رمضان **عيد الفطر** اول يوم من شوال **التروية** الثامن من ذي الحجة **عرفة** التاسع من ذي الحجة **عيد الاضحى** العاشر من ذىالحجة **غدير خم** الثامن عشر من ذى الحجة

الفارسي

النيروز اول يوم من فروردين ماه **نيروز الخاصة** السادس من فروردين ماه **المهرجان** السادس <عشر> من مهرماه **مهرجان الخاصة الصغير**[٥٤] الحادي و العشرون من مهر ماه **كاكيل**[٥٥] الخامس عشر من دى ماه **بهمنجنه** الثاني من بهمن ماه **السدق** ليلة العاشر من بهمن ماه **واذيره** الثاني والعشرون من بهمن ماه **كتب الرقاع** الخامس من اسفندارمذماه على ان المسترقة في آخر آبان ماه[٥٦]

الجاهنبارات الستة اولها **كو** من ارديبهشت ماه الثاني **كو** من تيرماه الثالث **يو** من شهريرماه الرابع **يه** من مهرماه الخامس **يا** من دى ماه السادس الخمسة المسترقة من اسفندارمذماه

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[٤٩] B, L, P, and Y om. مارت

[٥٠] B add. التجلي و هو

[٥١] P جمرات instead of جمار

[٥٢] L المعتضدي instead of المعتضد

[٥٣] C الهوى instead of الهواء

[٥٤] B, L, P, and Y om. الصغير

[٥٥] C تكاكيل instead of كاكيل found in B, L, P, and Y

[٥٦] C om. from here to the end of the section found in L and Y

الفصل الثاني في الجيوب والاوتار ستة ابواب

الباب الاول في مقدمة[1] لمعرفة الجيب

الجيب قانون يرجع اليه في وجود مقادير القسى كلها والجيب الاعظم و هو نصف قطر الدايرة اى جزء فرض جاز غير ان الاسهل و الاجمع للحساب ان تكون اجزاؤه من ستين و جيب تمام القوس هو جيب ما ينقص القوس من تسعين درجة كجيب تمام ستة و ثلثين يراد به جيب اربعة و خمسين و جيب تمام اربعة و خمسين يراد به جيب ستة و ثلثين و نكتفي بجيب اجزاء ربع الدايرة لان ما يجاوز الربع فجيبه مثل جيب اجزاء الربع راجعة من تسعين الى الواحد فجيب احد و تسعين كجيب تسعة و ثمانين و جيب اثنين و تسعين هو جيب ثمانية و ثمانين و على هذا الرسم حتى يفنى الجيب عند مائة و ثمانين ثم بعد ذلك ابتدا ثانياً على الرسم الاول الى ثلثمائة و ستين

و سهم القوس يبلغ مائة و عشرون درجة و هو قطر الدايرة و حيث ما قلنا في الحساب يضرب كذا في كذا منحطاً او يقسم كذا على كذا منحطاً فانا نعني به ان نحط ذلك العدد مرتبة فان كان درجاً اخذناه دقايق و ان كان دقايق اخذناه ثواني و على هذا الرسم

و من بعد ما تقدم ذلك فان جيب الدرجة الواحدة اما بالتحقيق فغير موجود و اما بالتقريب فقد استقصى في حسابه بحيث ليس بينه و بين تحقيقه فرق في شيء من الاعمال و هو على ما استخرجته بالاستقصا، **ا ب مط لح لا** وسنبين حسابه في باب البرهان

فاما جيب ما بعد الدرجة فالامر في حسابه قريب و يجب ان يتقدمه معرفة جيب تمام كل قوس معلومة الجيب و حسابه ان تنقص مربع الجيب المعلوم من مربع الجيب الاعظم و تأخذ جذر الباقي فيكون جيب تمام القوس المعلومة الجيب فعلى هذا الحساب يكون[2] جيب تمام الجزء الواحد و هو جيب **نط** جزء **نط نط كز و يب لط**

فاذا اردنا جيب اجزاء اخر ضربنا جيب جزء الذي قبله في جيب تمام الجزء الواحد منحطاً و نضرب جيب الجزء الواحد في جيب تمام الجزء الذي قبله منحطاً و نجمع المبلغين فيكون جيب الجزء الذي اردناه مثال ذلك انا نريد جيب **كد** فنضرب جيب **كج** في جيب تمام الجزء الواحد <منحطاً> ثم <نضرب جيب>[3] الجزء الواحد في جيب تمام **كج** منحطاً و نجمع المبلغين من كل واحد من الضربين فيكون جيب الجزء الذي اردنا و هو جيب **كد** و ليس انما

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[1] C مقدمات instead of مقدمة

[2] C كون instead of يكون

[3] Additions are found in L

يحصل جيب **كد** من الواحد و الثلثة والعشرون لكن من كل عددين مجموعها **كد** و اذا حسبنا حسابه كما قلنا في حساب الواحد و الثلثة و العشرون كالعشرة و الاربعة عشر < و الاثنى عشر و الاثنى عشر و الثمانية عشر>[٤] و الستة و ساير ما كان من ذلك

الباب الثاني في تعديل ما بين سطرى الجيب و ساير الجداول

الجداول كلها نسبة ما بين سطرى العدد منها الى ما بين سطرى الجدول منها كنسبة بعض ما بين سطرى العدد الى بعض ما بين سطرى الجدول فهذه اربعة اعداد متناسبة **ا** تفاضل ما بين سطرىالعدد **ب** تفاضل ما بين سطرى الجدول **ج** البعض من تفاضل سطرى العدد **د** البعض من تفاضل سطرىالجدول[٥] و المجهول المطلوب **ه**[٦] اما بعض ما بين سطرى الجدول و اما بعض ما بين سطرى العدد فان كان المطلوب بعض ما بين سطرى الجدول ضربنا **ج** البعض[٧] المعلوم مما[٨] بين سطرى العدد في **ب** تفاضل ما بين سطرى الجدول و قسمناه على **ا** تفاضل ما بين سطرى العدد و ان كان المطلوب بعض ما بين سطرى العدد ضربنا **د** البعض المعلوم مما بين سطرى الجدول في **ا** تفاضل ما بين سطرى العدد و قسمناه على **ب** تفاضل ما بين سطرى الجدول فيحصل المجهول المطلوب ثم ان وجب الزيادة على ما فى الجدول او العدد زدناه و ان وجب النقصان نقصناه[٩]

ا تفاضل سطرى العدد	**ب** <تفاضل> سطرى الجدول
ج بعض تفاضل سطرى العدد	**د** بعض تفاضل سطرى الجدول

[4] Missing words taken from L

[5] End of the missing fragment in F

[6] F om. the *abjad* notations found in C and Y for the quantities in this section

[7] C لبعض instead of البعض

[8] C فيما instead of مما

[9] C om. the following table

الباب الثالث في جيب القوس و قوس الجيب من الجدول

اذا اردنا جيب قوس مفروضة دخلنا بالقوس في سطر القوس و هو مقام سطر العدد و نأخذ ما بازائه من الجيب < و ان وجب نصحه> بما تقدم من تعديل ما بين السطرين و اذا اردنا قوس جيب مفروض دخلنا بالجيب في جدوله و اخذنا ما بازائه من القوس < وان وجب نصحه> بما تقدم من تعديل ما بين السطرين

الباب الرابع في سهم القوس و قوس السهم من جدوله و جدول الجيب

للسهم جدول موضوع يؤخذ منه سهم القوس و قوس السهم كما يؤخذ جيب القوس و قوس الجيب[10] من جدوله فان[11] اردنا سهم قوس من جدول الجيب نظرنا فان كانت القوس اقل من تسعين نقصناها من تسعين و اخذنا جيب الباقي و نقصناه من ستين و ان كانت القوس اكثر من تسعين نقصنا منها تسعين و اخذنا جيب الباقي و زدناه على ستين فان اردنا قوس سهم نظرنا فان كان السهم اقل من ستين نقصناه من ستين و اخذنا قوس الباقي و نقصناه من تسعين و ان كان السهم اكثر من ستين نقصنا منه ستين و اخذنا[12] قوس الباقي و زدناه على تسعين

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[10] F جيب القوس و جيب القوس instead of قوس الجيب و قوس الجيب found in C

[11] C فاذا instead of فان

[12] C نأخذ instead of اخذنا

الباب الخامس في وتر القوس و قوس الوتر من جدول الجيب

لسنا نحتاج في هذا الكتاب الى شيء من هذه الاوتار و انما ذكرناها لتمام الاعمال فاذا اردنا وتر قوس[١٣] نصفنا القوس و اخذنا جيبه و ضاعفناه و ان[١٤] اردنا قوس وتر نصفنا الوتر و اخذنا قوسه و ضاعفناه

الباب السادس في تصحيح الجيب اذا شككنا في شيء منه

جدول الجيب قد فرغ من حسابه و استقصى في صحته فلسنا نحتاج الى اعادة شيء منه و من حسابه الا انا اذا شككنا في جيب جزء من الاجزاء نظرنا فان كانت لتلك الاجزاء نصف صحيح اخذنا نصفه و ضربنا جيبه في جيب تمامه منحطاً و ضاعفنا المبلغ فيكون المبلغ المجتمع جيب الجزء المشكوك فيه مثاله انا اذا شككنا في جيب اربعة و عشرين ضربنا جيب اثنى عشر في جيب تمامه ثمانية و سبعين[١٥] منحطاً و ضاعفنا المبلغ فكان جيب اربعة و عشرين

و ان لم يكن لتلك الاجزاء نصف صحيح اخذنا قوسين مجموعهما مساوٍ لتلك الاجزاء ثم ضربنا جيب القوس الاصغر في جيب تمام القوس الاعظم منحطاً و ضربنا جيب القوس الاعظم في جيب تمام القوس الاصغر منحطاً و نجمع المبلغين فيكون جيب الجزء المشكوك فيه مثاله انا[١٦] شككنا في جيب خمسة و عشرين و كثير من القوسين يساوي خمسة و عشرين فلنأخذ منها عشرة و خمسة عشر ثم ضربنا جيب العشرة في جيب خمسة و سبعين منحطاً و ضربنا جيب الخمسة عشر في جيب ثمانين منحطاً و جمعنا المبلغين فكانا[١٧] جيب خمسة و عشرين و لو استخرجنا جيب اربعة و عشرين ايضاً على هذا الحساب و المثال لكان صواباً الا ان تلك الطريقة فى الاعداد الزوج اقرب

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[13] C القوس instead of قوس

[14] C اذا instead of ان

[15] F تسعين instead of سبعين

[16] C add. اذا

[17] C فكان instead of فكانا

الفصل الثالث في الاظلال[1] ثلثة ابواب

الباب الاول في حساب الظل الاول والثاني و قطريهما و قوسيهما

الظل الاول هو المأخوذ من المقاييس الموازية لسطح الافق و يقال له الظل المعكوس و هو الذي وضعناه في الجدول لحساب الابواب و الظل الثاني هو المأخوذ من المقاييس القائمة على سطح الافق و يقال له الظل المستوي و هو الذي وضعناه في الجدول لمعرفة الاصابع و الاقدام عند انتصاف النهار و يثبت في التقاويم

و المقياس[2] اى اجزاء[3] فرض جاز غير ان الاسهل في حساب الابواب ان تكون اجزاؤه ستين و لذلك وضعنا الظل الاول على ان يكون المقياس[4] ستون جزوًا و الظل الثانى على ان المقياس[5] اثنا عشر اصبعاً او سبعة اقدام و اذا كان اجزاء المقياس[6] اجزاء بعينها كان الظل الاول لكل قوس هو الظل الثاني لتمام تلك القوس و كل عدد فسواء[7] ضرب في ظل قوس[8] او قسم على ظل تمام القوس فان المبلغ من الضرب و الحاصل من القسمة شئ واحد

و قطرالظل هو الخط الواصل بين رأس المقياس و نهاية الظل و قوس الظل هو قوس الارتفاع الذي يزيد على[9] ظل الاشخاص و ينقص

[10] و من بعد ما تقدم ذلك فاذا اردنا الظل الاول لقوس قسمنا جيب القوس على جيب تمام القوس منحطاً فما حصل فهو الظل الاول على ان المقياس ستون جزءاً فان[11] اردنا قطره قسمنا الظل على جيب القوس منحطاً فما حصل فهو قطر الظل الاول و ان شئنا زدنا مربع الظل على مربع المقياس و اخذنا جذره و ان اردنا قوس الظل قسمنا الظل على قطره منحطاً

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[1] C الظلال instead of الاظلال

[2] C المقاييس instead of المقياس

[3] C جزء instead of اجزاء

[4] C ان المقاييس instead of ان تكون المقياس

[5] C المقاييس instead of المقياس

[6] C المقاييس instead of المقياس

[7] C سواء instead of فسواء

[8] C القوس instead of قوس

[9] Y به instead of على

[10] Fragment [4] is missing in F; it is found in C and Y.

[11] C فاذا instead of فان

فما حصل فهو جيب القوس[12] فان اردنا الظل الثاني لقوس قسمنا جيب تمام القوس على جيب القوس منحطاً فما حصل فهو الظل الثاني على ان المقياس ستون جزؤاً و ان اردنا قطره قسمنا الظل على جيب تمام القوس منحطاً فما حصل فهو قطر الظل الثاني و ان شئنا زدنا مربع الظل على مربع المقياس و اخذنا جذره فان[13] اردنا قوس الظل الثاني قسمنا الظل على قطره منحطاً فما حصل فهو جيب تمام القوس

الباب الثاني في ظل القوس و قوس الظل من الجدول

اذا اردنا ظل قوس[14] اخذنا ما بازا القوس من جداول[15] الظل كما تقدم في الجيب و ان [و ان] اردنا قوس ظل[16] اخذنا ما بازا الظل من القوس **فصل** انا وضعنا قوس الظل في الجدول الى خمسة و اربعين جزءاً[17] لان ما جاوز الخمسة و الاربعين يعظم فيه تفاضل ما بين السطرين فلا يصح العمل به الا بالقوه فاى عدد نريد ان نضربه في ظل قوس و القوس اكثر من خمسة و اربعين قسمنا العدد على ظل تمام القوس و اى عدد نريد ان نقسمه على ظل قوس و القوس اكثر من خمسة و اربعين ضربنا العدد في ظل تمام القوس و العدد هاهنا اما[18] جيب و اما ظل قوسه اقل من خمسة و اربعين فاما ضرب ظل قوس في ظل قوس و كلاهما اكثر من خمسة و اربعين او قسمة ظل قوس اكثر من خمسة و اربعين على عدد فلا و يقتصر عند ذلك على الجيب و ما يحصل[19] منه[20] من دون استعمال الظل

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[12] A marginal note in C mentions that this rule is valid both for the Tangent the Cotangent. Then, C adds the following fragment "from another manuscript" which is also found in Y:

فاذا اردنا الظل الثاني لقوس على ان المقياس اثني عشر اصبعاً او سبعة اقدام ضربنا جيب تمام القوس في اجزاء المقياس و قسمناه على جيب القوس فما حصل فهو الظل و ان اردنا قطره ضربنا الظل في اجزاء المقياس و قسمناه على جيب تمام القوس او زدنا مربع المقياس على مربع الظل و اخذنا جذره و ان اردنا قوسه ضربنا اجزاء المقياس في الظل و قسمناه على القطر فما حصل فهو جيب تمام القوس

[13] C و ان instead of فان

[14] C add. من جدول الظل

[15] C جدول instead of جداول

[16] C الظل instead of ظل

[17] C om. جزءاً

[18] C om. اما

[19] C حصل instead of يحصل

[20] C add. و

الباب الثالث في نقل الاظلال الى مقاييس مختلفة

نسبة اجزاء المقياس[٢١] الى اجزاء المقياس[٢٢] كنسبة الظل الى الظـــل <فـــهذه اربعـــة اعـــداد متناسبة>[٢٣] فليكن مقياس[٢٤] الظل المعلوم اولاً ا و مقياس[٢٥] الظل المجهول ثانيـــاً **ب** و الظـــل المعلوم ثالثاً **ج** و الظل المجهول[٢٦] رابعاً **د**[٢٧] فنضرب الثاني في الثالث و نقسمه علـــى الاول فيحصل الرابع و اما[٢٨] الاصابع و الاقدام فان الاصابع اذا ضربت في خمس[٢٩] و ثلثين دقيقـــة صارت اقداماً[٣٠] على ان المقياس[٣١] سبعة اجزاء و اذا قسمت الاقدام على خمسة و ثلثين دقيقـــة صارت اصابع على ان المقياس[٣٢] اثنا عشر جزءاً

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[21] C المقاييس instead of المقياس

[22] C المقاييس instead of المقياس

[23] Missing in F; recovered from C

[24] C المقاييس instead of المقياس

[25] C المقاييس instead of المقياس

[26] C المجهول من الظل instead of الظل المجهول

[27] F om. these *abjad* notations found in C

[28] C فاما instead of و اما

[29] C خمسة instead of خمس

[30] C اقدام instead of اقداماً

[31] C المقاييس instead of المقياس

[32] C المقاييس instead of المقياس

الفصل الرابع في تقويم الكواكب و[1] احوالها اثنا عشر باباً

الباب الاول في ذكر اصول و مقدمات[2] لاوساط الكواكب

انا لما تأملنا ارصاد القديمة و الحديثة التي في ايام المأمون وبعدها[3] و تصفحناها و امتحنّاهـــا بالقرانات و ارتفاعات نصف النهار و افنينا في البحث عن كل واحد منها سنين بعـــد اطـــراح الهوى و اجتناب الميل الى جانب و ترك التعصب لقوم دون قوم وجدنا رصد محمد بن جـــابر الحراني المعروف بالبتاني اكثرها صواباً و اقلها خللاً و تفاوتاً و اقربها الينا عهداً و صاحبـــه ادق نظراً و اشد استقصاء فيما نال من الرصد و كثيراً[4] مما يدرك بالرصد تركه على ارصـــاد بطلميوس و هو اكثر[5] ميلاً الى الصدق و اشد حباً للحق[6] فرصده لهذه الشرايط اولى بان يعتمد عليه و ان كان لايخلوا رصد من تفاوت و له ارصاد ببلاد الشام الا ان اعتماده على ارصـــاده التي كانت بالرّقه فوضع زيجاً و بنى تقويم كواكبه على تاريخ السريانيين و العرب و استعمال هذين التاريخين بالاضافة الى تاريخ الفرس صعب لما فيهما[7] من الكبايس و الكسور و اختلاف ايام الشهور فنقلنا اصول الاوساط الى تاريخ الفرس و قربنا العمل في التقويم و اصلحنا خلـــلاً وجدناه في تركيب بعض التعاديل و وضعه و سيأتي بذكره[8] في مقالة البرهان فان وجد بيـــن تقويم كوكب بهذا الزيج و بين تقويمه بزيج البتاني تفاوت فذاك من جهة اصلاح في تعديلـــه و اكثر ذلك في المريخ فانه يبلغ درجات لها قدر[9] فاما في ساير الكواكب فيســـير و الشـــمس و القمر لا يقع فيهما[10] شئ و قد نقصنا من اصول الاوساط[11] التي للرقّه مسير ساعة واحـــدة و سبع دقايق من ساعة لتكون موضوعة على طول تسعين من الجزاير[12] الخالدات فتكون ابيـــن

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[1] C في instead of و

[2] C مقدمات اصول instead of اصول و مقدمات

[3] C بعده instead of بعدها

[4] F فكثيراً instead of و كثيراً

[5] C اكثرهم instead of اكثر

[6] C اشدهم ميلاً الى الحق instead of اشد حباً للحق

[7] C فيها instead of فيهما

[8] C ذكره instead of بذكره

[9] C حدود درجتين و ربع instead of درجات لها قدر

[10] C فيها instead of فيهما

[11] C add. البتاني

[12] C جزاير instead of الجزاير

وضعاً و اقرب متناولاً و اوساط ما بين الطولين بين المغرب و[١٣] طول تسعين زايدة ابـــداً و الجزاير[١٤] الخالدات هى جزاير واغله في بحر المغرب[١٥] يذكر بطلميوس انها كانت عامره في قديم الدهر و بينها و بين ساحل البحر عشر درجات من دور الفلك اعني ثلثى ساعة

الباب الثاني في استخراج الاوساط من جداولها

اذا اردنا ذلك اخذنا سني يزدجرد مع السنة و الشهر و اليوم الذي نريد ثم ندخل بالسنين فـــي جدول السنين المجموعة و نأخذ ما بازاء اقرب عدد اليها مما هو اقل منها من الوسط و نثبتـــه على التخت و ما بقي من السنين نأخذ ما بازائها في جدول <السنين>[١٦] المبسوطة ثم نأخذ مـــا بازاء الشهر[١٧] و اليوم و نجمع كل ذلك فيكون الوسط لنصف نهار ذلك اليوم على طول تسعين فنعدله بتعديل ما بين الطولين على ما سنذكره من بعد فان كانت مع الايام ساعات تامة مـــن بعد نصف النهار اخذنا ما بازائها في[١٨] جدول الساعات و ان كانت مع الســـاعات كســـور و كانت الكسور دقايق اخذنا ما بازائها في[١٩] جدول الساعات منحطاً مرةً و ان كـــانت الكســـور ثواني اخذنا ما بازائها منحطاً مرتين و[٢٠] على هذا الرسم

الباب الثالث في نقل الاوساط من طول الى طول

قد تقدم القول بان هذه الاوساط تخرج للمواضع التي طولها من الجزاير[٢١] الخالدات التي فـــي بحر المغرب تسعون درجة فينبغي ان ننقله الى طول البلد الذي نحن فيه حتى يصـــح التقويـــم فاذا[٢٢] اردنا ذلك اخذنا الفضل[٢٣] بين طول بلدنا و طول تسعين و اخذنا لكـــل خمســـة عشـــر

---

13 C add. بين

14 C جزاير instead of الجزاير

15 C add. و

16 السنين added from C

17 C الشهور instead of الشهر

18 C من instead of في

19 C من instead of في

20 C om. و

21 C جزاير instead of الجزاير

22 C فان instead of فاذا

23 C add. الذي

جزءاً[٢٤] من الفضل ساعة واحدة و لكل درجة اربعة دقايق من ساعة فما بلغ فهو ساعات ما بين الطولين فان كان طول بلدنا اقل[٢٥] من تسعين زدنا ساعات ما بين الطولين على الوقت المفروض و ان كان بلدنا اكثر طولاً[٢٦] من تسعين نقصنا ساعات ما بين الطولين من الزمان المفروض فما بلغ او بقي فهو الوقت المعدل بفضل ما بين الطولين و عليه نستخرج الاوساط لبلدنا و طول البلدان اما ان نأخذه من الجدول الموضوع له و اما ان نستخرجه بالحساب على ما نذكره في الباب التاسع عشر من الفصل السادس

الباب الرابع في مواضع الاوجات و الجوزهرات و حركاتها

اما مواضع الاوجات لاول تاريخ يزدجرد فهى للشمس **ب يح لا** و لزحل **ح** δ **مه** و للمشتري **ه ى مه** و للمريخ **د ج يه** و للزهره **ب يح لا** و لعطارد **و يز مد** و اما حركاتها ففي كل اربع[٢٧] و عشرين الف سنة شمسية دور تام ففي[٢٨] كل سنة اربع و خمسين[٢٩] ثانية فاذا اردنا تعديلها اخذنا سني يزدجرد الماضية بعد الاوج المعدل المعلوم و نقصنا منها عشرها فما[٣٠] بقي فهو دقايق حركة الاوجات و ان شئنا اخذنا حركتها[٣١] من الجدول الموضوع لها و زدناها[٣٢] على مواضعها المعدلة من قبل

و اما الجوزهرات فلسنا نحتاج الى شئ منها في هذا الكتاب الا ان مواضعها لاول تاريخ يزدجرد زحل **ج ى مه** المشتري **ج** δ **مه** المريخ **ا ج يه** الزهره **يا يح لا** عطارد **ط يز مد** و حركاتها تابعة لحركات[٣٣] الاوجات و استخراج مواضعها هو ان ننقص من اوج زحل خمسين درجة ثم مما بقي تسعين درجة و نزيد على اوج المشتري عشرين درجة ثم ننقص مما بلغ تسعين درجة و ننقص من اوج المريخ و الزهره و من مقابلة اوج عطارد تسعين درجة فما بلغ فهو موضع الجوزهرات لذلك الوقت

---

[24] C درجة instead of جزءاً

[25] C add. طولاً

[26] C طول بلدنا اكثر instead of بلدنا اكثر طولاً

[27] C اربعة instead of اربع

[28] C وفي instead of ففي

[29] C خمسون instead of خمسين

[30] C و ما

[31] C حركاتها instead of حركتها

[32] C زدناه instead of زدناها

[33] C لحركة instead of لحركات

الباب الخامس في تعديل الايام بلياليها

لوقت تقويم النيرين خاصة تعديل يعرف بتعديل الايام بلياليها فاذا اردنا ذلك نقصنا من وسط الشمس للوقت عشرة بروج و ست عشرة[٣٤] درجة فما بقي فهو حاصل الوسط و نقصنا من مطالع تقويم الشمس للوقت بمطالع خط الاستواء[٣٥] عشرة بروج و اثنين و عشرين درجة و اربع دقايق فما بقي فهو حاصل المطالع ثم نأخذ فضل حاصل الوسط على حاصل المطالع و نضربه في اربعة[٣٦] ثم نأخذ عن الدرج دقايق و عن الدقايق ثواني فيكون دقايق من ساعة[٣٧] من تعديل الايام[٣٨] بلياليها فننقصها من الوقت المعدل بفضل ما بين الطولين فيكون الوقت المعدل بتعديل الايام وجه آخر نزيد على وسط[٣٩] الشمس للوقت ست درج[٤٠] و اربع دقايق و نأخذ الفضل بينه و بين مطالع تقويمه بمطالع خط الاستواء[٤١] و نضربه في[٤٢] اربعة ثم نأخذ ذلك منحطاً بان نحط مرتبة الدرج الى الدقايق و الدقايق الى الثواني و الثواني الى الثوالث فما بلغ يكون دقايق من ساعة و اجزاء من دقايق من ساعة من تعديل الايام بلياليها فننقصها من الوقت المعدل بفضل ما بين الطولين ابداً فيكون الوقت المعدل بتعديل الايام و على هذا الحساب وضعنا جدولاً كتبنا فيه وسط الشمس و بازائه دقايق و ثواني من ساعة[٤٣] من تعديل الايام[٤٤] بلياليها لئلا نحتاج ان نقوم الشمس مرتين على ان الاوج[٤٥] في اربعة و عشرين من الجوزا و ليس يقع من جهة حركة الاوج في هذا التعديل تأثير يحس[٤٦] الا في الدهور الطويلة و ليس لتقويم الكواكب الخمسة حاجة الى هذا التعديل بتّةً

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[34] C عشر instead of عشرة

[35] C الاستوى instead of الاستوا

[36] C om. from here to ثواني

[37] C ساعات instead of ساعة

[38] C om. from here to وجه آخر

[39] C om. وسط

[40] C بروج instead of درج

[41] C الاستوى instead of الاستوا

[42] C instead of the fragment from here to فننقصها : اربع دقايق فيكون دقايق من ساعة من تعديل الايام

[43] C ساعات instead of ساعة

[44] C om. from here to على

[45] C add. في الجوزا

[46] C om. يحس

الباب السادس في تقويم الشمس

نضع وسط الشمس في موضعين و ننقص الاوج المعدل للوقت من احد الموضعين فما بقي فهو الخاصة المعدلة[٤٧] فنأخذ ما بازائها من التعديل[٤٨] بعد ان نعدله بتعديل ما بين السطرين و نزيده على الوسط ابداً[٤٩] فما بلغ فهو التقويم

الباب السابع في تقويم القمر و جوزهره

نضع الوسط و الخاصة و المضاعف ثم نأخذ ما بازاء المضاعف من التعديل الاول و نزيده على الخاصة ابداً فما بلغ فهو التدوير فنأخذ ما بازائه من التعديل الثاني و نحفظه ثم نأخذ ما بازاء المضاعف من اختلاف البعد الاقرب و ما بازاء التدوير من دقايق النسب و نضرب بعضها في بعض فما بلغ[٥٠] نقسمه على ستين فما حصل فهو الاختلاف المعدل فان وقع التدوير في اعلى جدول دقايق النسب زدنا اختلاف المعدل على التعديل الثاني و ان وقع التدوير في اسفل جدول دقايق النسب نقصنا الاختلاف المعدل من التعديل الثاني فما بلغ او بقي من التعديل زدناه على الوسط ابداً فما بلغ فهو التقويم و الخاصة المعدلة و التدوير في جميع الكواكب بمعنى واحد **الجوزهر**[٥١] ننقص وسطه من الدور فما بقي فهو تقويم الرأس و الذنب في مقابلة موضع[٥٢] الرأس ابداً

الباب الثامن في تقويم الكواكب الخمسة

نضع الوسط والخاصة و ننقص الاوج المعدل للوقت من الوسط فما بقي فهو المركز فنأخذ ما بازائه من التعديل الاول و نزيده على المركز و ننقصه من الخاصة ابداً فما بلغ من المركز فهو المركز المعدل و ما بقي من الخاصة فهو التدوير[٥٣] فنأخذ ما بازائه من التعديل الثاني و

[47] C om. المعدلة

[48] C om. from here to و نزيده

[49] C puts ابداً before على

[50] C om. from here to فهو

[51] C جوزهره instead of الجوزهر

[52] C puts موضع before مقابلة

[53] C instead of the fragment from here to ثم : و الخاصة المعدلة ثم تأخذ ما بازاء التدوير من التعديل الثاني و تحفظه

نحفظه ثم ناخذ ما بازاء المركز المعدل من اختلاف البعد الابعد او[٥٤] الاقرب ايما نجده و ما بازاء التدوير من دقايق النسب و نضرب بعضها في بعض و نقسمه على ستين فما بلغ فهو الاختلاف المعدل فان وقع التدوير في اعلى جدول دقايق النسب زدنا الاختلاف المعدل على التعديل الثاني و ان وقع التدوير في اسفل جدول دقايق النسب نقصنا الاختلاف المعدل من التعديل الثاني فما بلغ او بقي من التعديل زدناه على المركز المعدل ابداً فما بلغ زدنا عليه[٥٥] الاوج فما بلغ فهو التقويم **فصل** هذا المركز المعدل غير حقيقي لانه بحسب وضع التعاديل في هذا الزيج فان اردنا حقيقته لاستعماله في العروض و معرفة المقام للرجوع و الاستقامة زدنا عليه لزحل سبع درجات و للمشتري اثنى عشر درجة و للمريخ سبعا و اربعين درجة و للزهره ثمانيا[٥٦] و اربعين درجة و لعطارد ستا و عشرين درجة

الباب التاسع في عرض القمر

ننقص الجوزهر المقوم من القمر المقوم او نزيد[٥٧] وسط الجوزهر على القمر المقوم فما بقي او حصل فهو حصة العرض فنأخذ ما بازائها من العرض فان كانت الحصة اقل من ثلثة بروج فالعرض شمالي صاعد زايد و ان كانت اكثر من ثلثة و اقل من ستة فالعرض شمالي ناقص هابط و ان كانت اكثر من ستة و اقل من تسعة فالعرض جنوبي هابط زايد و ان كانت اكثر من تسعةالى تمام الدور فالعرض جنوبي صاعد ناقص[٥٨] **حسابه** ننقص الجوزهر المقوم من القمر المقوم فما بقي فهو حصة العرض و نضرب[٥٩] جيبها في ظل العرض كله منحطاً فما بلغ فهو ظل العرض و العرض كله خمس درجات **وجه آخر** نضرب جيب حصة العرض في جيب العرض كله منحطاً فما بلغ فهو جيب عرض الحصة ثم نضرب جيب تمام الحصة في جيب العرض كله منحطاً فما بلغ فهو جيب عرض تمام الحصة فنقوسه و ناخذ جيب تمامه و[٦٠] نقسم جيب عرض الحصة منحطاً عليه فما بلغ فهو جيب العرض و اما الذي يقتصر عليه[٦١] اهل الصناعة كلهم من حسابه و هو انهم يضربون جيب حصة العرض في

[54] C و instead of او

[55] C زدناه على instead of زدنا عليه

[56] C ثمان instead of ثمانيا

[57] C from here to فهو : وسطه عليه فما حصل او بقي

[58] C ناقص صاعد instead of صاعد ناقص

[59] C فتضرب instead of و نضرب

[60] From here to فما taken from C; in F: تقسمه منحطاً على جيب عرض الحصة

[61] C om. عليه

جيب العرض كله منحطاً و يزعمون ان الذي يحصل هو[٦٢] جيب العرض[٦٣] فليس بجيب عرض القمر و انما هو جيب قوس قريبة من العرض

الباب العاشر في عروض الكواكب الخمسة

الكواكب العلوية نأخذ المركز المعدل الحقيقي المذكور في آخر الباب الثامن من هذا الفصل و نزيد عليه لزحل خمسين درجة و ننقص منه للمشتري عشرين درجة ونترك المريخ على حالته ثم ندخل به في سطرى العدد و نأخذ ما بازائه من دقايق حصص[٦٤] العرض فنثبته فان وقع المركز في النصف الاعلى من سطرى العدد اخذنا ما بازاء التدوير من عرض الكوكب في الشمال و ان وقع المركز في النصف الاسفل اخذنا ما بازاء التدوير من عرض الكوكب في الجنوب فما كان ضربناه في دقايق حصص العرض فما حصل فهو عرض الكوكب في الجهة الموجودة **الزهره و عطارد** نأخذ ما بازاء التدوير من الميل و الانحراف فنثبت كل واحد منهما على حدته فان كان المركز المعدل لعطارد خاصة يقع في النصف الاعلى من سطرى العدد نقصنا من انحرافه العشر و ان وقع في النصف الاسفل زدنا على انحرافه العشر فما كان فهو الانحراف المستعمل من دون الاول فنحفظه ثم نزيد على المركز المعدل <الحقيقي>[٦٥] للزهره ثلثة بروج و لعطارد تسعة[٦٦] بروج و نأخذ[٦٧] به دقايق حصص العرض[٦٨] و نضربه في الميل فما حصل فهو العرض الاول و هو ميل فلك التدوير فان كان المركز هنا الذي مع الزيادة و التدوير يقعان جميعاً في نصف واحد من سطرى العدد فالعرض الاول جنوبي و ان اختلف موقعهما فالعرض الاول شمالي[٦٩] ثم نأخذ المركز المعدل الحقيقي للزهره كما هو <و>[٧٠] لعطارد بزيادة ستة بروج و نأخذ به دقايق حصص العرض و

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[62] C فهو instead of هو

[63] C instead of here to the end of section: و ليس كذلك و انما يحصل حيب قوس اخرى قريبة من العرض و العرض كله خمس درجات

[64] C om. حصص

[65] الحقيقي added from C

[66] C تسع instead of تسعة

[67] C فما حصل من بعد الزيادة فتأخذ instead of و نأخذ

[68] C العروض instead of العرض

[69] C add. ثاني

[70] و added from C

نضعها[71] في موضعين و نضرب احد الموضعين في الانحراف فما حصل فهو العرض الثاني و هو الالتواء[72] فان كان هذا المركز الذي عرفنا به دقايق الحصص وقع في النصف الاعلى و التدوير اقل من ستة بروج فالعرض الثاني شمالي و ان كان التدوير اكثر فهو جنوبي و ان وقع المركز في النصف الاسفل و التدوير اقل من ستة بروج فالعرض الثاني[73] جنوبي و ان كان التدوير اكثر فالعرض الثاني[74] شمالي[75] ثم نأخذ الموضع الآخر من دقايق الحصص و نضربه للزهره في عشر دقايق و لعطارد في خمس و اربعين دقيقة فما حصل فهو العرض الثالث و هو ميل[76] الفلك الخارج المركز للزهره شمالي ابداً و لعطارد جنوبي ابداً فما وافق من هذه العروض الثلثة[77] في جهة واحدة جمعناه و ما خالف القينا الاقل من الاكثر و عرفنا جهة ما يحصل فهو عرض الكوكب في الجهة الحاصلة **الصعود و الهبوط** نقوم العرض لما بعد عشرة ايام فان كان في[78] الاول شمالياً[79] و زاد في الثاني عرضه فهو صاعد و ان نقص في الثاني فهو هابط و ان كان في الاول جنوبياً و زاد في الثاني فهو هابط و ان نقص في الثاني فهو صاعد و ان كان في الاول شمالياً و في الثاني جنوبياً فهو هابط في الشمال و ان كان في الاول جنوبياً و في الثاني شمالياً فهو صاعد في الجنوب و غاية العرض في الشمال لزحل **ج ب** و في الجنوب **ج ه** و للمشتري في الشمال **ب ه** و في الجنوب **ب ح** و للمريخ في الشمال **د كا** و في الجنوب **ز ن** و للزهره في الجهتين **و كب** و لعطارد في الجهتين **د ه**[80]

الباب الحادي عشر في رجوع الكواكب و استقامتها و رؤيتها و خفائها

نأخذ التعديل الاول بالمركز و نحفظه و نزيد على المركز وسط يوم و نأخذ تعديله ثانياً و ننقص اقل التعديلين من اكثرهما فما بقي ان كان التعديل زايداً زدناه على وسط اليوم و ان كان ناقصاً نقصناه فما بلغ او بقي فهو وسط اليوم المعدل ثم نأخذ التعديل الثاني بالتدوير و

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[71] C نضعهما instead of نضعها

[72] Marginal note in F: و هو ميل الالتواء

[73] C om. الثاني

[74] C om. الثاني

[75] C add. ثالث

[76] C om. ميل

[77] C الثالثة instead of الثلثة

[78] C om. في

[79] C شمالي instead of شمالياً

[80] C ديه | ده instead of ده

نحفظه و نزيد على التدوير خاصة يوم و نأخذ تعديله ثانيا و ننقص اقل التعديلين من اكثرهما فما بقي فهو تفاضل تعديل يوم فان كان التفاضل اقل من وسط اليوم المعدل فالكوكب مستقيم و ان كان اكثر فالكوكب راجع و ان كان مثله فالكوكب مقيم للرجوع او الاستقامة **وجه آخر** ندخل بالمركز المعدل في جدول المقام الاول و نأخذ ما بازائه و ننقص المقام الاول من الدور فما بقي فهو المقام الثاني ثم ننظر الى التدوير فان كان اقل من المقام الاول و اكثر من المقام الثاني فالكوكب مستقيم و ان كان اكثر من مقام الاول و اقل من المقام الثاني فالكوكب راجع و ان كان مساويا للمقام الاول فهو مقيم للرجوع و ان كان مساويا للمقام الثاني فهو مقيم للاستقامة و ان كان بينهما درجات يسيرة قسمناها على خاصة الكوكب ليوم فما حصل فهو المدة الى ان رجع[٨١] الكوكب او منذ رجوع[٨٢] او الى ان يستقيم او منذ استقام و خاصة الكوكب ليوم زحل[٨٣] δ **نز** المشتري δ **ند** المريخ δ **كح** الزهره δ **لز** عطارد **ج و** و قد اثبتنا الرجوع و الاستقامة و الظهور و الخفاء في مواضعها بالتقريب في[٨٤] جدول التعديل الثاني فنأخذ ما بازاء التدوير من هذه الاحوال و ان كان بين التدوير و بين موقع احد[٨٥] هذه الاحوال درجات يسيرة قسمناها[٨٦] على خاصة الكوكب ليوم كما قلنا فما كان فهو المدة الى ان يرجع او منذ رجع او الى ان يستقيم او منذ استقام او الى ان يظهر او منذ ظهر او الى ان يخفي او منذ خفى و اذا رئى[٨٧] الكوكب طالعا قبل طلوع الشمس فهو مشرق و اذا رئى[٨٨] غاربا بعد غروب الشمس فهو مغرب <و نهاية التشريق و التغريب للكواكب العلوية **س** درجة و للزهره **مز** درجة و لعطارد **كو** درجة و هو نهاية بعدهما عن الشمس>[٨٩] و احتراق الكواكب العلوية في منتصف ايام الاستقامة بالتقريب و مقابلتها للشمس في منتصف ايام الرجوع بالتقريب و احتراق الزهره و عطارد في منتصف ايام الاستقامة و منتصف ايام الرجوع بالتقريب.[٩٠]

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[81] C يرجع instead of رجع

[82] C رجع instead of رجوع

[83] C لزحل instead of زحل

[84] C من instead of في

[85] C احد مواضع instead of موقع احد

[86] C om. from here to: كما

[87] C راى instead of رئى

[88] C راى instead of رئى

[89] Addition from a marginal note in F

[90] A marginal note on this folio of F: و للرؤية و الخفا وجه آخر نذكر في الباب العشرين من الفصل السادس عند ذكرنا رؤية الهلال

الباب الثاني عشر في صعود الكواكب و هبوطها في افلاكها

الصعود و الهبوط يعني به في مناطق فلك الاوج و فلك التدوير اما في[٩١] فلك الاوج فمركز فلك التدوير و اما في[٩٢] فلك التدوير فجرم الكوكب عليه و قد اثبت مناطق فلك الاوج في جدول التعديل الاول ليؤخذ بالمركز و مناطق فلك التدوير في جدول التعديل الثاني ليؤخذ بالتدوير فاذا وجد[٩٣] المركز و التدوير فيما بين البعد الابعد و الاوسط[٩٤] على توالي البروج فمركز فلك التدوير او جرم الكوكب على فلك التدوير هابط الى البعد الاوسط من[٩٥] البعد الابعد و فيمابين البعد الاوسط و الاقرب هابط الى البعد الاقرب من البعد الاوسط و فيمابين البعد الاقرب و البعدالاوسط الثاني صاعد الى البعد الاوسط من البعد الاقرب و فيمابين البعد الاوسط هنا و بين البعد الابعد صاعد الى البعد الابعد من البعد الاوسط فاما صعود الكوكب <و هبوطها يعني صعود الكوكب>[٩٦] نفسه في فلك الاوج و هبوطه فيه فظاهر اذا كان موضع الاوج معلوماً

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[91] C om. في

[92] C om. في

[93] C وحدت instead of وجد

[94] C add. الاول

[95] C منه instead of من

[96] Addition from C

الفصل الخامس في اعمال طوالع النهار والليل اثنان و عشرون[1] باباً

الباب الاول في الميل الاول

نضرب جيب الاجزاء التي نريد ميلها في جيب الميل كله منحطاً فما حصل فهو جيب الميل الاول و الميل كله على ما وجدنا[2] بارصاد المتوالية ثلث و عشرون درجة و خمس و ثلثون دقيقة و على هذا الحساب له جدول موضوع

الباب الثاني في مطالع البروج بخط[3] الاستواء

نقسم جيب تمام الاجزاء التي نريد مطالعها على جيب تمام ميل الاجزاء منحطاً فما حصل فهو جيب تمام المطالع فنقوسه وننقصه من تسعين **وجه آخر** نقسم ظل ميل الاجزاء[4] على ظل الميل كله منحطاً فما حصل فهو جيب مطالع تلك الاجزاء **وجه آخر** اذا كان ميل الثاني معلوماً وهو ان نقوس الميل الاول لتلك الاجزاء في جدول الميل الثاني فما حصل فهو مطالع تلك الاجزاء و له جدول موضوع

الباب الثالث في الميل الثاني

[5] نقسم جيب ميل تلك الاجزاء على جيب تمام ميل تمام الاجزاء منحطاً فما حصل فهو جيب الميل الثاني **وجه آخر** نضرب جيب تلك الاجزاء في ظل الميل كله منحطاً فما حصل فهو ظل ميل الثاني و نهايته نهاية الميل الاول **وجه آخر** اذا كان مطالع خط الاستواء معلومة[6] و هو ان نقوس الاجزاء[7] في مطالع خط الاستواء فما كان فهو عكس المطالع فنأخذ[8] ميله الاول فيكون الميل الثاني لتلك الاجزاء و له جدول موضوع

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[1] F and C احد و عشرون instead of the correct number of sections اثنان و عشرون

[2] C وجد instead of وجدنا

[3] C لخط instead of بخط

[4] C add. التي تريد مطالعها

[5] C add. اذا اردنا ميل جزء من اجزاء فلك البروج

[6] C معلوماً instead of معلومة

[7] C add. التي تريد ميلها الثاني

[8] C و تأخذ instead of فنأخذ

الباب الرابع في بعد الكواكب[9] عن معدل النهار

ان كان عرض الكوكب و الميل الثاني لدرجته في جهة واحده جمعناهما و ان كانا مختلفين نقصنا الاقل من الاكثر و عرفنا جهة ما بقى ثم نضرب جيبه في جيب تمام الميل كله و نقسمه على جيب تمام الميل الثاني المأخوذ لدرجة الكوكب فما حصل فهو جيب بعد الكوكب عن معدل النهار و جهته الجهة التي عرفنا و هذا البعد للكوكب مثل الميل[10] الاول في الشمس حيث كان[11]

الباب الخامس في عرض البلد

نأخذ غاية ارتفاع الشمس في أي يوم كان بآلة[12] من آلات الارتفاع و نعرف ميل درجة الشمس فان كان الميل في الشمال نقصناه من غاية الارتفاع و ان كان في الجنوب زدناه على غاية الارتفاع فما حصل فهو تمام عرض البلد فان صار اكثر من تسعين نقصناه من مائة و ثمانين و ما بقى فهو تمام عرض البلد

الباب السادس في سعة مشرق الشمس و الكوكب

نقسم جيب ميل درجة الشمس او جيب بعد الكوكب عن معدل النهار على جيب تمام عرض البلد منحطاً فما حصل فهو جيب سعة المشرق[13] **وجه آخر** اذا كان نصف قوس نهار الدرجة او الكوكب معلوماً و هو ان نضرب جيب تمام ميل الدرجة او جيب تمام بعد الكوكب عن معدل النهار في جيب نصف قوس نهار الدرجة او الكوكب[14] منحطاً فما حصل فهو جيب تمام سعة المشرق فنقوسه و ننقصه من تسعين و [و] نصف قوس النهار في الباب العاشر من هذا الفصل

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[9] F الكواكب instead of الكواكب

[10] C في الكوكب كالميل instead of للكوكب مثل الميل

[11] C add. على الاطلاق

[12] C add. صحيحة

[13] C add. الشمس او الكوكب

[14] C نصف قوس النهار للدرجة او للكوكب instead of قوس نصف النهار الدرجة او الكوكب

الباب السابع في تعديل نهار الشمس و الكوكب

نقسم جيب تمام سعة مشرق الشمس[١٥] او الكوكب على جيب تمام ميل درجة الشمس او جيب تمام بعد الكوكب عن معدل النهار منحطاً فما حصل فهو جيب تمام تعديل النهار **وجه آخر** و هو ان نضرب جيب ميل الشمس او جيب بعد الكوكب عن معدل النهار في جيب عرض البلد و نقسمه على جيب تمام الميل او البعد فما حصل فهو الاصل ثم نقسم الاصل على جيب تمام عرض البلد منحطاً فما حصل فهو جيب تعديل النهار **وجه آخر** و هو ان نضرب ظل ميل الشمس او ظل بعد الكوكب عن معدل النهار في ظل عرض البلد منحطاً فما بلغ فهو جيب تعديل النهار و لظل الميل جدول موضوع **وجه آخر** <لاجزاء فلك البروج>[١٦] اذا كان تعديل نهار اول السرطان او الجدى معلوماً اعني تعديل النهار الكلي و هو ان نضرب جيب تعديل النهار الكلي في جيب مطالع الدرجة بخط الاستواء منحطاً فما بلغ فهو جيب تعديل نهار الدرجة و لتعديل نهار عرض **لو** جدول موضوع[١٧]

الباب الثامن في مطالع البلد

الدرجات الشمالية التي هى من اول الحمل الى آخر السنبله ننقص تعديل نهارها من مطالعها بخط الاستواء و الدرجات الجنوبية التي هى من اول الميزان الى آخر الحوت نزيد تعديل نهارها على مطالعها بخط الاستواء فما حصل فهو مطالع تلك الدرجة بمطالع البلد و لمطالع عرض **لو**[١٨] جدول موضوع

الباب التاسع في غاية ارتفاع الشمس و الكوكب[١٩]

ان كان ميل الشمس او بعد الكوكب عن معدل النهار شماليا زدناه على تمام عرض البلد و ان كان الميل او البعد جنوبيا نقصناه من تمام عرض البلد فما بلغ او بقى فهو غاية ارتفاع الشمس

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[15] C المشرق للشمس instead of مشرق الشمس

[16] Addition from C

[17] C mentions the same latitude 36° (لو); B, L, and Y give the table for the latitude 35;30° (له ل)

[18] C also mentions latitude 36°; B, L, and Y have the table for the latitude 35;30°

[19] C add. و هى ارتفاع اى جزء تريد من اجزاء فلك البروج

او الكوكب فان كان المبلغ اكثر من تسعين نقصناه من مائة و ثمانين و ما بقـــى فـــهو غايـــة الارتفاع من جهة الشمال

الباب العاشر في نصف قوس نهار الشمس و الكوكب[٢٠]

ان كان ميل الشمس او بعد الكوكب عن معدل النهار شمالياً زدنا تعديل نهاره على تســـعين و ان كان الميل او البعد جنوبياً نقصنا تعديل نهاره من تسعين فما بلغ او بقى فهو نصف قـــوس نهار الشمس او الكوكب **وجه آخر** ننقص مطالع الدرجة من مطالع نظيرها بمطالع البلد فمـــا بقى فهو قوس النهار واذا نقص قوس النهار للشمس او الكوكب[٢١] من ثلثمائة و ســـتين كـــان مابقى قوس الليل

الباب الحادي عشر في ساعات نهار الشمس و[٢٢] الكوكب و اجزاء ساعاتهما[٢٣]

نضرب تعديل نهار الشمس او الكوكب في ثمان دقايق ثم ان كان ميل درجة الشمس او بعـــد الكوكب عن معدل النهار شمالياً زدناه على اثنى عشر و ان كان الميل او البعد جنوبياً نقصنـاه من اثنى عشر فما بلغ او بقى فهو ساعات نهار الشمس او الكوكب و نضرب تعديل النهار في عشر دقايق ثم ان كان الميل او البعد شمالياً زدناه على خمسة عشر و ان كان الميل او البعـــد جنوبياً نقصناه من خمسة عشر فما بلغ او بقى فهو اجزاء ساعات نهار الشـــمس او الكوكـــب **وجه آخر** نقسم قوس نهار الشمس او الكوكب على خمسة عشر فيحصــــل ســـاعات النـــهار المستوية و نقسمه ايضاً على اثنى عشر فيحصل اجزاء ساعات النهار الزمانية و اذا نقصـــت [اجزاء][٢٤] ساعات النهار من اربع[٢٥] و عشرين كان ما بقى ساعات الليل و اذا نقصت اجـــزاء ساعات النهار من ثلثين كان ما بقى اجزاء ساعات الليل **فصل** اذا زيد على ســـاعات النـــهار

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[20] C او الكوكب و هو باب معرفة نهار اى جزء تريد من اجزاء فلك البروج instead of و الكوكب

[21] C نهار الشمس او الكوكب فاذا نقصته instead of النهار و اذا نقص قوس النهار للشمس او الكوكب

[22] C او instead of و

[23] C add. و هو باب معرفة ساعات اى جزء تريد من اجزاء فلك البروج و اجزاء ساعات الزمانية

[24] C om. اجزاء

[25] C اربعة instead of اربع

المستوية ربعها كان ما بلغ اجزاء ساعات النهار الزمانية و اذا نقص من اجـــزاء[٢٦] ســـاعات النهار <الزمانية>[٢٧] خمسها كان ما بقى ساعات النهار المستوية

الباب الثاني عشر في درجة ممر الكوكب بنصف النهار

ان لم يكن للكوكب عرض فدرجة ممره درجة طوله و ان كان له عرض ضربنا جيب تمـــام العرض للكوكب[٢٨] في جيب بعد الدرجة[٢٩] من المنقلب القريب منها متقدماً او متأخراً و نقسمه على جيب تمام بعد الكوكب عن معدل النهار فما حصل فهو جيب البعد المعدل من المنقلـــب فنقوسه و نزيده على اول المنقلب ان كان بعد الكوكب منه على توالي البروج و ننقصه منه ان كان البعد منه[٣٠] على خلاف التوالي فما حصل فهو مطالع درجة الممر من اول الحمل بمطالع خط الاستواء فنقوسه في المطالع <بخط الاستواء من اول الحمل>[٣١] فما كان فهو الدرجة التـي يتوسط[٣٢] السماء مع الكوكب

الباب الثالث عشر في درجة طلوع الكوكب و غروبه[٣٣]

ان كان بعد الكوكب عن معدل النهار شمالياً نقصنا تعديل نهاره من مطالع درجة ممره بخـــط الاستواء و ان كان البعد جنوبياً زدنا تعديل نهاره على مطالع درجة الممر[٣٤] فما حصل فـــهو مطالع الدرجة التي تطلع مع الكوكب بمطالع البلد و نزيد قوس نهار الكوكب علــــى مطـــالع درجة الطلوع فما بلغ نقوسه في مطالع البلد و نـــأخذ مقابلته فما كان فهوالدرجة التي تغيـــب مع الكوكب

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[26] C om. اجزاء

[27] Addition from C

[28] C عرض الكوكب instead of العرض للكوكب

[29] C درجته instead of الدرجة

[30] C om. منه

[31] Addition from margin of F; missing in C

[32] C تتوسط instead of يتوسط

[33] C add. و هو معرفة الدرجة التي تطلع مع الكوكب من فلك البروج و الدرجة التي تغيب معه

[34] C ممره بخط الاستواء instead of ممر

الباب الرابع عشر في الداير من الفلك لطلوع الشمس او الكوكب من ارتفاع الوقت

نضرب جيب ارتفاع الوقت في سهم نصف قوس النهار و نقسمه على جيب غاية الارتفاع فما حصل فهو جيب ترتيب الداير فننقصه من سهم نصف قوس النهار فما بقى فهو سهم فضل الداير فنقوسه فيكون فضل الداير فان كان ارتفاع الوقت شرقيا نقصنا فضل الداير من نصف قوس النهار و ان كان الارتفاع غربيا زدنا فضل الداير على نصف قوس النهار فما حصل فهو الداير من الفلك

الباب الخامس عشر في الساعات من الداير

نقسم الداير من الفلك على خمسة عشر فما حصل فساعات مستوية منذ طلعت الشمس او الكوكب[٣٥] و نقسم[٣٦] الداير على اجزاء ساعات جزء الشمس او الكوكب فما حصل فساعات زمانية لطلوع الشمس او الكوكب

الباب السادس عشر في الطالع من الداير بالنهار و الليل

نزيد الداير من طلوع الشمس او الكوكب على مطالع جزء الشمس او مطالع الدرجة التي تطلع مع الكوكب فما بلغ فهو مطالع الطالع بمطالع البلد فنقوسه في جدول المطالع فما كان فهو الطالع

الباب السابع عشر في الداير من الطالع

ننقص من مطالع الطالع مطالع جزء الشمس او مطالع الدرجة التي تطلع مع الكوكب فما بقى فهو الداير من الفلك لطلوع الشمس او الكوكب

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[35] Marginal note in F: و ما بقى ضربناه في اربعة فيكون الدقايق من ساعة

[36] C فتقسم instead of و نقسم

الباب الثامن عشر في ارتفاع الوقت من الداير

نأخذ الفضل بين الداير و بين نصف قوس النهار فما كان فهو فضل الداير فننقص سهمه من سهم نصف قوس النهار فما بقى فهو جيب ترتيب الداير فنضربه في جيب غاية الارتفاع و نقسمه على سهم نصف قوس النهار فما حصل فهو جيب ارتفاع الوقت للشمس او الكوكب عند الداير المفروض فنقوسه فيكون الارتفاع

الباب التاسع عشر في الداير لمغيب الشمس من الطالع

ننفص من مطالع الطالع مطالع نظير جزء الشمس لوقت القياس فما بقى فهو الداير من الفلك لمغيب الشمس

الباب العشرون في الطالع من الداير لمغيب الشمس[٣٧]

نزيد الداير من الفلك لمغيب الشمس على مطالع نظير جزء الشمس فما بلغ فهو مطالع الطالع فنقوسه في جدول المطالع فيكون الطالع

الباب الحادي و العشرون في اصل يعم اكثر الاعمال النهار[٣٨] والليل

نضرب جيب تمام ميل جزء الشمس في جيب تمام عرض البلد منحطاً مرتين فما بلغ[٣٩] فهو الاصل **جيب الترتيب من ارتفاع الوقت**[٤٠] نقسم جيب ارتفاع الوقت على الاصل فيحصل جيب الترتيب **الارتفاع من جيب الترتيب**[٤١] نضرب الاصل في جيب ترتيب الداير فيحصل جيب الارتفاع **سهم نصف قوس النهار** و يسمى جيب النهار نقسم جيب غاية الارتفاع على الاصل فيحصل جيب النهار **ارتفاع نصف النهار من جيب النهار** نضرب الاصل في جيب النهار

[37] In the list of contents, this title is written as في الطالع لمغيب الشمس من الداير

[38] F اعمال بالنهار instead of الاعمال النهار

[39] C حصل instead of بلغ

[40] C add. و

[41] C add. و

فيحصل جيب ارتفاع نصف النهار[٤٢] **سهم فضل الداير** نقسم فضل مابين جيبي ارتفاع الوقت و ارتفاع نصف النهار على الاصل فما حصل فهو سهم فضل الداير **ارتفاع الوقت مـــن ســـهم الفضل <الداير>**[٤٣] نضرب سهم فضل الداير في الاصل فما بلغ ننقصه من جيب ارتفاع نصف النهار فما بقى فهو جيب الارتفاع **تعديل النهار** نصف قوس النـــهار معلـــوم مـــن ســـهمه و الفضل[٤٤] بين نصف قوس النهار و بين تسعين تعديل النهار **الداير من الفلـــك** فضـــل الدايـــر معلوم من سهمه و نصف قوس النهار معلوم من سهمه فان كان الارتفاع شرقيا نقصنا الفضـــل من نصف قوس النهار و ان كان الارتفاع غربيا زدنا الفضل على نصف قوس النهار فما بلـــغ او بقى فهو الداير من الفلك

الباب الثاني و العشرون في تسوية البيوت

نأخذ اجزاء ساعات درجة الطالع و نضاعفه و نحفظه و ننقص هذا المضاعف من ستين فمـــا بقى فهو اجزاء ساعات نظير درجة الطالع مضاعفه فنحفظه ثم ننقص من مطـــالع الطـــالع[٤٥] تسعين درجة فما بقى فهو مطالع العاشر بمطالع خط الاستواء ثم نضع مطالع الطالع بمطـــالع خط الاستواء في موضعين و ننقص من احد الموضعين اجزاء ساعات الطـــالع المضاعفـــه و نزيد على الآخر اجزاء ساعات النظير المضاعفه فيحصل من الناقص مطالع الثاني عشـــر و من الزايد مطالع الثاني بمطالع خط الاستواء فننقص من الناقص ما نقصنا و نزيد على الزايـــد ما زدنا فيحصل من الناقص مطالع الحادي عشر و من الزايد مطالع الثـــالث بمطـــالع خـــط الاستواء فنقوس كل واحد من هذه المطالع فيحصل درجات البيوت ثم الرابع نظير العاشـــر و الخامس نظير الحادي عشر و السادس نظير الثاني عشر و السابع نظير الطالع و الثامن نظير الثاني و التاسع نظير الثالث و ان اردنا ان نمتحن العمل و نعرف هل اصبنا او اخطانا فاننـــا[٤٦] ننقص من مطالع الحادي عشر ما نقصنا من اجزاء ساعات الطالع المضاعفه و نزيـــد علـــى مطالع الثالث ما زدنا من اجزاء ساعات النظير المضاعفه فان حصل من الناقص مثل مطـــالع العاشر و من الزايد ما يقابله فقد اصبنا و الا قد[٤٧] اخطانا فنعيد العمل **وجه آخـــر** و هـــو ان

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42 C add. فنقوسه فيكون الارتفاع

43 Addition from C; C also add. و

44 C فالفضل instead of والفضل

45 C add. بالبلد

46 C فانا instead of فاننا

47 C فقد instead of قد

نضع مطالع العاشر بمطالع خط الاستواء في موضعين و نزيد على احد الموضعين اجزاء ساعات الطالع مضاعفه[48] و ننقص من الاخرا اجزاء[49] ساعات النظير مضاعفه فالزايد مطالع الحادي عشر و الناقص مطالع التاسع بمطالع خط الاستواء فنزيد على الزايد[50] ما زدنا و[51] ننقص من الناقص ما نقصنا فيحصل من الزايد مطالع الثاني عشر و من الناقص مطالع الثامن بمطالع خط الاستواء و قسى[52] هذه المطالع درجات البيوت و نظايرها هى على ما تقدم القول فيها[53]

[48] C المضاعفه instead of مضاعفه

[49] C جزء instead of اجزاء

[50] C و نزيد على instead of فنزيد على الزايد

[51] C او instead of و

[52] C قوس instead of قسى

[53] C نظيرها على ما تقدم instead of نظايرها هى على ما تقدم القول فيها

الفصل السادس في الكسوفات و ما يليق بها عشرون باباً

الباب الاول في مسير النيرين ليوم و ساعة

مسير اليوم هو ما ينقص تقويم احد النيرين ليوم ما من تقويم غده او امسه و يسمى بهت يوم و مسير الساعة هو ما يقسم بهت اليوم على اربعة و عشرين و يسمى بهت ساعة[1] او نقوم احد النيرين للوقت المفروض ثم لما بعده او قبله بست ساعات و نأخذ الفضل بين التقويمين و نضربه في عشر دقايق و له جدول موضوع و اذا نقص بهت ساعة الشمس من بهت ساعة القمر كان ما بقي البهت المعدل وسمى[2] سبق القمر

الباب الثاني في مقدار قطر النيرين و قطر الظل

اما قطر الشمس فنضرب مسير يومها في ثلث[3] و ثلثين دقيقة او نضرب مسير ساعتها في ثلث[4] عشر درجة و خمس فما بلغ فهو قطرها بحسب بعدها من الارض و اما قطر القمر فنضرب مسير يومه في دقيقتين و ست و عشرين ثانية او نضرب مسير ساعته في ثمان و خمسين دقيقة و خمس و عشرين ثانية فما بلغ فهو قطره بحسب بعده من الارض و اما قطر الظل فنضرب قطر القمر في اثنين و ثلثة[5] اخماس فما بلغ فهو قطر الظل بحسب بعد القمر من الارض و الشمس في بعدها الابعد فان اردنا المبالغة في التدقيق اخذنا ما يزيد من مسير ساعة الشمس على دقيقتين و ثلث و عشرين ثانية[6] فنضربه في عشر درجات و ننقصه من قطر الظل الحاصل فما بقي فهو قطر الظل المعدل بحسب بعد الشمس ايضاً من الارض و لهذه الاقطار جدول موضوع مع مسير ساعة النيرين

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[1] C الساعة instead of ساعة

[2] C يسمى instead of سمى

[3] C ثلاثة instead of ثلث

[4] C ثلاثة instead of ثلث

[5] C ثلاثة instead of ثلثة

[6] P gives 25 seconds instead of 23 seconds; C om و عشرين ثانية .

الباب الثالث في جزء الاجتماع و الاستقبال و ساعاتهما و طوالعهما[٧]

نقوم النيرين لنصف نهار اقرب يوم الى الاجتماع او الاستقبال و نأخذ[٨] البعد بين التقويمين اما في الاجتماع فمن تقويمها ذاك و اما في الاستقبال فبعد ان نزيد على موضع القمر[٩] ستة بروج و ننظر لايهما البعد ثم نضرب البعد في خمس دقايق فما كان سميناه جزء البعد و نحفظه و نزيده على البعد فما بلغ فهو البعد و جزء البعد ثم ننظر فان كان البعد للشمس زدنا البعد و جزوه على القمر و زدنا جزو البعد على الشمس و ان كان البعد للقمر نقصنا البعد و جزؤه من القمر و نقصنا جزء البعد من الشمس فيجتمعان او يتقابلان في ثانية واحدة **الساعات**[١٠] ثم نعرف مسير ساعة النيرين و ننقص مسير ساعة الشمس من مسير ساعة القمر فمابقي فهو سبق القمر فنقسم البعد على سبق القمر فما حصل فهو ساعات البعد فان كان البعد للشمس زدنا ساعات البعد على ساعات نصف النهار فان كان المبلغ[١١] اقل من ساعات النهار كله فهو الساعات الماضية من النهار و ان[١٢] كان اكثر من ساعات النهار نقصنا منه[١٣] ساعات النهار و ما بقي فهو الساعات الماضية من الليلة المقبلة و ان كان البعد للقمر و ساعات البعد اقل من ساعات نصف النهار نقصنا ساعات البعد من ساعات نصف النهار و ما بقي فهو الساعات الماضية من النهار و ان كان اكثر من ساعات نصف النهار نقصنا ساعات البعد من ساعات نصف النهار و ساعات الليل مجموعين فما بقي فهو الساعات الماضية من الليلة الماضية ثم نقوم النيرين على الساعات الحاصلة فان اتفقا في الجزء الحاصل من قبل فالساعات صحيحة و ان اختلف موضعهما اخذنا الفضل بينهما و عملنا به كعملنا بتقويم نصف النهار و البعد الذي كان بين النيرين عملاً سواء فما حصل في المرة الثانية فهو جزء الاجتماع و[١٤] الاستقبال و ساعاته بالاستقصاء و على ما يحصل من الساعات نقيم الطالع فيكون طالع الاجتماع و الاستقبال

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[٧] C ساعاتهما و طوالعهما instead of ساعتهما و طالعهما

[٨] C فتأخذ instead of و نأخذ

[٩] C موضعه instead of موضع القمر

[١٠] C om. الساعات

[١١] C البعد instead of المبلغ

[١٢] C فان instead of و ان

[١٣] C منها instead of منه

[١٤] C او instead of و

الباب الرابع في اصابع الخسوف[١٥] مطلقة و معدلة

نتأمل عرض القمر عند الاستقبال فان كان اكثر من ثلث و ستين دقيقة شمالياً او جنوبياً لم ينخسف القمر و ان كان اقل من ذلك امكن ان ينخسف فنعرف قطر القمر و قطر الظل و نجمعها و ننصف المبلغ و هو نصف القطرين فان كان عرض القمر اكثر من نصف القطرين او مثله لم ينخسف القمر و ان كان العرض اقل خسف[١٦] و ما فضل من نصف القطرين على العرض فهو دقايق الخسوف فان كانت هي[١٧] اكثر من قطرالقمر خسف كله و مكث فيه زماناً و ان كانت[١٨] مثل قطر القمر خسف كله و لم يمكث في الخسوف و ان كانت اقل من قطر القمر خسف بعضه فنضرب دقايق الخسوف في اثنى عشر و نقسمه على قطر القمر فما حصل فهو اصابع الخسوف مطلقة و هو على ان قطره اثنا[١٩] عشر اصبعاً و تعديله ان ننقص دقايق الخسوف من قطر القمر و من قطر الظل[٢٠] و نجمع الباقيين ثم نضرب ما بقي من قطر القمر في دقايق الخسوف و نقسمه على مجموع الباقيين فما حصل فهو سهم الظل فننقصه من دقايق الخسوف فما بقي فهو سهم القمر ثم ننقص سهم القمر من قطر القمر و نضرب ما بقي في سهم القمر و نأخذ جذر المبلغ فما بلغ[٢١] فهو الجيب المطلق فنحفظه[٢٢] ثم نضرب الجيب المطلق في ستين و نقسمه على <نصف>[٢٣] قطر القمر فما حصل فهو الجيب المعدل فنقوسه فان كان سهم القمر اقل من نصف قطره فالقوس قوس القمر و ان كان السهم اكثر من نصف القطر نقصنا القوس من مائة[٢٤] و ثمانين و ما بقي فهو قوس القمر ثم نضرب قطر القمر في اثنين و عشرين و نقسمه على سبعه فما حصل فهو محيط دايرة القمر فنضرب نصفه في نصف قطر القمر فما بلغ فهو تكسير سطح دايرة القمر ثم نضرب محيط الدايرة في القوس و نقسمه على ثلثمائة و ستين فما حصل فهو نصف قوس القطاع فنضربه في نصف قطر القمر فما بلغ فهو قطاع القمر ثم نأخذ الفضل بين السهم و نصف القطر و نضربه في الجيب المطلق فما بلغ فهو مثلثة القمر فان كان السهم اقل من نصف القطر نقصنا المثلثة من القطاع و ان

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[١٥] C add القمر

[١٦] C انخسف instead of خسف

[١٧] C دقايق الخسوف instead of هي

[١٨] C كان instead of كانت

[١٩] C اثنى instead of اثنا

[٢٠] C كاملاً من قطر الظل و كاملاً من كل واحد على حدته instead of من قطر الظل

[٢١] C حصل instead of بلغ

[٢٢] C فتحققه instead of فنحفظه

[٢٣] Missing in F: added from C

[٢٤] F ثانية instead of مائة found in C

كان السهم اكثر زدناه عليه فما بلغ او بقى فهو قطعة القمر ثم نعيد العمل من حيث الجيب المطلق و نستعمل الظل في جميع ما استعملنا[٢٥] الا ان سهم[٢٦] الظل لا يبلغ مقدار[٢٧] نصف قطره فاذا حصل قطعة الظل اضفناها[٢٨] الى قطعة القمر فما بلغ فهو دقايق الخسوف معدلة فنضربها في اثنى عشر و نقسم المبلغ على تكسير سطح دايرة القمر فما حصل فهو اصابع الخسوف معدلة[٢٩] على ان صفحة دايرته اثنى[٣٠] عشر اصبعاً و هذا الباب كاف في تعديل اصابع كسوف الشمس ايضاً اذا اقمنا دايرة الشمس في ذاك[٣١] مقام دايرة القمر في هذا و دايرة الظل في هذا مقام دايرة القمر في ذاك و نحتفظ فيها بالشريطة التي اشرطناها في القمر و سهمه و قوسه و مثلثته و لتعديل الكسوفين جدول موضوع بالتقريب

الباب الخامس في ازمان الخسوف[٣٢] مطلقة و معدلة

ساعات الاستقبال هى ساعات وسط الخسوف و الازمان الباقية هى بدو الخسوف و بدو المكث و بدو الانجلاء و تمام الانجلاء فان لم يكن المكث[٣٣] فبدو الخسوف و تمام الانجلاء فننقص مربع عرض القمر لوسط الخسوف من مربع نصف القطرين و نأخذ جذره فيكون دقايق السقوط من بدو الخسوف الى وسطه كان له مكث او لم يكن[٣٤] فنقسمه على سبق القمر فما حصل فهو ساعات السقوط من البدو الى الوسط فننقصها من ساعات وسط الخسوف و نزيدها عليها فالناقص ساعات البدو و[٣٥] الزايد ساعات تمام الانجلاء فان كان له مكث فانا ننقص نصف قطر القمر من نصف قطر الظل و ما بقي ننقص من مربعه مربع العرض لوسط الخسوف و نأخذ جذر الباقي فيكون دقايق السقوط من بدو المكث الى الوسط فنقسمها على سبق القمر فما حصل فهو ساعات السقوط من بدو المكث الى الوسط[٣٦] فننقصها من ساعات الوسط و نزيدها عليها فالناقص ساعات بدو المكث و الزايد ساعات بدو الانجلاء و تعديله ان

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[٢٥] C في جميع استعمالنا القمر instead of جميع ما استعملنا

[٢٦] F نصف instead of سهم found in C

[٢٧] C مقداره instead of مقدار

[٢٨] C ضفناها instead of اضفناها

[٢٩] F مطلقة instead of معدلة found in C

[٣٠] C اثنا instead of اثنى

[٣١] C add و.

[٣٢] C om الخسوف.

[٣٣] C مكث instead of المكث

[٣٤] C ان كان له مكث و ان لم يكن instead of كان له مكث او لم يكن

[٣٥] C om و.

[٣٦] C om. from فنقسمها up to here

ننقص مربع عرض القمر لبدو الخسوف من مربع نصف القطرين و[٣٧] نزيد الباقي من مربع نصف القطرين[٣٨] على مربع ما بين عرض القمر لبدو الخسوف و بين عرضه لوسط الخسوف فما بلغ نأخذ جذره فما كان فهو دقايق السقوط من البدو الى الوسط معدله و نقسمها على سبق القمر فما حصل فهو ساعات السقوط معدله فننقصها من ساعات وسط الخسوف فما بقي فهو ساعات البدو المعدلة[٣٩] ثم ننقص مربع عرض القمر ايضاً لتمام الانجلاء من مربع نصف القطرين[٤٠] و ما بقي نزيده على مربع ما بين عرض القمر لتمام الانجلاء و بين عرضه لوسط الخسوف فما بلغ نأخذ جذره فيكون دقايق السقوط الثانية معدلة و هو[٤١] من الوسط الى تمام الانجلاء فنقسمها على سبق القمر فما حصل فهو ساعات السقوط الثانية معدلة فنزيدها[٤٢] على وسط الخسوف فما بلغ فهو ساعات تمام الانجلاء معدلة فاما الزمانان الباقيان فلا فايدة في تعديلهما

الباب السادس في تصوير الخسوف

نخط خطاً مستقيماً باى قدر كان و نقسمه بعدد دقايق نصف القطرين ثم ندير دايرة نصف قطرها مساوٍ لهذا الخط فتكون دايرة نصف القطرين و نأ خذ من الخط بقدر نصف قطر الظل و ندير ببعده دايرة على مركز دايرة الاولى فتكون دايرة الظل و نخرج قطرى الدايرتين يتقاطعان عند المركز على زوايا قائمة و نكتب على اطرافها الجهات الاربع المشرق بازاء المغرب و الشمال بازاء الجنوب ثم نأخذ من الخط بقدر عرض القمر لوسط الخسوف و نضع احدى رجلى البركار على مركز الدايرتين و الاخرى حيث وقعت من خط الشمال او الجنوب بحسب جهة العرض و نعلم عليه علامة فيكون مركز القمر لوسط الخسوف ثم نأخذ من الخط بقدر نصف قطر القمر و ندير ببعده دايرة على مركز القمر فتكون دايرة القمر لوسط الخسوف فما وقع منها في دايرة الظل فهو مقدار ما ينخسف من جرم القمر

[٣٧] C او instead ofو

[٣٨] C omمن مربع نصف القطرين .

[٣٩] F معدلة instead of المعدلة found in C

[٤٠] C addو تأخذ جذر الباقي فان (فما؟) كان يزيد على مربعه بوسط الخسوف .

[٤١] C هى instead ofهو

[٤٢] C و نزيدها instead ofفنزيدها

الباب السابع في بعد القمر من الارض

نبتدى فننظر فان كان البعد المضاعف صفراً فبعد مركز فلك التدوير من مركز الارض ستون جزؤاً[٤٣] و ان كان المضاعف ستة بروج سوا فبعد المركز تسعة و ثلثون جزؤاً[٤٤] و ثلث و ان كان المضاعف ثلثة[٤٥] بروج او تسعة بروج سوا نقصنا مربع عشرة اجزاء و ثلث من مربع تسعة و اربعين جزءاً و ثلثين[٤٦] و اخذنا جذر الباقي فيكون بعد المركز ثمانية و اربعين جزؤاً[٤٧] و ثلث و ربع بالتقريب و ان كان المضاعف فيمابين ذلك ضربنا كل واحد من جيبه و جيب تمامه في عشر دقايق و ثلث و نقصنا مربع ما حصل من مربع[٤٨] تسعة و اربعين جزؤاً[٤٩] و ثلثى و اخذنا جذر الباقي فان كان المضاعف اقل من ثلثة[٥٠] بروج او اكثر من تسعة[٥١] زدنا على الجذر ما حصل[٥٢] من جيب تمام المضاعف[٥٣] و ان كان المضاعف اكثر من ثلثة بروج[٥٤] و[٥٥] اقل من تسعة نقصنا من الجذر ما حصل من جيب تمام المضاعف فما كان فهو بعد مركز فلك التدوير من مركز الارض **جرم القمر** ثم نأخذ ما بازاء المضاعف في[٥٦] جداول[٥٧] التعديل من اختلاف البعد الاقرب و ما بازاء التدوير من دقايق النسب و نضرب بعضها في بعض و نزيده على خمسة اجزاء و دقيقة واحدة فما بلغ نأخذ جيبه فما كان فهو نصف قطر فلك التدوير المعدل ثم ان كان التدوير صفراً زدنا على بعد مركز فلك التدوير نصف قطر فلك التدوير المعدل فما كان فهو بعد القمر من مركز الارض و ان كان التدوير ستة بروج سواء نقصنا نصف قطر فلك التدوير من بعد مركز فلك التدوير فما[٥٨] بقي فهو بعد القمر من مركز الارض و ان كان التدوير ثلثة بروج[٥٩] او تسعة بروج[٦٠] سواء زدنا على

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٤٣ C جزءاً instead of جزؤاً

٤٤ C جزءاً instead of جزؤاً

٤٥ C ثلاثة instead of ثلثة

٤٦ C ثلثى instead of ثلثين

٤٧ C جزءاً instead of جزؤاً

٤٨ C مربع ما حصل من instead of من مربع

٤٩ C add جزء .

٥٠ C ثلاثة instead of ثلثة

٥١ C add بروج .

٥٢ C يحصل instead of حصل

٥٣ C add و هو قوس الكبرى .

٥٤ C ابراج instead of بروج

٥٥ F او instead of و found in C

٥٦ C من instead of في

٥٧ F جدول instead of جداول found in C

٥٨ C و ما instead of فما

٥٩ C om بروج .

مربع بعد المركز مربع نصف قطر فلك التدوير المعدل و اخذنا جذره و هو بعد القمـــر و ان كان التدوير فيما بين ذلك ضربنا كل واحد من جيب التدوير و جيب تمامه في نصف قطر فلك التدوير المعدل منحطاً فان كان التدوير اقل من ثلثة[٦١] بروج او اكثر من تسعة زدنا ما حصل من جيب التمام على بعد المركز و ان كان التدوير اكثر من ثلثة بروج[٦٢] و[٦٣] اقل من تســعة نقصنا ما حصل من جيب التمام من بعد المركز فما بلغ او بقى زدنا على مربعه مربـــع مـــا حصل من الجيب و اخذنا جذره فما كان فهو بعد القمر من مركز الارض و لبعد القمر بمقدار ما نحتاج اليه في الكسوفات الشمسية و رؤية الاهلة جدول موضوع <و يؤخذ من الجدول بعد القمر بالبعد المضاعف عرضاً و بالخاصة المعدلة طولاً و نكتفي بذلك عن العمل بالحساب>[٦٤] و لسنا نحتاج الى حساب بعد الشمس احتياجاً ضرورياً و حسابه كحساب بعد القمر علـــى ان نستعمل خاصتها بدلاً من التدوير و درجتين و دقيقة واحدة بدلاً من نصف قطر فلك التـــدوير المعدل و الستين بدلاً من بعد مركز فلك التدوير ثم ما حصل من البعد نضربه في ثمانية عشر و اربعة اخماس فيكون البعد من المركز الارض و ابعد بعدها الـــف و مائتـــان و خمســـة و خمسون[٦٥] جزءاً بالتقريب و اوسط بعدها الف و مائتان و ثمانية اجزاء بالتقريـــب و اقـــرب بعدها الف و مائة و احد و ستون جزؤاً بالتقريب

الباب الثامن في ارتفاع قطب فلك البروج المسمى عرض اقليم الرؤية

نقسم جيب ارتفاع درجة عاشر الوقت على جيب القوس التي بين عاشر الوقت و طالعه مـــن فلك البروج منحطاً فما حصل فهو جيب تمام ارتفاع القطب فنقوسه و ننقصه من تسعين فيكون ما يبقي[٦٦] ارتفاع القطب

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٦٠ C omبروج .

٦١ C ثلاثة instead ofثلثة

٦٢ C ابراج instead ofبروج

٦٣ F او instead of و found in L

٦٤ Add. from C

٦٥ C عشرون instead ofخمسون

٦٦ C بقي instead ofيبقي

الباب التاسع في ارتفاع اى درجة نريد من درجات فلك البروج

نضرب جيب القوس التي بين الدرجة و بين الطالع او الغارب في جيب ارتفاع العاشر[٦٧] و نقسمه على جيب القوس التي بين العاشر و الطالع او الغارب فما حصل فهو جيب ارتفاع الدرجة و[٦٨] ارتفاع كل كوكب لا عرض له

الباب العاشر في البعد بين نصف النهار و مطالع نقطة معلومة من فلك البروج

ان كانت النقطة المعلومة فيمابين العاشر و الطالع نقصنا من مطالع النقطة بمطالع خط[٦٩] الاستواء مطالع العاشر بمطالع خط الاستواء فما[٧٠] بقي فهو بعد النقطة من نصف النهار و ان كانت النقطة المعلومة فيمابين السابع و العاشر نقصنا من مطالع العاشر بمطالع خط الاستواء مطالع النقطة بمطالع الاستواء فما بقي فهو بعد النقطة من نصف النهار و ان كان الباقي اكثر من تسعين نقصناه من مائة و ثمانين فما بقي فهو البعد

الباب الحادي عشر في اختلاف منظر النيرين في دايرة ارتفاع

نأخذ كل واحد من جيب ارتفاع درجة القمر و جيب تمام الارتفاع منحطاً فما حصل من جيب الارتفاع نقصناه من بعد القمر من الارض و ما بقي زدنا على مربعه مربع ما حصل من جيب تمام الارتفاع فما بلغ اخذنا جذره ثم نقسم ما حصل من جيب تمام الارتفاع على هذا الجذر منحطاً فما حصل فهو جيب اختلاف المنظر من القمر في دايرة الارتفاع فان كان القمر على الافق زدنا على مربع بعد القمر من الارض مربع نصف قطر الارض و هو جزو[٧١] واحد و اخذنا جذره ثم نقسم نصف قطر الارض[٧٢] على هذا الجذر منحطاً فما حصل فهو جيب اختلاف المنظر[٧٣] فاذا نقصنا اختلاف المنظر[٧٤] من ارتفاع درجة القمر بالحساب كان ما

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[٦٧] C القطب instead of العاشر

[٦٨] C add على هذا يحسب .

[٦٩] C om خط .

[٧٠] C و ما instead of فما

[٧١] C جزء instead of جزو

[٧٢] Marginal note in C نصف قطر الارض اعنى درجة واحدة :

[٧٣] C add القمر .

[٧٤] C add القمر .

بقي الارتفاع المرئي من ظهر الارض **فصل** و بمثل ذلك يستخرج اختلاف منظر الشمس على ان يستعمل بعدها الاوسط من الارض[٧٥] فليس يختلف اختلاف منظرها فيمابين الابعاد[٧٦] بشئ له قدر و غاية اختلاف منظرها حدود ثلث دقايق و نحتاج الى ذلك لننقص من اختلاف منظر القمر هذا فيبقى اختلاف منظر القمر في دايرة الارتفاع معدلاً و ذلك عند المبالغة في تـدقيق الكسوفات الشمسية و له جدول موضوع يؤخذ منه بحسب تمام ارتفاع الشمس

الباب الثاني عشر في الزوايا الست التي يحتاج اليها في الكسوفات الشمسية

الزاوية الاولى هى التي تكون موضع القمر اول الحمل او اول الميزان و هو درجـة طـالع الوقت و هى بمقدار تمام ارتفاع رأس السرطان او الجدى ايهما كان على دايرة نصف النهار و هى زاوية العرض و تمامها زاوية الطول **الزاوية**[٧٧] **الثانية** هى التي تكون موضـع القمـر اول الحمل او الميزان و هو درجة[٧٨] عاشر الوقت وهى بمقدار تمام الميل كله و هى زاويـة العرض و تمامها زاوية الطول **الزاوية**[٧٩] **الثالثة** هي التي تكون موضع القمر غير اول الحمل او[٨٠] الميزان و هو درجة طالع الوقت و هى بمقدارارتفاع قطب فلك البروج في الوقت و هى زاوية العرض و تمامها زاوية الطول **الزاوية**[٨١] **الرابعة** هى التي تكون موضـع القمـر اول السرطان او الجدى و هو درجة عاشر الوقت و هى زاوية قائمة وليس هنا لك[٨٢] للطول زاوية **الزاوية**[٨٣] **الخامسة** هى التي تكون موضع القمر غير نقط[٨٤] الاعتدال و الانقلاب و هو درجة عاشر الوقت فننظر الى ميل درجة العاشر و الى عرض البلد فان كان الميل شـمالياً نقصـنا الاقل من الاكثر و ان كان الميل جنوبياً زدناه على عرض البلد فما بلغ او بقى فهو بعد فلـك البروج عن سمت الرأس فنقسم جيب ارتفاع قطب فلك البروج على جيب بعد فلـك البـروج عن[٨٥] سمت الرأس منحطاً فما حصل فهو جيب زاوية العرض فنقوسه فتكون زاوية العـرض فتمامها زاوية الطول **وجه آخر** نقسم جيب مطالع بعد نقطة الاعتدال التي فـوق الارض مـن

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[٧٥] Marginal note in C: البعد الاوسط اذا كانت الخاصة المعدلة في كز درجة من الجوزى

[٧٦] C الابعد instead of الابعاد

[٧٧] C om الزاوية .

[٧٨] C درجات instead of درجة

[٧٩] C om الزاوية .

[٨٠] C و instead of او

[٨١] C om الزاوية .

[٨٢] C هناك instead of هنا لك

[٨٣] C om الزاوية .

[٨٤] C نقطة instead of نقط

[٨٥] C من instead of عن

نصف النهار على ما في بابه على جيب مابين العاشر و نقطة الاعتدال[86] من فلك البروج منحطاً فما حصل فهو جيب زاوية العرض فنقوسه فتكون زاوية العرض و تمامها زاوية الطول **الزاوية**[87] **السادسة** هى التي تكون موضع القمر اى درجة كانت و هو فيمابين الطالع و الغارب فنقسم جيب ارتفاع [ارتفاع] قطب فلك البروج على جيب تمام ارتفاع درجة القمر منحطاً فما حصل فهو جيب زاوية العرض فنقوسه فتكون زاوية العرض و تمامها زاوية الطول

الباب الثالث عشر في اختلاف منظر القمر طولاً و عرضاً من هذه الزوايا

نضرب كل واحد من جيب زاوية العرض و جيب زاوية الطول في اختلاف المنظر من دايرة الارتفاع منحطاً فما حصل من زاوية العرض فهو اختلاف المنظر في العرض و ما حصل من زاوية الطول فهو اختلاف المنظر في الطول فان كان بعد القمر عن سمت الرأس عند بلوغه دايرة نصف النهار الى الجنوب و عرض القمر جنوبي او <البعد>[88] الى الشمال و عرض القمر شمالي[89] زدنا اختلاف <المنظر في>[90] العرض على العرض و ان اختلفا نقصنا اقلهما من اكثرهما فما كان فهو العرض المرئي و جهته جهة مجموع العرض و الاختلاف او جهة الاكثر منهما و محاز القمر في اكثر البلدان الشمالية جنوبي عن سمت الرأس

الباب الرابع عشر في اصابع كسوف الشمس مطلقة و معدلة

ان كان عرض القمر عند الاجتماع جنوبياً و في اكثر من خمسة[91] و ثلثين دقيقة او شمالياً و في اكثر من خمسة[92] و تسعين دقيقة لم تنكسف الشمس و ان كان العرض اقل من ذلك امكن ان تنكسف فان امكن عرفنا ساعات الاجتماع و طالعه و اختلاف منظر القمر في الطول و عرضه المرئي ثم نقسم الاختلاف في الطول على سبق القمر فما حصل فهو ساعات الاختلاف فان كان بعد درجة الاجتماع من الطالع اقل من تسعين جزءاً نقصنا ساعات الاختلاف من

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[86] C add من نصف النهار التي فوق الارض.

[87] C om الزاوية.

[88] F om البعد. found in C

[89] C شمال instead of شمالي

[90] F om المنظر في. found in C

[91] C خمس instead of خمسة

[92] C خمس instead of خمسة

ساعات الاجتماع و دقايق اختلاف الطول[٩٣] من درجة الاجتماع و من حصة العرض لنعـرف منها العرض و ان كان بعد درجة الاجتماع من الطالع اكثر من تسعين زدنا ساعات الاختلاف على ساعات الاجتماع و دقايق الاختلاف على درجة الاجتماع و على حصة العرض فما بلغ او بقى من [درجة][٩٤] ساعات الاجتماع فهو ساعات الاجتماع المرئي و ما بلغ او بقـى مـن درجة الاجتماع فهى درجة الاجتماع المرئي و نستخرج[٩٥] الطالع من ساعات الاجتماع المرئي و نستخرج من هذا الطالع و من درجة الاجتماع المرئي عرض القمـر المرئـي و اخـتلاف منظره في الطول ثم نقسم الاختلاف على سبق القمر فما حصل فهو ساعات الاختلاف ثانيـاً فان كان بعد درجة الاجتماع الاول من طالع الاجتماع المرئي اقل من تسعين نقصنا سـاعات الاختلاف من ساعات الاجتماع الاول و دقايق الاختلاف من درجة الاجتماع الاول و ان كان بعد درجة الاجتماع الاول من طالع الاجتماع المرئي اكثر من تسعين زدنا ساعات الاخـتلاف على ساعات الاجتماع الاول و دقايق الاختلاف على درجة الاجتماع الاول فما بلغ او بقى من الساعات فهى ساعات الاجتماع المرئي المعدل و هى ساعات وسط الكسوف و ما بلغ او بقى من درجة الاجتماع فهو موضع القمر لوسط الكسوف و ذلك لانه ان اسـتخرجنا مـن طـالع ساعات الاجتماع هذه و من موضع القمر فيها اختلاف المنظر في الطول وجدناه مساوياً لمـا خرج في المرة الثانية او قريباً منه بما لايحس فاذا حصل ساعات وسط الكسـوف و طالعـه جمعنا نصف قطرى الشمس و القمر سميناه نصف القطرين فان كان العرض المرئـي مثـل نصف القطرين او اكثر منه لم تنكسف الشمس و ان كان اقل فانها تنكسف و ما فضـل مـن نصف القطرين على العرض المرئي فهو دقايق الكسوف فنضرب دقايق الكسوف في اثنـى[٩٦] عشر و نقسمه على قطر الشمس فما حصل فهو اصابع الكسوف و هو ما ينكسف من قطرها على ان القطر اثنا عشر اصبعاً فان اتفق ان يكون الاجتماع قبل طلوع الشمس فتطلع الشـمس منكسفة استعملنا وتد الارض بدلاً من وسط السماء و بدلنا الطالع بالغارب في جميع الاعمـال المتعلقة بالكسوف[٩٧] الشمس و اذا[٩٨] انتهى بنا العمل الى ساعات الاختلاف و دقايق الاخـتلاف <نقصنا ساعات الاختلاف من ساعات الاجتماع و دقايق الاختلاف>[٩٩] من درجة الاجتماع ابداً

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[٩٣] C للطول instead of الطول

[٩٤] Abundant word درجة is crossed out in C

[٩٥] C فنستخرج instead of و نستخرج

[٩٦] C اثنا instead of اثنى

[٩٧] C بكسوف instead of بالكسوف

[٩٨] C فاذا instead of و اذا

[٩٩] Missing in F recovered from C

و تعديل اصابع الكسوف كتعديل اصابع خسوف[١٠٠] القمر حساباً و جدولاً على ان نقيم دايرة القمر في هذا مقام دايرة الظل في ذاك و دايرة الشمس في هذا مقام دايرة القمر في ذاك

الباب الخامس عشر في ازمان الكسوف مطلقة و معدلة

ننقص مربع العرض المرئي لوسط الكسوف من مربع نصف القطرين و نأخذ جذر الباقي فما حصل فهو دقايق السقوط فنقسمها على سبق القمر فما حصل فهى ساعات السقوط فننقصها من ساعات وسط الكسوف و نزيدها عليها فالباقي[١٠١] منها ساعات بدو الكسوف و المبلغ ساعات تمام الانجلاء و تعديل هذين الزمانين كتعديل ازمان خسوف القمر اذا اقمنا العرض المرئي في هذا مقام العرض المطلق في ذاك[١٠٢] و ليس لكسوف الشمس مكث

الباب السادس عشر في تصوير الكسوف

نخط خطاً مستقيماً باى قدر كان[١٠٣] و نقسمه بعدد دقايق نصف القطرين و ندير ببعده دايرة نصف قطرها مساوٍ لهذا الخط فتكون دايرة نصف القطرين و نخرج قطريها[١٠٤] يتقاطعان عند المركز على زوايا قائمة و نكتب على اطرافها الجهات الاربع المشرق بازاء المغرب و الشمال بازاء الجنوب ثم نأخذ من الخط مثل نصف قطر الشمس و ندير ببعده دايرةً على مركز دايرة[١٠٥] نصف القطرين و هى دايرة الشمس ثم نأخذ من الخط مثل العرض المرئي و نضع احدى رجلى البركار على مركز الدايرتين و الاخرى[١٠٦] حيث وقعت[١٠٧] من خط الشمال او[١٠٨] الجنوب بحسب جهة العرض المرئي و نعلم عليها علامة تكون مركز القمر لوسط الكسوف[١٠٩] ثم نأخذ من الخط مثل نصف قطر القمر و نجعل العلامة مركزاً و ندير عليه دايرة القمر فما وقع من دايرة الشمس في دايرة القمر فهو مقدار ما ينكسف منها

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[١٠٠] C الخسوف instead of خسوف

[١٠١] C الباقي instead of فالباقي

[١٠٢] C هناك instead of في ذاك

[١٠٣] C om. باى قدر كان

[١٠٤] F قطريهما instead of قطريها found in C

[١٠٥] C om. دايرة

[١٠٦] C الآخر instead of الاخرى

[١٠٧] C وقع instead of وقعت

[١٠٨] C و instead of او

[١٠٩] C om. from تكون to here

الباب السابع عشر في ارتفاع القمر بحسب عرضه

ان بطلميوس و من شايعه[١١٠] من اهل الصناعة كلهم حسبوا حساب اختلاف منظر القمر في دايرة الارتفاع و مقادير الزوايا الست التي ذكرناها على ان القمر لا عرض له بته و استخرجوا اختلاف المنظر في الطول <و العرض>[١١١] بان اقاموا خطوطاً مستقيمةً مقام قسي الصغيرة و ليس فيما فعلوه ضرر في العرض بشئ محسوس الا ان للتحقيق فضلاً على التقريب و موقعاً من القلوب خلاف موقع التقريب و قد تيسرت[١١٢] لنا طريقة مبرهنة ليس بينها و بين الاولى كبير[١١٣] فرق في الصعوبة و الطول يتبين منها[١١٤] ارتفاع القمر بحسب عرضه و عرضه[١١٥] المرئي و اختلاف منظره في الطول و هى ان[١١٦] نضرب جيب تمام العرض في جيب تمام بعد درجته من طالع الوقت او غاربه ايهما كان اقل من تسعين منحطاً فما حصل فهو جيب نقوسه و ننقصه من تسعين فما بقى فهو القوس الاول ثم نقسم جيب العرض على جيب القوس الاول منحطاً فما حصل فهو جيب قوس الثاني فنقوسه فان كان العرض شمالياً زدنا هذه القوس على تمام ارتفاع قطب فلك البروج[١١٧] و ان كان العرض جنوبياً نقصناها منه فما بلغ او بقى فهو الحاصل من تمام ارتفاع القطب ثم نضرب جيب القوس الاول في جيب الحاصل من تمام ارتفاع القطب منحطاً فما حصل فهو جيب الارتفاع بحسب عرض للقمر[١١٨] و ساير الكواكب ذوات العروض[١١٩] فيستخرج من هذا الارتفاع اختلاف المنظر في دايرة الارتفاع[١٢٠]

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١١٠ C تابعه instead of شايعه

١١١ F om. و العرض found in C, Y, and L

١١٢ C تيسر instead of تيسرت

١١٣ C طريق instead of كبير

١١٤ F منهما instead of منها found in C

١١٥ C منها عرض القمر instead of منهما ارتفاع القمر بحسب عرضه و عرضه

١١٦ C om. و هى ان

١١٧ Marginal note in C: و هو ارتفاع وسط سماء الطالع

١١٨ C القمر instead of للقمر

١١٩ C العرض instead of العروض

120 There is "another method" here in C and F which has been omitted because it was not authentic.

الباب الثامن عشر في اختلاف منظر القمر طولاً و عرضاً بطريقة[١٢١] مبرهنة

قد قلنا في الباب الحادي عشر[١٢٢] من هذا الفصل ان الارتفاع الحاصل من الحساب هو الارتفاع الحقيقي الذي كنا نجده لو رصدناه من مركز دايرة الارتفاع[١٢٣] و المنقوص منه اختلاف المنظر هو الارتفاع المرئي من ظهر الارض[١٢٤] و من بعد ما تقدم ذلك فان هذا الباب يقع على خمسة اوجه **الاول** ان يكون ارتفاع درجة عاشر الوقت تسعين جزؤاً و ليس للقمر عرض و اختلاف المنظر في دايرة الارتفاع هو اختلاف المنظر في الطول وحده فليس له اختلاف منظر في العرض[١٢٥] **الثاني** ان يكون بعد درجة القمر من طالع الوقت تسعين جزءاً كان للقمر عرض او لم يكن فاختلاف المنظر في دايرة الارتفاع هو العرض المرئي وحده و ليس له اختلاف منظر في الطول[١٢٦] **الثالث** ان يكون ارتفاع عاشر الوقت تسعين جزؤاً و للقمر عرض اما العرض المرئي فنضرب جيب عرض القمر في جيب تمام الارتفاع المرئي و نقسمه على جيب تمام الارتفاع الحقيقي فما حصل فهو جيب العرض المرئي و جهته جهة عرض القمر و اما اختلاف الطول فنقسم جيب الارتفاع المرئي على جيب تمام العرض المرئي منحطاً فما حصل فهو جيب فنقوسه[١٢٧] و ننقصه من بعد درجة القمر من الطالع او الغارب[١٢٨] فما بقى فهو اختلاف <المنظر في>[١٢٩] الطول **الرابع** ان يكون ارتفاع عاشر الوقت اقل من تسعين و ليس للقمر عرض اما العرض المرئي فنضرب جيب اختلاف المنظر من دايرة الارتفاع في جيب ارتفاع قطب فلك البروج و نقسمه على جيب تمام الارتفاع الحقيقي فما حصل فهو جيب العرض المرئي في الجنوب و اما اختلاف المنظر في[١٣٠] الطول فنقسم جيب تمام اختلاف المنظر من دايرة الارتفاع على جيب تمام العرض المرئي منحطاً فما حصل فهو جيب تمام اختلاف المنظر في الطول **الخامس** ان يكون ارتفاع عاشر الوقت اقل من تسعين و للقمر عرض اما العرض المرئي فنضرب جيب تمام عرض القمر في جيب تمام القوس التي بين درجته و بين طالع الوقت او غاربه منحطاً[١٣١] ايهما كان اقل من تسعين[١٣٢]

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[١٢١] C بطريق instead of بطريقة
[١٢٢] F العاشر instead of الحادي عشر found in C
[١٢٣] C om. from الذي to here
[١٢٤] C om. من ظهر الارض
[١٢٥] C om. from فليس to here
[١٢٦] C om. from. و ليس to here
[١٢٧] C نقوسه instead of فنقوسه
[١٢٨] C om. او الغارب
[١٢٩] Addition from C
[١٣٠] C om. المنظر في
[١٣١] C om. منحطاً

فما حصل فهو جيب قوس اول فنضربه في جيب تمام الارتفاع المرئي و نقسمه علـــى جيـــب تمام الارتفاع الحقيقي فما حصل فهو جيب قوس ثان[١٣٣] فنقوسه ثم نقسم جيب تمام[١٣٤] الارتفاع المرئي على جيب تمام القوس الثاني منحطاً فما حصل فهو جيب قوس ثالث فنقوسه و نأخـــذ الفضل بينه و بين تمام ارتفاع قطب فلك البروج فما كان فهو[١٣٥] قوس رابع ثم نضرب جيـــب القوس الرابع في جيب تمام القوس الثاني منحطاً فما حصل فهو جيب العرض المرئي فان كان القوس الثالث اكثر من تمام ارتفاع قطب فلك البروج فجهة العرض الشمال و ان كان القـــوس الثالث[١٣٦] اقل فجهة العرض الجنوب و اما اختلاف الطول فنقسم جيب القوس الثاني على جيب تمام العرض المرئي منحطاً فما حصل فهو جيب قوس اول فنقوسه و نحتفظ به[١٣٧] ثم نـــنقص ما بين درجة القمر و بين الطالع او الغارب ايهما كان اقل من تسعين من تسعين[١٣٨] و ما بقى ننقصه من القوس[١٣٩] الاول المحفوظ[١٤٠] فما بقى فهو اختلاف المنظر في الطول

الباب التاسع عشر في استخراج طول البلدان

نحسب كسوفاً شمسياً على طول تسعين و نعرف ساعات البدو او ساعات تمام الانجـــلاء ثـــم نرصد احدى[١٤١] هاتين الساعتين في بلدنا بما امكن من الاستقصاء و نأخذ الارتفاع لذلك الوقت و نستخرج منه الساعات فان كانت الساعات المرصودة اكثر من المحسوبة فبلدنا شرقي عـــن طول تسعين و ان كانت الساعات المرصودة اقل فبلدنا غربي عن طول تسعين و الفضل بين الساعات المحسوبة و المرصودة[١٤٢] ساعات ما بين الطولين فنضربها في خمسة عشر فيكـــون طول ما بين البلدين فان كان بلدناشرقياً زدناه على طول تسعين و ان كان غربياً نقصناه مـــن طول تسعين فما بلغ او بقى فهو طول بلدنا فاما خسوف القمر فان ارتفـــاع القمـــر لايصـــح لاختلاف منظره و ارتفاع الكواكب الثابتة يتعسر وجوده بالاستقصاء و لايوثـــق بمواضـــعها

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١٣٢ C add. منحطاً

١٣٣ C ثاني instead of ثان

١٣٤ C om. تمام

١٣٥ C add. جيب

١٣٦ C om. الثالث

١٣٧ C om. و نحتفظ به

١٣٨ C om. من تسعين

١٣٩ C om. القوس

١٤٠ C om. المحفوظ

١٤١ C احد instead of احدى

١٤٢ C المرصودة و المحسوبة instead of المحسوبة و المرصودة

الحقيقية فان اخذنا[١٤٣] ارتفاع احد السيارة التي نعرف حقيقة[١٤٤] مواضعها كان ذلك الكوكـــب و الشمس فيما نطلبه سواء **وجه آخر** و هو ان نقوم الشمس لنصف نهار يوم ما على تسعين ثم نرصد ارتفاعها لنصف نهار ذلك اليوم بآلة صحيحة دقيقة[١٤٥] من آلات الارتفاع فـــان كانـــت الشمس في البروج الشمالية نقصنا تمام عرض بلدنا[١٤٦] من الارتفـــاع الموجـــود و ان كانـــت الشمس في البروج الجنوبية نقصنا الارتفاع الموجود من تمام عرض البلد فما بقى فهو ميـــل الشمس فنقوسه في جدول الميل من الربع الذي فيه الشمس فما كان فهو موضع الشـــمس فـــي بلدنا فنأخذ الفضل بينه و بين التقويم الاول و ندخل به في وسط ساعات الشمس و نأخـــذ مـــا بازائه من الساعات فما كانت فهى ساعات ما بين الطولين فنضربها في خمسة عشر فيكـــون درجات ما بين الطولين فان كان موضع الشمس في بلدنا اقل من موضعها الاول فبلدنا شرقي عن طول تسعين فنزيد[١٤٧] ما بين الطولين على تسعين و ان كان موضعها في بلدنا اكثر فبلدنا غربي عن طول تسعين فننقص ما بين الطولين من تسعين فما بلغ او بقى فهو طول بلـــدنا و كلما كانت الشمس اقرب من نقطتى الاعتدالين[١٤٨] كان اصح لان الميل هناك ابين و تفاضـــله اكثر[١٤٩] **وجه آخر** استعملته القدماء بالتقريب و عليه طول اكثر البلدان في الكتب و الجداول و هو ان ننظر الى ما[١٥٠] بين بلدنا و بلد معلوم الطول و العرض من الفراسخ و ايام المسير ثم نأخذ لكل مسيره يومين[١٥١] او لكل عشرين فرسخاً جزءاً واحداً و نضربه في مثله و ننصـــف المبلغ و نحفظه[١٥٢] فان كان عرض البلدين متساويين اخذنا جذر المبلغ المنصف بالتقريب فما كان فهو طول ما بين المدينتين و ان كان عرضا البلدين مختلفين نقصنا الاقل من الاكثـــر و ضربنا الباقي في مثله و ننقصه من المبلغ المنصف فما بقى اخذنا جذره فما كان فهو طول ما بين المدينتين و هذا شئ مأخوذ بالتقريب <لايستند الى برهان و لبعض البلدان جدول موضوع باطوالهما و عروضهما و اثبتنا فيه بلدان مشهورة لتكون معلومة بالتقريب>[١٥٣]

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[١٤٣] F احذ instead of اخذنا found in C

[١٤٤] C om. حقيقة

[١٤٥] C om. دقيقة

[١٤٦] C البلد instead of بلدنا

[١٤٧] C و نزيد instead of فنزيد

[١٤٨] C الاعتدال instead of الاعتدالين

[١٤٩] C تفاضل الميل هنا ابين instead of الميل هناك ابين و تفاضله اكثر

[١٥٠] C om. ما

[١٥١] C يوم instead of يومين

[١٥٢] C om. و نحفظه

[١٥٣] Missing in F, added from C

الباب العشرون في رؤية[١٥٤] الهلال و الكواكب من جهة قسى محدودة لها

اما رؤية الهلال فلم يتكلم فيها احد من القدماء لان اوايل الشهور القمرية كانت معلومة لهم[١٥٥] من الاجتماعات و المحدثون لما احتاجوا الى رؤية الهلال للعبادات في شريعة الاسلام عمل كل واحد منهم في ذلك باباً و حساباً على ما غلب عليه ظنه و ليس فيها شئ يطرد[١٥٦] فيعتمد و لا حسابهم فيها يرجع الى قانون و اصل صحيح و الذي لايكاد يخطى مع صفا الجو و حدة البصر في اكثر المعمورة[١٥٧] هو ان نستخرج البعد بين النيرين بحسب العرض و هو الباب الخامس من الفصل الثامن[١٥٨] و البعد بين الشمس و الجزء الذي يغيب معه القمر باجزاء المغارب اما حد القوس الاول فعشرة اجزاء و حد القوس الثاني ثمانية اجزاء ثم نعرف ارتفاع القمر بحسب عرضه عند المغيب الشمس[١٥٩] او طلوعها فما كان ننقص منها[١٦٠] اختلاف المنظر في دايرة الارتفاع و <الباقي>[١٦١] هو قوس الرؤية و حده ست درجات فان كانت القسى الثلث[١٦٢] بحدودها المذكورة فما فوقها فان الهلال يرى و ان كانت[١٦٣] اقل فلايرى و ان شهد اثنان منها على الرؤية فالحكم عليهما و صعوبة الرؤية من جهة القوس الناقص و ان كان قوس الرؤية فمن جهة الارتفاع و ان كان البعد بين النيرين بالسواء فمن جهة قلة[١٦٤] الضوء و ان كان البعد بينهما بالمغارب فمن جهة قلة[١٦٥] مكثه فوق الارض و سرعة غروبه **وجه آخر** نضرب جيب ما بين جزء الشمس و الجزء الذي يغيب معه القمر من فلك البروج في جيب تمام ارتفاع قطب فلك البروج منحطاً فما حصل فهو جيب قوس الرؤية تحت الارض فان كان قوس الرؤية مع البعد بين النيرين بحسب العرض ثمنية[١٦٦] عشر فما فوقه رئى[١٦٧] الهلال و ان كان اقل فلا يرى و هكذى[١٦٨] حساب رؤية الكواكب الا ان في رؤيتها لا نحتاج الى اختلاف

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١٥٤ C om. رؤية

١٥٥ C عندهم instead of لهم

١٥٦ C مطرد instead of يطرد

١٥٧ C المعمور instead of المعمورة

١٥٨ Marginal note in C: كما سيأتي

١٥٩ Marginal note in C: كما تقدم

١٦٠ C منه instead of منها

١٦١ Missing in F, recovered from C

١٦٢ C الثلاثة

١٦٣ C كان instead of كانت

١٦٤ C قلت instead of قلة

١٦٥ C قلت instead of قلة

١٦٦ C ثمانية instead of ثمنية

١٦٧ C رؤى instead of رئى

١٦٨ C هكذا instead of هكذى

المنظر و لا الى البعد بينهما و بين الشمس و انما يعرف ارتفاعها بحسب عروضها فان كـــان ذلك مثل قوس الرؤية رئى[١٦٩] و ان كان اقل فلايرى و قوس الرؤية على ما وجـــدت قـــديماً لزحل **يا** و للمشتري **ى** و للمريخ **يا ل** و للزهره ه و للعطارد **ى**

[١٦٩] C رؤي instead of رئى

الفصل السابع في أعمال تتعلق بالأحكام ستة[1] أبواب

الباب الأول في ساعات بعد درجة الكوكب من الأوتاد

ان كان الكوكب فوق الأرض أخذنا بعده من العاشر متقدماً أو متأخراً[2] بمطالع خط الاستواء و ان كان تحت الأرض أخذنا بعده من الرابع متقدماً أو متأخراً[3] بمطالع خط الاستواء ثم ان كان الكوكب فوق الأرض قسمنا البعد على أجزاء ساعات درجة الكوكب و ان كان الكوكب تحت الأرض قسمنا البعد على أجزاء ساعات نظير درجة الكوكب فما حصل فهو ساعات بعد الكوكب من وتد العاشر أو الرابع متقدماً أو متأخراً[4] بالساعات الزمانية و إذا نقصت هذه الساعات من ستة[5] بقيت ساعات البعد من الطالع أو الغارب

الباب الثاني في مطرح الشعاع بدرج السواء

مطرح الشعاع بدرج السواء قسي من دايرة فلك البروج مقدارها ستون و تسعون و مائة و عشرون و مائة و ثمانون جزءاً فإذا كان للكوكب عرض أخذت[6] هذه القسي من دايرة تمر بالكوكب ثم ينقل الى دايرة فلك البروج و هو ان نقسم جيب تمام الستين أعنى جيب الثلثين[7] على جيب تمام عرض الكوكب منحطاً فما حصل فهو جيب الفضل بين التسعين و قوس التسديس أو[8] التثليث فنقوسه و ننقصه من تسعين فيبقى قوس التسديس و نزيده على تسعين فيبلغ قوس التثليث فأما التربيع فهو أبدا تسعون و المقابلة مائة و ثمانون

الباب الثالث في مطرح الشعاع بدرج المطالع

هذا على حساب تسوية البيوت إلا انه بمطالع أفق الكوكب كما ان تسوية البيوت هى بمطالع أفق البلد و الأحكاميون مجمعون على هذه التسوية فان صحت فخليق ان يصح مطرح

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[1] C ست instead of ستة
[2] C متوخراً instead of متأخراً
[3] C متوخراً instead of متأخراً
[4] C متوخراً instead of متأخراً
[5] C ست instead of ستة
[6] C أخذة instead of أخذت
[7] C ثلاثين instead of ثلثين
[8] C و instead of أو

الشعاع على ذلك الحساب و نحتاج في ذلك الى معرفة أجزاء ساعات درجة الكوكب بحسب موضعه و هو ان ننظر فان كانت درجة الكوكب هى درجة العاشر أو الرابع فأجزاء ساعاتهما خمسة عشر و ان كانت هى درجة الطالع أو الغارب فأجزاء ساعاتهما أجزاء ساعات درجة[9] الطالع أو الغارب و ان[10] كانت فيما بين وتدين أخذنا الفضل بين[11] أجزاء ساعات درجته[12] تلك و بين خمسة عشر و ضربناه في بعد الدرجة من وتد العاشر أو الرابع[13] و قسمناه على ستة فما حصل فهو التعديل <فان كانت الدرجة فيما بين العاشر و الطالع أو في نظير هذا الربع و كان الفضل لخمسة عشر نقصنا منه التعديل و إلا زدنا عليه التعديل>[14] و ان كانت الدرجة فيما بين الطالع و الرابع أو في نظير هذا الربع و كان الفضل لاجزاء ساعات الدرجة نقصنا منها التعديل و إلا زدنا عليها التعديل فما حصل فهو أجزاء ساعات درجة الكوكب بحسب موضعها ثم نأخذ مطالع درجة الكوكب بمطالع خط الاستواء و ننقص منها أجزاء ساعاتها مضروبة في أربعة و ما بقي نقوسه في مطالع خط الاستواء فما كان فهو موضع التسديس الأيسر و يقابله[15] التثليث الأيمن و ننقص أيضا من مطالع درجة الكوكب بمطالع خط الاستواء أجزاء ساعاتها مضروبة في ستة و ما بقي نقوسه في مطالع خط الاستواء فما كان فهو موضع التربيع الأيسر و يقابله[16] التربيع الأيمن ثم ننقص أجزاء ساعات الدرجة من ثلثين و ما بقي نضربه في أربعة و نزيده على مطالع درجة الكوكب بمطالع خط الاستواء فما بلغ نقوسه في هذه المطالع فما كان فهو موضع التسديس الأيمن و يقابله[17] التثليث الأيسر و ليس لهذا اللفظ أعنى مطرح الشعاع معنىً يصح غير أحد هذين البابين

الباب الرابع في التسييرات

التسييرات على أربعة أوجه أحدها في السنة الشمسية ثلثة عشر برجاً و هو التسيير الأصغر لأنه أسرع سيراً و الثاني في السنة الشمسية برج واحد و هو التسيير الأوسط و الثالث في السنة الشمسية درجة واحدة مطلعية و هو التسيير الأعظم لأنه أبطأ سيراً و الرابع تسيير

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[9] C om. درجة

[10] C فان instead of و ان

[11] C من instead of بين

[12] C درجة instead of درجته

[13] C الطالع instead of الرابع

[14] Missing in F, retrieved from C

[15] C مقابله instead of يقابله

[16] C مقابله instead of يقابله

[17] C مقابله instead of يقابله

ادلاء[18] التحويل بمثل وسط مسير الشمس و هو التسيير التحويلي فأما الأصغر و الأوسط فقد وضعنا لهما جدولين يؤخذ منهما[19] ما بازاء الشهور و الأيام المفروضة من أجزاء الجدول أو[20] يؤخذ ما بازاء الأجزاء المفروضة من الأيام و الشهور[21] و التسيير التحويلي معلومة[22] من جهة[23] جدول وسط الشمس فأما التسيير الأعظم فنحتاج الى عمل و[24] **حسابه** ان ننظر فان كانت الدرجة التي نريد ان نسيّرها هي درجة العاشر أو الرابع أو درجة كوكب فيهما نقصنا مطالع درجة العاشر أو الرابع بمطالع خط الاستواء من مطالع الدرجة التي نسيّر إليها فما بقي فكل[25] درجة سنة و كل[26] دقيقة ستة أيام فإلى[27] تلك السنين و الأيام تنتهي الدرجة المسيّرة الى ان[28] نسيّر إليها و ان كانت الدرجة التي نسيّرها هي درجة الطالع أو درجة كوكب فيه نقصنا مطالع الطالع بمطالع البلد من مطالع الدرجة التي نسيّر إليها فما بقي فكل درجة سنة و كل دقيقة ستة أيام و ان كانت الدرجة هي درجة الغارب أو درجة كوكب فيه نقصنا مطالع الطالع بمطالع البلد من مطالع نظير الدرجة التي نسيّر إليها فما بقي فكل درجة سنة و كل دقيقة ستة أيام و ان كانت الدرجة التي نسيّرها فيما بين وتدين أخذنا مطالع تلك الدرجة[29] بمطالع خط الاستواء و بمطالع[30] البلد و نضرب الفضل بين المطالعين في ساعات بعد درجة الكوكب من الوتد المتقدم في الطلوع و نقسمه على ستة فما حصل فهو التعديل فان كانت الدرجة[31] فيما بين العاشر و الطالع أو في نظير هذا الربع و كان الفضل لمطالع خط الاستواء نقصنا منه التعديل و الا زدنا عليه التعديل[32] و ان[33] كانت الدرجة فيما بين الطالع و الرابع أو في نظير هذا الربع و كان الفضل لمطالع البلد نقصنا منه التعديل و إلا زدنا عليه

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[18] C om. ادلاء

[19] C منها instead of منهما

[20] C ان instead of أو

[21] F adds a fragment here "from another manuscript" for explaining the application of the relevant tables and provides an example, but it does not seem to be authentic.

[22] C معلوم instead of معلومة

[23] C om. جهة

[24] C om. و

[25] C فهو لكل instead of فكل

[26] C لكل instead of كل

[27] C و الى instead of فإلى

[28] C التي instead of ان

[29] C الدرجات instead of الدرجة

[30] C مطالع instead of بمطالع

[31] C add. التي نسيّرها

[32] C زدناه عليه instead of زدنا عليه التعديل

[33] C فان instead of و ان

التعديل[34] فما كان فهو مطالع الدرجة بحسب موضعها ثم نستخرج مطالع الدرجة التي نسيّر إليها بمثل هذا العمل إلا أنا نستعمل فيها أيضا ساعات بعد الدرجة الأولى[35] التي سيّرناها[36] من الوتد المستعمل من[37] قبل و نستعمل المطالع أيضا كما استعملناه[38] فيه ثم ننقص مطالع الدرجة التي نسيّرها من مطالع الدرجة التي نسيّر إليها فما بقي فلكل درجة سنةً و كل دقيقة ستة أيام فإن كان الزمان معلوماً و نريد ان نعلم أين بلغ الانتهاء[39] من درجة مفروضة في مدة ذلك الزمان **حسابه** ان كانت الدرجة المفروضة هي درجة العاشر أو الرابع أو درجة كوكب فيهما زدنا على مطالعها بخط الاستواء عن الزمان المعلوم لكل سنة درجةً و لكل ستة أيام دقيقةً[40] فما بلغ فنقوسه في مطالع خط الاستواء فما كان فهو الانتهاء[41] من تلك الدرجة و ان كانت الدرجة المفروضة هي درجة الطالع أو الغارب أو درجة كوكب فيهما زدنا على مطالع الطالع عن الزمان المعلوم لكل سنة درجةً و لكل ستة أيام دقيقةً فما بلغ نقوسه في مطالع البلد فما كان فهو الانتهاء[42] من درجة الطالع و نظير هذا الانتهاء[43] هو الانتهاء[44] من درجة الغارب و ان كانت الدرجة المفروضة فيما بين وتدين زدنا على مطالع الدرجة بمطالع خط[45] الاستواء و مطالع البلد عن الزمان المعلوم لكل سنة درجةً و لكل ستة أيام دقيقةً[46] و نقوس كل واحد منهما في مطالعه ثم نأخذ الاختلاف بين القوسين و نضربه في ساعات بعد الدرجة من الوتد المتقدم في الطلوع و نقسمه على ستة فما حصل فهو التعديل فان كانت الدرجة بين[47] العاشر و الطالع أو في نظيره و كان الفضل لقوس مطالع خط الاستواء نقصنا منه التعديل و إلا زدنا عليه التعديل و ان كانت الدرجة فيما بين الطالع و الرابع أو في نظيره و كان الفضل لقوس مطالع البلد نقصنا [نقصنا] منه التعديل و إلا زدنا عليه التعديل فما حصل فهو الانتهاء[48] من تلك الدرجة[49] مثال ذلك الطالع الحوت

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[34] C زدناه عليه instead of زدنا عليه التعديل

[35] C om. الأولى

[36] C سيّرها (؟) instead of سيّرناها

[37] C om. من

[38] C استعملنا instead of استعملناه

[39] C الانتهى instead of الانتهاء

[40] C لكل درجة سنةً و لكل دقيقة ستة أيام instead of لكل سنة درجةً و لكل ستة أيام دقيقةً

[41] C الانتهى instead of الانتهاء

[42] C الانتهى instead of الانتهاء

[43] C الانتهى instead of الانتهاء

[44] C الانتهى instead of الانتهاء

[45] C بخط instead of بمطالع خط

[46] C دقيقة ستة أيام instead of ستة أيام دقيقةً

[47] C فيما بين instead of بين

[48] C الانتهى instead of الانتهاء

[49] C om. This example, but L contains it

أربع درجات العاشر القوس **يه** درجة الزهرة في الجدي **كد** درجة و المريخ في الدلو **ك** درجة سيّرنا الزهرة الى درجة المريخ فانتهت إليها في ثلث و عشرين سنة و مائة و خمسين يوماً و أردنا ان نعلم أين يبلغ الانتهاء من الزهرة عند انقضاء هذا الزمان فكان قد بلغ الدلو عشرين درجة و ثلث و عشرين دقيقة

الباب الخامس في تحاويل السنين و طوالعهما

نحتاج في هذا الباب الى وسط الشمس للتحويل و الى ساعات التقويم و هى التي ينبغي ان نقوم عليها كواكب التحويل و الى ساعات التحويل و طالعه و ليعلم أنا إذا نقصنا سنة الأصل لابتداء ما من السنة التي يقع فيها[50] التحويل من سني يزدجرد كان ما بقي سنين تامة أتت على ذلك الابتداء و التحويل هو دخول السنة القابلة بعودة الشمس الى موضعها الأصلي مثال ذلك ابتداءً اتفق في السنة الثانية و الثلثين و الثلثمائة فأردنا تحويل سنة في السنة التاسعة و الثمانين فنقصنا اثنين و ثلثين[51] من تسعة و ثمانين فيبقى سبعة و خمسون و هى سنون تامة أتت على الابتداء و التحويل دخول السنة الثامنة و الخمسين بعودة الشمس الى موضعها الأصلي[52] **وسط التحويل** فنضع الشمس المقومة للأصل ناحية على التخت ليكون معلوماً ثم نضعها في ثلثة[53] مواضع و ننقص الاوج المعدل لوقت التحويل من الموضع الأول فما بقي فهو الخاصة فنأخذ بها التعديل و ننقصه من الخاصة و من الموضع الثاني و الثالث ثم نأخذ بهذه الخاصة التعديل و نزيده على الموضع الثاني و ننظر فان زاد على تقويم الأصل نقصنا الزيادة من الخاصة و من الموضع الثالث و ان نقص عن تقويم الأصل زدنا النقصان على الخاصة و على الموضع الثالث و نجعل الثاني مثل الثالث ثم نأخذ بهذه الخاصة أيضا التعديل و نزيده على الموضع الثاني و ننظر فان زاد على تقويم الأصل[54] نقصنا الزيادة من الخاصة و من الموضع الثالث و ان نقص عن تقويم الأصل زدنا النقصان على الخاصة و على الموضع الثالث و نجعل الثاني مثل الثالث ثم نأخذ بهذه الخاصة أيضا التعديل و نزيده على الموضع الثاني و ننظر فان زاد على تقويم[55] الأصل نقصنا الزيادة من الخاصة و من

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[50] C om. فيها

[51] C ثلاثين instead of ثلثين

[52] C add. أما

[53] C ثلاثة instead of ثلثة

[54] Beginning of lacuna in C

[55] F om. تقويم

الموضع الثالث و ان نقص عن تقويم الأصل[56] زدنا النقصان على الخاصة و على الموضع الثالث فما حصل من الخاصة في هذه الكرة فهى خاصة التحويل و ما حصل من موضع الثالث فهو وسط التحويل **ساعات التقويم** و أما ساعات التقويم فنستخرج وسط الشمس من الجدول لأول[57] السنة التي يقع فيها التحويل ثم للشهور[58] و الأيام من السنة و للساعات و كسورها حتى تستوي مع وسط التحويل فما حصل من الشهور و الأيام و الساعات فهى ساعات الداير[59] من بعد نصف النهار **ساعات التقويم من ساعات[60] الداير** ان كان البلد اقل طولاً من تسعين[61] أخذنا[62] الفضل بين ساعات ما بين الطولين و ساعات تعديل الأيام و ان[63] كان الفضل لما بين الطولين زدناه على ساعات الداير و ان كان الفضل لتعديل الأيام نقصناه من ساعات الداير فما بلغ أو بقي فهو ساعات التقويم من النهار أو الليل فنعرف بعدها من نصف النهار و ان كان البلد اكثر طولاً من تسعين جمعنا ساعات ما بين الطولين و ساعات تعديل الأيام و نقصنا المبلغ من ساعات الداير فما بقي فهو ساعات التقويم من النهار أو الليل فنعرف بعدها من نصف النهار **فصل في ساعات التحويل**[64] و أما ساعات التحويل فان كان البلد اقل طولاً من تسعين نقصنا ساعات ما بين الطولين من ساعات التقويم و ان كان البلد اكثر طولاً من تسعين زدنا ساعات ما بين الطولين على ساعات التقويم فما بلغ أو بقي زدنا عليها ساعات تعديل الأيام بلياليها فما بلغت فهى ساعات التحويل من بعد نصف النهار[65] فان كانت اقل من ساعات نصف النهار زدناها على ساعات نصف النهار فما بلغت فهى الساعات الماضية من اليوم الذي نحن فيه[66] و ان[67] كانت اكثر من ساعات نصف النهار نقصنا منها ساعات نصف النهار و ما بقي فهو الساعات الماضية من الليلة[68] المقبلة و ان كانت اكثر من ساعات نصف النهار و ساعات الليلة المقبلة مجموعين نقصنا منها ساعات نصف النهار و ساعات[69] الليلة المقبلة و ما بقي فهو الساعات الماضية من اليوم المقبل **الطالع** اى[70] ذلك

[56] End of lacuna in C

[57] F الأول instead of لأول found in C

[58] C الشهور instead of للشهور

[59] C فهو ساعات التقويم instead of فهى ساعات الداير

[60] C om. ساعات

[61] C om. ان كان البلد اقل طولاً من تسعين

[62] C نأخذ instead of أخذنا

[63] C فان instead of و ان

[64] C add. و هو ساعات الداير من النهار و الليل

[65] Beginning of lacuna in C

[66] End of lacuna in C

[67] C فان instead of و ان

[68] C الليلة instead of الليلة

[69] C om. ساعات

حصل ضربناه في خمسة عشر فيكون الداير من الفلك من وقت[71] طلوع الشمس أو غروبها الى وقت التحويل فان كان نهاراً زدناه على مطالع جزء الشمس و ان كان ليلاً زدناه على مطالع نظير جزء الشمس بمطالع البلد فما بلغ قوسناه في جدول المطالع فما كان فهو الطالع و على هذا العمل في حلول الشمس اى جزء أردناه من فلك البروج و عودة الشمس الى موضعها بعد دورة واحدة و زيادة قوس بدور[72] من الفلك مقدارها فو درجة لو دقيقة

الباب السادس في نقل طالع سنة العالم[73] من بلد الى بلد

أخذنا[74] أجزاء ما بين طول البلدين و هو الداير فان كان البلد الثاني اكثر طولاً زدنا الداير على مطالع طالع البلد الأول و ان كان الثاني اقل طولاً نقصنا الداير من مطالع طالع البلد الأول فما بلغ أو بقي قوسناه في مطالع البلد الثاني فما كان فهو الطالع في البلد الثاني

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[70] C فأى instead of اى

[71] C om. وقت

[72] C بدورة instead of بدور

[73] C om. العالم

[74] C نأخذ instead of أخذنا

الفصل الثامن في أعمال يقل الاحتياج إليها عشرة أبواب

الباب الأول في عرض البلد من ساعات النهار الأطول

نضرب نصف ساعات النهار الأطول في خمسة عشر فيكون نصف قوس النهار فيستخرج منه سعة المشرق فيكون أول السرطان ثم نقسم جيب الميل كله على جيب سعة المشرق منحطاً فما حصل فهو جيب تمام عرض البلد **وجه آخر**[1] او نعرف نصف قوس النهار و نقصانه من تسعين او زيادته على تسعين هو[2] تعديل النهار فنقسم ظل ميل الدرجة على جيب تعديل النهار[3] منحطاً فما حصل فهو جيب[4] تمام عرض البلد و يطرد هذا الحساب في ساعات أيام السنة كلها إذا استعملنا ميل الشمس في ذلك اليوم

الباب الثاني في الارتفاع الذي لا سمت له

نقسم جيب ميل الشمس او جيب[5] بعد الكوكب عن معدل النهار على جيب[6] عرض البلد منحطاً فما حصل فهو جيب الارتفاع الذي لا سمت له و يوجد هذا الارتفاع ما دامت الشمس او الكوكب يطلع من ناحية الشمال عن معدل النهار اعني عن مطلع أول الحمل او الميزان و يمر بدايرة نصف النهار في الجنوب عن سمت الرأس

الباب الثالث في سمت اى ارتفاع نفرض

نضرب جيب تمام ميل جزء الشمس في جيب مطالع بعد جزء الشمس و نقسمه على جيب تمام الارتفاع فما حصل فهو جيب تمام السمت فان كانت الشمس في البروج الشمالية و ارتفاع الوقت اقل من الارتفاع الذي لا سمت له فالسمت شرقي او غربي[7] شمالي و ان كان ارتفاع الوقت اكثر من الارتفاع الذي لا سمت له و كان الارتفاع شرقياً او غربياً فالسمت جنوبي و

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[1] C om. وجه آخر

[2] C om. هو

[3] C التعديل instead of تعديل النهار

[4] This جيب is incorrect and the correct word is ظل. See commentary.

[5] C om. جيب

[6] C om. جيب

[7] C و كان الارتفاع شرقياً أو غربياً فالسمت instead of فالسمت شرقي أو غربي

ان كانت الشمس في البروج الجنوبية فالسمت جنوبي و ما اقل ما يحتاج إلى هذا الباب في الكواكب فان احتيج إليه فبعد الكوكب عن معدل النهار بدلٌ من ميل الشمس و درجة ممره بدلٌ من جزء الشمس **وجه آخر** نضرب جيب الارتفاع في جيب عرض البلد و نقسمه على جيب تمام عرض البلد فما حصل فهو حصة السمت فان كان الميل جنوبياً زدنا حصة السمت على جيب سعة المشرق و ان كان الميل شمالياً نقصنا الأقل من الأكثر فما بلغ او بقى فهو تعديل السمت فنقسمه على جيب تمام الارتفاع منحطاً فما حصل فهو جيب السمت ثم ان كانت حصة السمت اكثر من جيب سعة المشرق فالسمت جنوبي و ان كانت[8] اقل منه فالسمت شمالي

الباب الرابع في الارتفاع من السمت

نضرب جيب تمام عرض البلد في جيب تمام السمت منحطاً فما بلغ فهو جيب القوس الأول فنقوسه ثم نقسم جيب عرض البلد على جيب تمام القوس الأول منحطاً فما حصل فهو جيب القوس الثاني و يسمى تمام حصة الارتفاع ثم نضرب جيب ميل الشمس في جيب القوس الثاني و نقسمه على جيب عرض البلد فما حصل فهو جيب القوس الثالث فنقوسه و يسمى تعديل الارتفاع فان كان الميل جنوبياً نقصنا القوس الثالث من تمام القوس الثاني و ان كان الميل او البعد شمالياً زدنا القوس الثالث على تمام القوس الثاني فما بلغ او بقى فهو الارتفاع فاما ان كانت السمت شمالياً فانا ننقص تعديل الارتفاع من حصة الارتفاع ابداً[9] **فايدة** هذين البابين انه اذا اتفقت ولادة مولود في وقت من اوقات النهار و خط على استقامة امتداد ظل شخص قائم على سطح الافق خطاً ثم رصد عود ذلك الظل سمته الأول في اى يوم كان و اخذ عند ذلك ارتفاع الشمس و استخرج السمت من ذلك الارتفاع كان سمت ارتفاع وقت الولادة فيستخرج[10] ارتفاع هذا السمت في يوم الولادة و موضع شمسه فيكون ارتفاع الشمس في وقت الولادة فيستخرج منه الطالع و ما يحتاج إليه

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[8] C كان instead of كانت

[9] Marginal note in C: فايدة و هي حاشية فان كان السمت متناقصاً و هو شرقياً تنقص الحصة من تعديل الارتفاع و إن كان السمت متزايداً و هو شرقياً تزيد الحصة على تعديل الارتفاع يحصل الارتفاع

[10] C و يستخرج instead of فيستخرج

الباب الخامس في البعد بين كوكبين[11] لأحدهما عرض

نضرب جيب تمام الأجزاء التي بين الكوكبين في جيب تمام عرض الكوكب الذي له عـــرض منحطاً فما حصل فهو جيب تمام ما بين الكوكبين[12]

الباب السادس في البعد بين كوكبين ذوى عرض

نضرب جيب تمام عرض الكوكب الأقل في الطول[13] في جيب ما بين الكوكبين من الأجـــزاء منحطاً فما حصل فهو جيب قوس أول فنقوسه ثم نقسم جيب هذا العرض علـــى جيـــب تمـــام القوس الأول منحطاً فما حصل فهو جيب قوس ثان فنقوسه و نزيد عليـــه عـــرض الكوكـــب الأكثر في الطول ان كان العرضان في جهتين و ان كانا في جهة واحدة اخذنا الفضل بين هـذا العرض و القوس الثاني فما كان فهو القوس الثالث ثم نضرب جيب تمام القـــوس الأول فـــي جيب تمام القوس الثالث منحطاً فما حصل فهو جيب تمام ما بين الكوكبين

الباب السابع في استخراج خط نصف النهار

نسوّي موضعاً من الأرض حتى يصير سطحه موازياً للأفق و ندير فيه دايرةً و نغـــرز فـــي المركز إبرة مستوية القامة و نقدر قيامها على السطح من ثلثة مواضع متباعدة على[14] محيـــط الدايرة ثم إذا كان بالقرب من نصف النهار رصدنا رأس ظل الإبرة و هو متناقص بـلن نعلم[15] على مواقعه كما تدور علامات[16] متقاربةٍ جداً برأس إبرة اخرى و نستقصى فيه حتـــى يـــأخذ الظل في الزيادة ثم نصل بين اقرب العلامات إلى المركز و بين المركز بخط مستقيم فيكـــون خط نصف النهار[17] **وجه آخر** و هو ان نسوّى الأرض و الدايرة و الشخص كما قلنـــا إلا ان الدايرة تكون مساوية لدايرة الارتفاع التي على ظهر الأُم من اصطـــرلاب يحضـــر و طـــول

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[11] F الكوكبين instead of كوكبين found in C

[12] C add. و بالله التوفيق

[13] C طولاً instead of طولاً

[14] C عن instead of على

[15] F يعلم instead of نعلم found in C

[16] C علاماة instead of علامات

[17] C add. ثم يجعل مكان الإبرة شخصاً و نعدل قيامه كما عدلنا قيام الإبرة

الشخص بحيث لا ينقص ظله عن محيط دايرة عند[18] نصف النهار ثم نستخرج سمت ارتفاعها عن إحدى جنبتى نصف النهار[19] و نعلم عند وجود ذلك الارتفاع على موقع الظل من محيط الدايرة علامةً[20] و نأخذ من دايرة الارتفاع على الاصطرلاب بالبركار مثل تمام السمت و نضع إحدى رجلي البركار على العلامة و الرجل الاخرى حيث وقعت من محيط الدايرة في[21] جهة الارتفاع شرقياً كان او غربياً و نخرج من موقعه خطا إلى مركز الدايرة فيكون خط نصف النهار فان كان الارتفاع هو الارتفاع الذي لا سمت له كان سمت الظل على خط المشرق و المغرب و الخط الخارج من منتصف نهايتيه[22] إلى مركز الدايرة خط نصف النهار و لاستخراج هذا الخط وجوه كثيرة[23] إلا ان كلها دون هذين الوجهين في الاستقصاء و القرب من الصواب إذا اخذناه من حيث العمل فاما من حيث العلم فكلها صحيحة مبرهنة

الباب الثامن في انحراف البلدان المعلومة الطول و العرض عن نصف[24] النهار بلدنا

هذا الانحراف يسمى سمت البلدان فليكن البلد المطلوب السمت[25] مكه فنضرب جيب تمام عرض مكه في جيب ما بين الطولين منحطاً فما حصل فهو جيب تعديل الطول فنقوسه ثم نقسم جيب عرض مكه على جيب تمام تعديل الطول منحطاً فما حصل فهو جيب تعديل العرض فنقوسه فان كان هذا القوس اقل من عرض بلدنا نقصناه من عرض البلد فما[26] بقي فهو عرض البلد المعدل جنوبياً و ان[27] كان مثله سواء فسمت مكه خط المشرق و المغرب و ان كان اكثر نقصنا منه عرض البلد و ما بقي فهو عرض البلد المعدل شمالياً ثم نضرب جيب تمام تعديل الطول في جيب تمام عرض البلد المعدل منحطاً فما حصل فهو جيب تمام البعد بين البلدين ثم نقسم جيب تعديل الطول على جيب البعد بين البلدين منحطاً فما حصل فهو

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[18] F om. عند which is found in C

[19] C om. ثم نستخرج سمت ارتفاعها عن إحدى جنبتي نصف النهار

[20] C add. و ناخذ من دايرة علامةً

[21] C من instead of في

[22] C نهايته instead of نهايتيه

[23] C كثير instead of كثيرة

[24] C om. نصف

[25] C فليكن السمت المطلوب سمت مكة

[26] C و ما instead of فما

[27] C فان instead of و إن

جيب انحراف مكه **جهة الانحراف** فأما جهة الانحراف فننظر إلى ما بين الطوليـــن و إلـــى عرض البلد المعدل فان وقع ما بين الطولين في الربع الشرقي الجنوبي و عرض البلد المعـدل جنوبي فالانحراف شرقي جنوبي و ان كان عرض البلد المعدل شـــمالياً فـــالانحراف شـــرقي شمالي و ان وقع ما بين الطولين في الربع الغربي الجنوبي و عرض البلد المعـــدل جنوبـــي فالانحراف غربي جنوبي و ان كان عرض البلد المعدل شمالياً[٢٨] فالانحراف غربي شـــمالي و لما عملنا ذلك لبلد الرى على ان طوله من المغرب **فه** و عرضه **لو** و طـــول مكـــه **عـــز** و عرضه **كا** كان الانحراف إلى المغرب **كز لو**

الباب التاسع في ذكر الكواكب الثابتة و علامات[٢٩] بعضها لتعرف بالعيان

انا وضعنا في الجدول من هذه الكواكب[٣٠] ما نحتاج إليها[٣١] في الأكثر و أثبتنا مواضعـــها لأول سنة إحدى و ثلاثمائة ليزدجرد و تعديلها[٣٢] تعديل الأوجات و وضعنا بازائـــها عروضـــها و مقاديرها و مزاجاتها من[٣٣] السيارة[٣٤] و لانا نحتاج إلى معرفة كوكبٍ و[٣٥] كوكبين بالعيان فـــي كل ربع و أرباع فلك البروج لنأخذ ارتفاعها بالليل لمعرفة الطالع و الوقت ذكرنا علامـــات[٣٦] بعضها لتمييز للناظر إليها[٣٧] فمن ذلك **الكف[٣٨] الخضيب** كوكب في الحمل من القدر الثالث فـــي الشمال على سنام الصورة المعروفة عند العامة بالناقة و تحته كوكبان من قدره هما و[٣٩] هـــذا الكوكب على صورة مثلثة **عين الثور** و يسمى الدبران كوكب احمر في الثـــور مـــن القـــدر الأول في الجنوب خلف الثريا فيما بين كواكب على صورة الدال **العيوق** كوكب كبـــير فـــي الجوزاء من القدر الأول في الشمال على طرف المجرة خلف ثلثة[٤٠] كواكب مصطفة[٤١] يطلـــع

---

[28] C شمالي instead of شمالياً

[29] C علامة instead of علامات

[30] C om. من هذه الكواكب

[31] C إليه instead of إليها

[32] C (؟) تعدليها instead of تعديلها

[33] C من instead of إلى

[34] C add. و اما حركاتها و هي مثل حركة الأوجات

[35] C آو instead of و

[36] C علامة instead of علامات

[37] C om. اليها

[38] C كوكب instead of الكف

[39] C من قدرها و instead of كوكبان من قدره هما و

[40] C ثلاثة instead of ثلثة

مع الثريا **منكب الجوزاء**[42] كوكب احمر في الجوزاء من القدر الأول في الجنوب و هو في موضع المنكب[43] من صورة إنسان قائم **الشعرى اليمانية** كوكب ابيض كبير في أول السرطان من القدر الأول في الجنوب خلف كواكب الجوزاء **الشعرى الشامية** كوكب[44] في السرطان من القدر الأول في الجنوب و هي دون اليمانية في الكبر و شمالياً[45] عنه بازائه **قلب الأسد** كوكب في الأسد من القدر الأول في المنطقة بالتقريب[46] و على الطرف الجنوبي من أربعة[47] كواكب معترضة من الجنوب إلى الشمال على سطر منعوج[48] **الصرفه** و تسمى ذنب الأسد كوكب في السنبلة على ذنب الأسد من القدر الأول في الشمال بينه و بين قلب الأسد كوكبان نيّران يسميان الزبره **السماك الرامح** كوكب في الميزان من القدر الأول في الشمال امامه إلى المغرب كوكب اصغر منه به سمى الرامح[49] **السماك الأعزل** كوكب في الميزان من القدر الأول في الجنوب بازاء الرامح **المنير من الفكه** كوكب في[50] الميزان من القدر الثاني في الشمال فيما بين كواكب مستديرة خلف السماك الرامح تسميها العامة قصعة المساكين **قلب العقرب** كوكب احمر في العقرب من القدر الثاني في الجنوب فيما بين كوكبين نيرين على خط فيه تقويس **النسر الواقع** كوكب في آخر القوس من القدر الأول في الشمال مجراه قريب من سمت الرأس تحته كوكبان صغيران هما و هذا الكوكب على صورة مثلثة تسميها العامة الاثافي **النسر الطائر** كوكب في الجدى من القدر الثاني في الشمال ما بين كوكبين نيرين على خط مستقيم **ذنب الدجاجة** و يسمى[51] الردف كوكب في الدلو من القدر الثاني في الشمال خلف كواكب[52] نيّرة يقطعن المجرة عرضاً **منكب الفرس**[53] كوكب في الحوت[54] من القدر الثاني في الشمال شمالي عن كوكب آخر من قدره يسميان الفرغ المقدم من منازل القمر

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[41] C add. و

[42] C add. اليمنى

[43] C منكب instead of المنكب

[44] C om. كوكب

[45] F شمالية instead of شمالياً found in C

[46] C في التقريب instead of في المنطقة بالتقريب

[47] C أربع instead of أربعة

[48] C عن صدر متعرج instead of على سطر منعوج

[49] C يسمى الرمح instead of به سمى الرامح

[50] C من instead of في

[51] C تسمى instead of يسمى

[52] C كوكب instead of كواكب

[53] F منكب القوس instead of منكب الفرس found in C

[54] C الجنوب instead of الحوت

الباب العاشر في أسماء منازل القمر و أيام طلوعها[55]

هذه المنازل[56] ثمانية و عشرون[57] و أسماؤها على الترتيب[58]

الشرطين[59] **ا**	البطين **ب**	الثريا **ج**	الدبران **د**	الهقعه **ه**
ك من نيسان	**ج**[60] من أيار	**يو** من أيار	**كط** من أيار	**يا** من حزيران
الهنعه **و**	الذراع **ز**	النثره **ح**	الطرفه[61] **ط**	الجبهه **ى**
كه[62] من حزيران	**ح**[63] من تموز	**ك** من تموز	**ب** من آب	**يه** من آب
الزبره[64] **يا**	الصرفه **يب**	العوّا **يج**	السماك **يد**	الغفر **يه**
كح من آب	**ى** من ايلول	**كج** من ايلول	**و** من تشرين الأول[65]	**ك** من تشرين الأول[66]
الزباني[67] **يو**	الإكليل **يز**	القلب **يح**	الشوله **يط**	النعائم **ك**
ب من تشرين الثاني[68]	**يه** من تشرين الثاني[69]	**كح** من تشرين الثاني[70]	**يا** من كانون الأول[71]	**كد** من كانون الأول[72]

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55 C طولعها (؟) instead of طلوعها
56 C المنازل عددها instead of هذه المنازل
57 C add. منزلة
58 C add. في الجدول الآتي في الصفحة الآتية و الله اعلم
59 F الشرطان instead of الشرطين
60 C د instead of ج
61 C الطرف instead of الطرفه
62 C كد instead of كه
63 C ز instead of ح
64 C الخردان (؟) زبره instead of الزبره
65 C أول instead of الأول
66 C أول instead of الأول
67 C الزبانا instead of الزباني
68 C ثاني instead of الثاني
69 C ثاني instead of الثاني
70 C ثاني instead of الثاني
71 C أول instead of الأول
72 C أول instead of الأول

البلده **كا**	سعد الذابح **كب**	سعد بلع **كج**	سعد السعود **كد**	سعد الاخبيه **كه**
و من كانون الثاني[٧٣]	**يط** من كانون الثاني[٧٤]	ا من شباط	**يد** من شباط	**كز** من شباط

الفرغ المقدم **كو**	الفرغ المؤخر **كز**	بطن الحوت **كح**
يب من آذار	**كه**[٧٥] من آذار	ز من نيسان

[٧٦]أقسام هذه المنازل من دايرة فلك البروج متساوية <و> مأخوذة من النقطة التـــي هـــي أول الحمل و صور ها من الكواكب الثابتة مختلفة المقدار و المواضع من فلك البروج فأمـــا أيـــام طلوعها اعني ظهور ها من تحت الشعاع فان الشرطين يطلع في حدود سنة الف و ثلاثمائـة و عشرين لذي القرنين في العشرين من نيسان ثم كل ثلثة عشر يوماً طلوع منزله اخرى حتـــى إذا طلع السماك اخذنا لطلوع الغفر بعده أربعة عشر يوماً لجبر الكسور التي مع الثلثة عشـــر يوماً ثم إلى آخر المنازل على الرسم الأول و بعد ست و ستين سنة يطلـــع الشـــرطين[٧٧] فـــي الحادي و العشرين من نيسان و يتأخر كل منزله كذلك بيومٍ على هـــذا النســـق و إذا طلعـــت منزله غابت نظيرتها و هي الخامسة عشر منها كما انه إذا طلع الشرطين غـــاب الغفـــر و لا يبعد ان يكون بين ظهور ها بالحقيقة و بين الذي حددناه يوم و يومان فليس في ذلك من دقيـــق الأرصاد ما نريد الاختلاف بالكلية لما لم تدعهم الضرورة إلى تحقيقه و من بعد ما وفينا بمـــا أثبتنا في صدر المقالة من الأبواب و بذلنا في تقريبها و الاستقصاء في تحقيقها الإمكـــان فانـــا نختم المقالة الأولى بهذا الباب و بالله الاستعانة و عليه التكلان و يتلوها المقالـــة الثانيـــة فـــي الجداول **[و فرغ من نسخها محمود بن احمد بن الحسين في الليلة ه من شهر رمـــضان ســـنة ثمه للهجرة]**

[73] C ثاني instead of الثاني

[74] C ثاني instead of الثاني

[75] C كد instead of كه

[76] From here to the end of this chapter/section is missing in C. However, C contains in one page miscellaneous fragments from Kūshyār and from other sources. At the end of this section in C, we read:

و كان الفراغ من تسويده في اليوم العاشر من الشهر السابع من السنة التاسعة من العشر السابع من المائة الثاني(الثانية؟) من الألف الثاني من الهجرة النبوية على صاحبها افضل الصلاة و أتم التحية و سلم تم

[77] F شرطان instead of شرطين

بسم اللّه الرحمن الرحيم[1]

المقالة الرابعة من الزيج الجامع

قال الكيا ابو الحسن[2] كوشيار بن لبان بن باشهري الجيلي نور الله ضريحه[3] و لما فرغت من المقالة الثالثة في علم الهيئة بدأت بالمقالة الرابعة هذه في البرهان على ترتيب[4] ابواب المقالة الاولى[5] فالبرهان[6] الهندسي دليل لايقبل في معني الصحة الزيادة و النقصان فيتساوي في معرفة المدلول و العلم به كل عارف بذلك البرهان و هذه المقالة آخر مقالات الكتاب و سألت الله في اتمامها العصمة و الكفاية و التوفيق و الهداية انه ولي ذلك[7]

ترجمة الابواب و هى ثمانية[8] فصول و سبعون[9] باباً

الفصل الاول في الاوتار و الجيوب: احد عشر باباً

ا في صفة الوتر و الجيب

ب في وجود كمية[10] وتر تمام القوس اذا كان وتر القوس معلوماً

ج في وجود كمية وتر الربع

د في وجود كمية وتر الثلث

ه في وجود كمية وتر العشر و الخمس

---

[1] V add. العزة لله الحمدلله على آلائه و له الشكر على نعمائه و السلام على خاتم انبيائه محمد و اوليائه

[2] V om. الكيا ابو الحسن

[3] V om. نور الله ضريحه

[4] V om. ترتيب

[5] V add. و على ذلك الترتيب

[6] F و البرهان instead of فالبرهان found in V

[7] F add. و القادر عليه

[8] V ثمنية instead of ثمانية

[9] F ستون and V ستة و ستون instead of سبعون

[10] F om. كمية

و في مقدمة لما بعد

ز في وجود كمية وتر[11] فضل ما بين قوسين معلومى الوتر[12]

ح في وجود كمية وتر نصف قوس معلوم الوتر

ط في وجود كمية وتر مجموع قوسين معلومى الوتر[13]

ى في مقدمة لما بعد

يا في تقدير وتر جزء واحد و تركيب الاوتار

الفصل الثاني في الاظلال: ثلثة ابواب

ا في صفة الظل الاول و الثاني

ب في وجود الظل الاول

ج في وجود الظل الثاني

الفصل الثالث في مقدمات يستند اليها[14] البراهين: سبعة[15] ابواب

ا في مقدمة كلية لاكثر البراهين[16]

ب في مقدمة اخرى هى من فروع المقدمة الاولى

ج في تذكرة من[17] خواص المقادير المتناسبة

د في مقدمة اخرى هى من فروع المقدمة الاولى[18]

ه[19] في مقدمة يتعلق بالظل تنوب عن المقدمة الاولى في كثير من البراهين

و في تذكرة من خواص الظل

ز في تذكرة اخرى هى ايضاً من خواص الظل[20]

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[11] F om. وتر

[12] V معلوم الوترين instead of معلومى الوتر

[13] V معلوم الوترين instead of معلومى الوتر

[14] V om. اليها

[15] V اربعة instead of سبعة

[16] V البرهان instead of البراهين

[17] V om. من

[18] V om. this chapter in this list

[19] V د instead of ه

[20] V om. the last two chapters in this list

الفصل الرابع في تقويم الكواكب و احوالها: عشرة[21] ابواب

ا[22] في تعديل الايام بلياليها

ب في تعديل الشمس

ج في التعديل الاول للقمر

د في التعديل الثاني للقمر و الكواكب

ه في اختلاف نصف قطر فلك التدوير فيما بين البعد الابعد و الاقرب

و في التعديل الاول لعطارد

ز في التعديل الاول لباقي الكواكب

ح في عرض القمر

ط في عروض الكواكب

ى في رجوع الكواكب

الفصل الخامس في اعمال طوالع النهار و الليل: ستة عشر باباً

ا في الميل الاول

ب في مطالع البروج بخط الاستواء

ج في الميل الثاني

د في بعد الكواكب[23] عن معدل النهار

ه في عرض البلد

و في سعة مشرق الشمس و الكواكب[24]

ز في تعديل نهار الشمس و الكوكب

ح في مطالع البلد

ط في غاية الارتفاع الشمس و الكوكب

ى في نصف قوس نهار الشمس و الكوكب

يا في درجة ممر الكوكب بنصف النهار

يب في درجة طلوع الكوكب و غروبه

يج في الداير من الفلك لطلوع الشمس و الكوكب من ارتفاع الوقت و الارتفاع من [من] الداير

[21] V تسعة instead of عشرة

[22] V om. ا and hence the following chapters in this section are numbered one less in this list in V

[23] V الكوكب instead of الكواكب

[24] V الكوكب instead of الكواكب

يد في الطالع من الداير و الداير من الطالع
يه في البرهان على اصل يقم الداير و ما يتعلق به
يو في تسوية البيوت

الفصل السادس في الكسوفات و ما يتعلق[25] بها: اربعة[26] عشر باباً
ا في اصابع خسوف القمر مطلقة و معدلة
ب في ازمان الخسوف مطلقة
ج في تعديل الازمان
د في تصوير الخسوف
ه في بعد القمر من الارض
و في ارتفاع قطب فلك البروج
ز في ارتفاع اية[27] درجة نريد[28] من درجات فلك البروج
ح في اختلاف منظر النيرين من دايرة الارتفاع
ط في الزوايا الست التي يحتاج اليها في الكسوفات الشمسية
ى في اختلاف منظر القمر طولاً و عرضاً من هذه الزوايا
يا في تصوير الكسوف
يب في ارتفاع القمر بحسب عرضه
يج في اختلاف منظر القمر طولاً و عرضاً بطريقة مبرهنة
يد[29] في قوس الرؤية

الفصل السابع فيما يتعلق بالاحكام و هو باب واحد
ا في مطرح الشعاع بحسب عرض الكوكب

---

[25] V يليق instead of يتعلق
[26] V خمسة instead of اربعة
[27] V اى instead of اية
[28] V تريد instead of نريد
[29] V add. في البعد بين الكوكبين عرض لاحدهما

الفصل الثامن في اعمال يقل الاحتياج اليها: ثمانية[30] ابواب

ا في عرض البلد من ساعات النهار الاطول و الاقصر

ب في ارتفاع الذي لا سمت له

ج في سمت اى ارتفاع يفرض

د في الارتفاع من السمت

ه في البعد بين كوكبين لاحدهما عرض

و في البعد بين كوكبين ذوي عرض[31]

ز في استخراج خط نصف النهار

ح في انحراف البلدان عن نصف نهار بلدنا

هذه الابواب كافية في براهين ابواب المقالة الاولى لان ما عساه شذ عنها فيبرهن لمن تقدم في[32] الهيئة و الهندسة بمؤنة خفيفة و فكرة قريبة و الله الموفق و المعين

[30] V سبعة instead of ثمانية

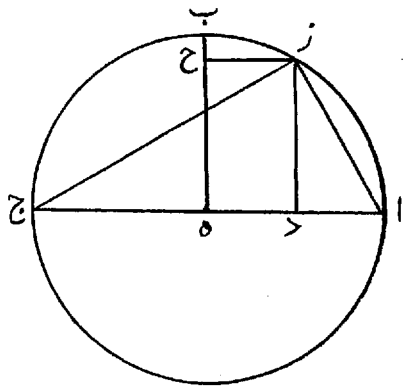
[31] V ذي عرضين instead of ذوي عرض

[32] V لمن تقدمته instead of تقدم في

الفصل الاول في الاوتار و الجيوب احد عشر باباً

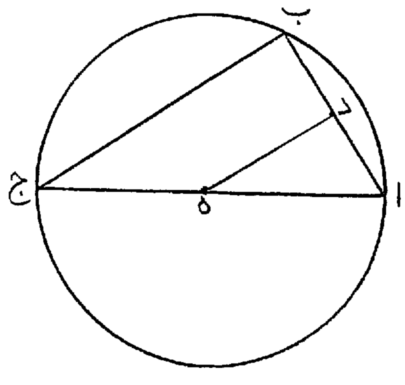
الباب الاول في صفة الوتر و الجيب

اب ج دايرة و[1] مركزها ه و قطرها اج و نخرج ه ب على زاوية قائمة و نفرض قوس از و نصل خط از و نخرج زد عموداً على اج و زح عموداً على ب ه و نصل زج فخط از وتـــر قوس از و زج وتر تمامها و زد جيب قوس از و زح جيب تمامه[2] و هو مساوٍ لخط ده و اد سهم قوس از و ب ح سهم قوس زب و قوس زب تمام قوس از من ربع دايرة و قوس زب ج تمام قوس از من نصف دايرة و ذلك ما اردنا ان نصف



الباب الثاني في وجود كمية وتر تمام القوس اذا كان وتر القوس معلوماً

لتكن اب ج دايرة و قطرها اج و نفصل[3] منها قوس اب و نصل خطى اب ب ج و نجعل وتر اب معلوماً فاقول ان وتر ب ج معلوم



[1] V om. و

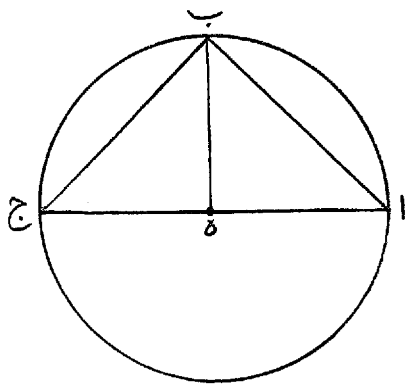
[2] V تمامها instead of تمامه

[3] V نفرض instead of نفصل

برهانه زاوية **اب ج** قائمة لانها في نصف الدايرة فمربع **اج** مثل مربعى **اب ب ج** فاذا نقصنا مربع **اب** من مربع **اج** بقى مربع **ب ج** معلوماً و جذره[4] هو وتر **ب ج** معلوم و ذلك ما اردنا ان نبين و هنا لك استبان ان نسبة كل وتر الى قطر الدايرة كنسبة جيب نصف قوس الوتر الى نصف قطر الدايرة و ذلك انا اذا قسمنا وتر **اب** بنصفين على **د** و وصلنا **ده** و **ه** مركز الدايرة كان **ده** موازياً[5] لـ**ب ج** و صار **اد** جيب نصف قوس **اب** فنسبة **ب ا** الى **اج** كنسبة **دا** الى **اه** فكل حساب يحسب على الوتر و القطر فهو مطرد[6] على جيب نصف قوس الوتر و نصف القطر و ذلك ما اردنا ان نبين[7]

الباب الثالث في وجود كمية وتر الربع

لتكن **اب ج** دايرة مركزها **ه** و قطرها **اج** و نخرج **ه ب** على زاوية قائمة و نصل **اب ب ج** فكل واحدة من قوسى[8] **اب ب ج** ربع دايرة و كل واحد من خطى **اب ب ج** وتر الربع فاقول انهما معلومان



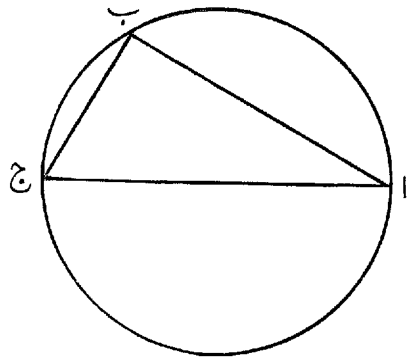
برهانه زاوية **اه ب** قائمة فمربع **اب** مثل مربعى **اه ه ب** و كل واحد من **اه ه ب** نصف القطر فمجموع مربعهما[9] معلوم و جذره معلوم فوتر **اب** معلوم و ذلك ما اردنا ان نبين و

[4] V فجذره instead of و جذره
[5] V موازٍ instead of موازياً
[6] V يطرد instead of مطرد
[7] V نذكر instead of نبين
[8] F قوس instead of قوسى found in V
[9] V مربعيهما instead of مربعهما

هنالك استبان ان مربع وتر الربع مثلا مربع نصف القطر و مربع القطر اربعة امثال مربع نصف القطر لان مربع **اج** مثل مربعى **اب ب ج** و كل واحد من مربعى **اب ب ج** مثلا مربع **اه** فمربع **اج** اربعة امثال مربع **اه** و ذلك ما اردنا ان نبين[10]

الباب الرابع في وجود كمية وتر الثلث

لتكن **اب ج** دايرة و قطرها **اج** و نصل **ب ج** مثل نصف قطر الدايرة و هو وتر السدس و نصل **اب** فاقول ان وتر الثلث معلوم



برهانه زاوية **اب ج** قائمة لانها في نصف الدايرة فمربع **اج** مثل مربعى **اب ب ج** و مربع **اج** معلوم و مربع **ب ج** و هو وتر السدس معلوم فمربع **اب** الباقي من مربع **اج** معلوم فجذره معلوم و هو وتر **اب** فوتر **اب** معلوم و ذلك ما اردنا ان نبين و هنالك استبان ان مربع وتر الثلث ثلثة امثال مربع نصف القطر[11] و وتر **ب ج** مثل نصف القطر فاذا نقص من مربع **اج** مربع **ب ج** بقي من[12] مربع **اج** ثلثة امثال مربع نصف القطر و هو مثل مربع وتر **اب** و ذلك ما اردنا ان نصف[13]

الباب الخامس في وجود كمية وتر العشر و الخمس

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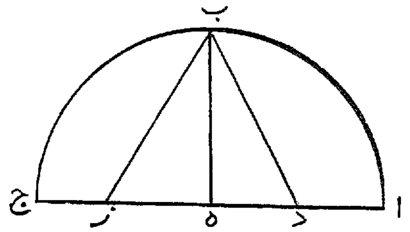
[10] V تذكر instead of نبين

[11] V add. و ذاك ان مربع القطر اربعة امثال مربع نصف القطر

[12] F om. من found in V

[13] V om. و ذلك ما اردنا ان نصف

لتكن اب ج نصف دايرة و مركزها ه و قطرهـا اج و ه ب عمـود علـى اج[14] و نقسـم اه بنصفين على د و نصل ب د و نجعل دز مثل ب د فاقول ان ه ز مساو لوتر عشر الـدايرة و ب ز مساو لوتر خمسها



برهانه اه قسم بنصفين على د و زيد فيه ه ز فضرب از في زه مع مربع ده مساو لمربع دز و دز مثل دب و مربع دب مثل مربعي ده ه ب فضرب از في زه مع مربع ده مثل مربعي ده ه ب فتلقى مربع ده المشترك فيبقى[15] ضرب از في زه مثل مربع ه ب و ه ب مثـل ه ا فـاز مقسوم[16] على نسبة ذات وسط و طرفين و قسمة الاعظم اه و اه وتـر السـدس فـه ز وتر العشر و لان مربعي ب ه ه ز مثل مربع ب ز و ب ه وتر السدس و ه ز وتر العشر[17] ب ز وتر الخمس و ذلك ما اردنا ان نبين

الباب السادس في مقدمة لما بعد

كل ذي اربعة الاضلاع يحيط به دايره فان ضرب اضلاعه المتقابلة كل واحد منهما في الذي يقابله اذا جمع مساو لضرب قطريه احدهما في الآخر فلتكن دايرة اب ج[18] فيهـا ذو اربعـة الاضلاع اج ب د[19] فاقول ان ضرب اب في ج د و اد في ج ب اذا جمع كان مثل ضرب اج في ب د

برهانه انا نجعل زاوية دج ه مثل زاوية ب ج ا و لان زاوية دج ه مثل زاويـة ب ج ا[20] و زاوية اج ه مشتركة تكون زاوية دج ا مثل زاوية ب ج ه و زاوية ج اد مثل زاويـة ج ب د لانهما على قوس ج د فيبقى زاوية ادج مثل زاوية ب ه ج[21] فنسبة ج ب الى ب ه كنسبة ج ا الى اد فضرب ج ب في اد مثل ضرب ج ا في ب ه و ايضاً زاوية دج ه مثل زاوية ب ج ا و زاوية ج دب مثل زاوية ج اب لانهما على قوس ب ج فيبقى زاوية ج ه د مثل زاويـة اب

---

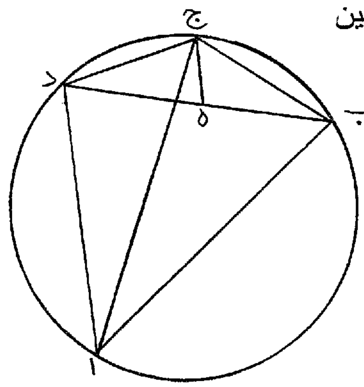
[14] V عيه instead of على اج

[15] V يبقى instead of فيبقى

[16] V repeats مقسوم

[17] F add. و

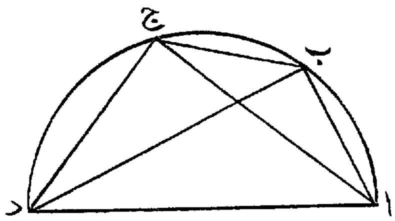
ج فنسبة **ج د** الى **ده** كنسبة **ج ا** الى **اب** فضرب **ج د** في **اب** مثل ضرب **ج ا** في **ده** و قد تبين ان ضرب **ج ب** في **اد** مثل ضرب **ج ا** في **ب ه** فضرب **اج** في **ب د** مثل ضرب **ج ب** في **اد** و **ج د** في **اب** و ذلك ما اردنا ان نبين



الباب السابع في وجود كمية وتر فضل ما بين قوسين معلومى الوتر[22]

فلتكن[23] **اب ج د** نصف دايرة و قطرها **اد** و وترا[24] **اب اج** فيها معلومان[25] و نصل **ب ج** فاقول ان **ب ج** معلوم

برهانه انا نصل **ب د ج د** فانهما[26] معلومان لانهما وترا تمامى[27] **اب اج** فعلى ما تبين في المقدمة ضرب **اج** في **ب د** مثل مجموع ضرب **اب** في **ج د** و **اد** في **ج ب**[28] و ضرب **اج** في **ب د** معلوم و قطر **اد** معلوم فوتر **ب ج** معلوم و ذلك ما اردنا ان نبين



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18 V اب ج د instead of اب ج

19 V اب ج د instead of اج ب د

20 F om. و لان زاوية د ج ه مثل زاوية ب ج ا found in V

21 Marginal note in V: يوجب ان يكون مثلث ب ه ج مشابه لمثلث اد ج

22 V معلوم الوترين instead of معلومى الوتر

23 V لتكن instead of فلتكن

24 F and V وتر instead of وترا found in L

25 V معلومين instead of معلومان

26 V فهما instead of فانهما

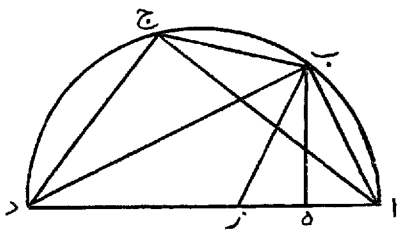
27 V تمام instead of تمامى

28 V ب ج instead of ج ب

الباب الثامن في وجود كمية وتر نصف قوس معلوم الوتر

لتكن **اب ج د** نصف دايرة قطرها **اد** و نفرض وتر **اج** معلوماً و نقسم قوس **اج** بنصفين على **ب** و نصل **اب ب ج** فاقول ان **اب** معلوم

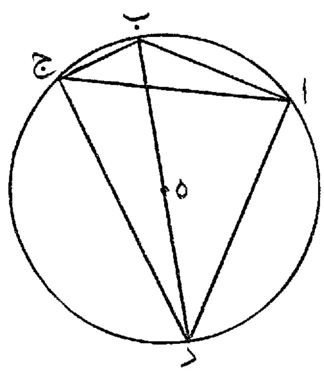
برهانه انا[29] نصل **ج د** و نجعل **دز** مثل **ج د** و نصل **ب د ب ز** و نخرج **ب ه** عموداً على **از** فـ**ج د** مثل **دز** و **دب** مشترك فـ**ج د دب** مثل **زد دب** و زاوية **زدب** مثل زاوية **ب دج** لانهما على قوسين متساويتين فقاعدة **ب ج** مثل قاعدة **ب ز** و **اب** مثل **ب ج** فـ**اب** مثل **ب ز** فمثلث **اب ز** متساوي الساقين و خرج من زاوية **اب ز** عمود **ب ه** فـ**اه** مثل **ه ز** و لان مثلث **اب د** قائم الزاوية و خرج من زاويته القائمة عمود **ب ه** فمثلثا **اب د اب ه** متشابهان فنسبة **دا** الى **اب** كنسبة **ب ا** الى **اه** فضرب **دا** في **اه** مثل مربع **اب** و كل واحد من **دا اه** معلوم فمربع **اب** معلوم فجذره و هو وتر **اب** معلوم و ذلك ما اردنا ان نبين



الباب التاسع في وجود كمية وتر مجموع قوسين معلومى الوتر[30]

لتكن **اب ج د** دايرة مركزها **ه** و نفرض فيها وترى **اب ب ج** معلومين و نصل **اج** فاقول ان **اج** معلوم

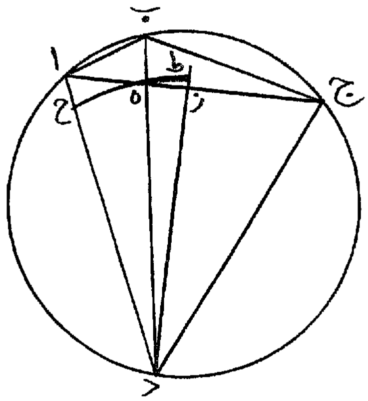
برهانه انا نخرج من **ب** قطر **ب د** و نصل **اد دج** فـ**اد** وتر تمام **اب** و **ج د** وتر تمام **ب ج** و هما معلومان فضرب **اب** في **ج د** و **ب ج** في **اد** مثل ضرب **ب د** في **اج** و كل واحد من **اب ج د ب ج اد** معلوم و قطر **ب د** معلوم فوتر **اج** معلوم و ذلك ما اردنا ان نبين



[29] V ان instead of انا

[30] V معلوم الوترين instead of معلومى الوتر

اذا كان في دايرة وتران غير متساويين فان نسبة الوتر الاعظم الى الوتر الاصغر اقل من نسبة قوس الوتر الاعظم الى قوس الوتر الاصغر فلتكن دايرة عليهما اب ج د و فيها وترا اب ب ج و ب ج اعظمها فاقول ان نسبة وتر ب ج الى وتر ب ا اقل من نسبة قوس ب ج الى قوس ب ا



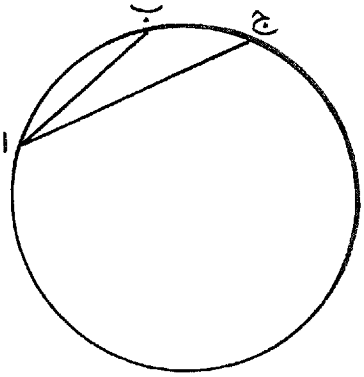
برهانه انا نقسم زاوية اب ج بنصفين بخط ب د و نصل اج اد ج د فلان زاوية اب ج قسمت بنصفين بخط ب د يكون خط ج د مثل خط اد و خط ج ه اطول من خط ه ا و نخرج من د الى خط اج عمود دز فلان اد اطول من ه د و ه د اطول من دز تكون الدايرة المخطوطة على مركز د و ببعد ده تقطع اد و تجوز دز فنرسم عليها ح ه ط و نخرج دز الى ط فلان قطاع ده ط اعظم من مثلث ده ز و مثلث ده ا اعظم من قطاع ده ح تكون نسبة قطاع ده ط الى قطاع ده ح اعظم من نسبة مثلث ده ز الى مثلث ده ا و نسبة مثلث ده ز الى مثلث ده ا كنسبة خط ه ز الى ه ا و نسبة قطاع ده ط الى قطاع ده ح كنسبة زاوية زده الى زاوية ه دا فنسبة خط زه الى خط ه ا اقل من نسبة زاوية زده الى زاوية اده[31] فاذا ركّبنا كانت نسبة[32] خط زا الى خط ه ا اقل من نسبة زاوية زدا الى زاوية اده و نسبة الانصاف كنسبة الاضعاف فنسبة ضعف از و هو ج ا الى اه اقل من نسبة ضعف زاوية زدا و هو زاوية ج دا الى زاوية اده و اذا فصّلنا فنسبة خط ج ه الى ه ا اقل من نسبة زاوية ج ده الى زاوية ه دا و نسبة ج ه الى ه ا كنسبة وتر ج ب الى وتر ب ا و نسبة زاوية ج دب الى زاوية ب دا كنسبة قوس ج ب الى قوس ب ا فنسبة وتر ج ب الى وتر ب ا اقل من نسبة قوس ج ب الى قوس ب ا و ذلك ما اردنا ان نبين

[31] V ه دا instead of اده

[32] V و اذا ركبنا فنسبة instead of فاذا ركبنا كانت نسبة

الباب الحادي عشر في تقدير وتر جزء واحد باقرب قرب و تركيب الاوتار

قد تبين من الباب السابع[33] كيف تعرف وتر[34] فضل ما بين سدس الدايرة و خمسها و هو وتر اثنا عشر جزءاً و من الباب الثامن وتر نصفه و نصف نصفه حتى ينتهي الى وتر جــزء و نصف جزء و وتر نصف و ربع جزء[35] و من بعد ذلك فانا نخط دايرة عليها **اب ج** و نجعل خط **اب** اولا يوتر من الدايرة قوس نصف و ربع جزء و خط **اج** وتر قوس جزء واحد فنسبة



وتر **اج** الى وتر **اب** اقل من نسبة قوس **اج** الى قوس[36] **اب** و قوس **اج** مثل و ثلث قــوس **اب** فوتر[37] **اج** اقل من مثل و ثلث وتر **اب** و مثل و ثلث وتر **اب** **ا ب مط نب** و ايضاً[38] نجعــل في هذه الدايرة خط **اب** وتر قوس جزء واحد و خط **اج** وتر قوس جزء و نصــف جــزء[39] فقوس **اج** مثل و نصف قوس **اب** فوتر **اج** اقل من مثل و نصف وتر **اب** فوتر **اب** اعظم مــن ثلثي وتر **اج** و ثلثا[40] وتر **اج** **ا ب مط مح** فاذا كان وتر الجزء الواحد مرة اقل و مرة اكثــر من شئ واحد بعينه كان ذلك التفاوت مما لا قدر له فاذا اخذ نصف التفاوت و زيد على الاقل حصل وتر جزء واحد باقرب تقريب **ا ب مط ن** و من بعد ما عرفنا ذلك فقد تبين من البـــاب التاسع وتر مجموع قوسين فوتر الجزء معلوم فوتر مجموع الجزؤين معلوم و ايضـــاً وتـــر الجزء معلوم[41] و[42] وتر الجزؤين معلوم فوتر ثلثة اجزاء معلوم و ايضاً وتر الجزء معلوم و

---

33 F الخامس instead of السابع

34 F om. وتر

35 F om. جزء و وتر نصف و ربع جزء

36 F om. قوس

37 V فقوس instead of فوتر

38 F انا instead of ايضاً

39 F om. جزء

40 V ثلثي instead of ثلثا

41 F om. فوتر مجموع الجزؤين معلوم و ايضاً وتر الجزء معلوم

42 V om. و

وتر الثلثة الاجزاء معلوم فوتر اربعة اجزاء معلوم و على هذا نركّب اوتار الاجزاء الــى[43] تسعين جزؤاً و نضعها في الجدول و ذلك ما اردنا ان نبين[44]

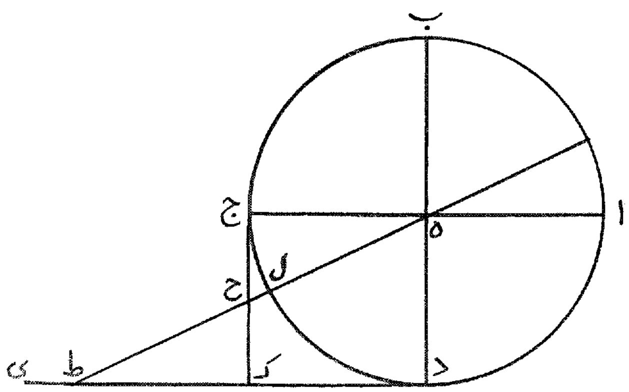
[43] F التي instead of الى

[44] V add. و نختم الفصل الاول بهذا الباب و الله محمود

الفصل الثاني في الاظلال ثلثة ابواب

الباب الاول في صفة الظل الاول و الثاني

لتكن[1] **اب ج د** دايرة الارتفاع و مركزها **ه** و **دى** الفصل المشترك بين سطح دايرة الارتفاع و دايرة الافق و **ده** المقياس القائم[2] على زوايا قائمة عند نقطة **د** و **ج ك** الفصل المشترك بين سطح دايرة الارتفاع[3] و السطح القائم على الافق على زوايا قائمة و **ج ه** المقياس الموازي لسطح الافق قائم على السطح المذكور على زوايا قائمة عند نقطة **ج** و نفرض **از** قوس الارتفاع و نصل **زه ط** و هو الشعاع الواصل بين رأس المقياس و طرف الظل و **دط**[4] ظل مقياس[5] **ده** و هو الظل المستوي و الظل الثاني لارتفاع **از** و **ج ح** ظل مقياس **ج ه** و هو الظل المعكوس و الظل الاول لارتفاع **از** و اذا فرضنا الارتفاع **ب ز** كان المقياس الظل



المستوي[6] **ج ه** و مقياس الظل المعكوس **ده** فيكون **دط** الظل الاول لارتفاع **ب ز** و **ج ح** الظل الثاني له و **ب ز** تمام **از** فالظل الاول لكل ارتفاع هو الظل الثاني لتمام ذلك الارتفاع و الظل الثاني لكل ارتفاع هو الظل الاول لتمام ذلك الارتفاع[7] و سمى الظل المعكوس اولاً لانه يبتدى بالظهور و الزيادة مع ابتداء ارتفاع الشمس و زيادته و الظل الثاني يتناقص بزيادة الارتفاع و ذلك ما اردنا ان نبين[8]

---

[1] V add. دايرة

[2] V add. عليه

[3] F and V erroneously الافق instead of الارتفاع found in Y

[4] V فدط instead of ودط

[5] V المقياس instead of مقياس

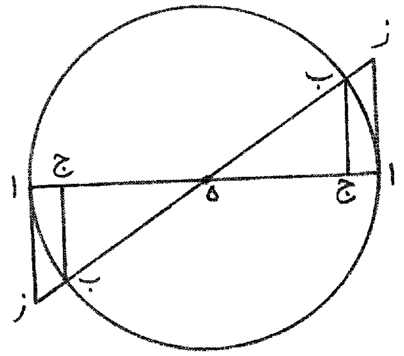
[6] V add. له

[7] F om. و الظل الثاني لكل ارتفاع هو الظل الاول لتمام ذلك الارتفاع

[8] V اعنى الاول و الثاني (؟) instead of و ذلك ما اردنا ان نبين

الباب الثاني في وجود كمية الظل الاول

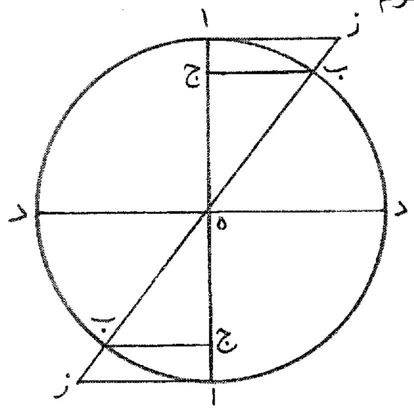
لتكن **اب** دايرة الارتفاع على مركز **ه** و قطرها **ا ه ا** و **اب** قوس الارتفاع و نخرج **ه ب ز**[9] و نقيم **از** عموداً على **اه** و نخرج **ب ج** عموداً على **اه** ايضاً فـ**از** هو الظل الاول لارتفاع **اب** فاقول انه معلوم



برهانه ان[10] **زا ب ج** عمودان على **اه** فهما متوازيان فنسبة **زا** الى **اه** كنسبة **ب ج** الى **ج ه** و **اه** نصف القطر و مساوٍ للمقياس باى اجزاء فرض و **ب ج** جيب قوس **اب** و **ج ه** مثل جيب تمامها فـ**از** معلوم و ذلك ما اردنا ان نبين

الباب الثالث في وجود كمية الظل الثاني

لتكن **اب** دايرة الارتفاع على مركز **ه** و قطراها **اه ا ده د** و نفرض قوس **دب** الارتفاع و نخرج **ه ب ز** و نقيم **از** عموداً على **اه** و نخرج **ب ج** عموداً على **اه** ايضاً فـ**از** هو الظل الثاني لارتفاع **دب** فاقول انه معلوم



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[9] V ه ب instead of ه ب ز

[10] V om. ان

برهانه ان[11] **زا ب ج** عمودان[12] على **اه** فهما متوازيان فنسبة **زا** الى **اه** كنسبة **ب ج** الى **ج ه** و **اه** نصف القطر و مساو للمقياس باى الاجزاء[13] فرض و **ب ج** جيب تمام الارتفـاع و **ج ه** مثل جيب الارتفاع فـ**از** معلوم و ذلك ما اردنا ان نبين[14]

[11] V om. ان

[12] V عموادن (؟) instead of عمودان

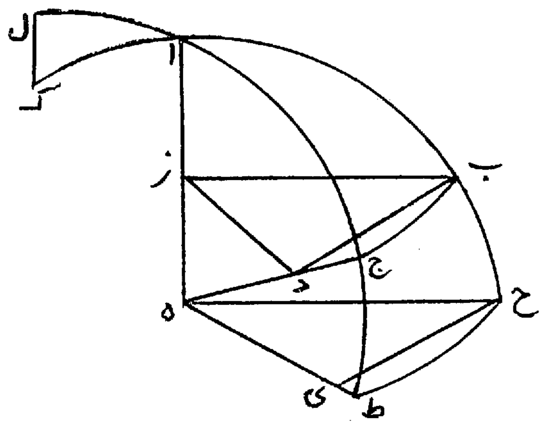
[13] V اجزاء instead of الاجزاء

[14] V add. و نختم الفصل الثاني بهذا الباب و الله محمود

الفصل الثالث في مقدمات تستند اليها البراهين سبعة ابواب

الباب الاول في مقدمة كلية لاكثر البراهين[1]

كل مثلث من قسي دواير عظام فی الكرة فيه زاوية قائمة و فرضت فيه زاوية اخرى فان نسبة جيب وتر الزاوية القائمة الى جيب وتر الزاوية المفروضة كنسبة الجيب الاعظم الى جيب الزاوية المفروضة فليكن المثلث **اب ج** و الزاوية القائمة منه زاوية **ج** والمفروضة **ب اج** فاقول ان نسبة جيب[2] قوس **اب** الى جيب قوس **ب ج** كنسبة الجيب الاعظم الى جيب زاوية **ب اج**



برهانه ان مركز الكرة ه و نصل اه و نتمم كل واحد من قوسى **اب اج** ربع دايرة و هما **اح اط**[3] و نجعل نقطة **ا** قطباً و ندير ببعد ضلع المربع قوس **ح ط** فزاوية **ح ط ج** قائمة و نخرج **ج ه ط ه** كل واحد منهما[4] نصف قطر دايرة **اج ط** فهما في سطح الدايرة و نخرج **ب د**[5] عموداً على **ج ه** و **ح ى** عموداً على **ط ه** فهما[6] عمودان على سطح دايرة **اج ط** و نخرج **ب ز** عموداً على اه و كذلك **ح ه** عموداً عليه فهما في سطح دايرة **اب ح** و نصل **دز** فب **ز** جيب قوس **اب** و **ب د** جيب قوس **ب ج** و **ح ه** الجيب الاعظم و **ح ى** جيب قوس **ح ط** و هو جيب زاوية **ب اج** و لان **ب د ح ى** عمودان على سطح دايرة **اج ط** فكل[7] خط يخرج من نقطتى **د ى**[8] يحيط مع العمود بزاوية قائمة فزاويتا **دى** قائمتان فب **ز ح ه** متوازيان و **ب د**

---

[1] Marginal note in V: هذا هو الشكل المغني

[2] F om. جيب

[3] V repeats و هما اح اط

[4] V منها instead of منهما

[5] F دب instead of ب د

[6] F و هو instead of فهما

[7] F و كل instead of فكل

[8] F د ط instead of د ى

ح ى متوازيان فـزب ب د موازيان له ح ح ى فزاوية زب د مثل زاوية ه ح ى و زاويتا د ى قائمتان فزاويتا ز ه من المثلثين متساويتان فمثلثا زب د ه ح ى متشابهان فنسبة زب الى ب د كنسبة ه ح الى ح ى و قد تقدم ان زب جيب قوس اب و ب د جيب قـوس ب ج و ه ح الجيب الاعظم و ح ى جيب زاوية ح اط فنسبة جيب قوس اب الى جيب قوس ب ج كنسـبة الجيب الاعظم الى جيب زاوية ح اط و ذلك ما اردنا ان نبين

و هنا لك استبان ان كل[9] مثلثين فى الكرة على زاويتين متساويتين و فيهما زاويتان قائمتـان فان نسبة جيب الوتر الزاوية القائمة من مثلث الى جيب وتر الزاوية المتساوية كنسبة جيـب وتر الزاوية القائمة من المثلث الآخر الى جيب[10] وتر الزاوية النظيرة للاولى

و على هذا القانون[11] لان مثلث اك ل اذا كانت زاوية ل منه[12] قائمة و ركّبنا قـوس ال علـى قوس اج تركيب قوس[13] اك على قوس اب لان زاويتى ا متساويتان و صارت نسـبة جيـب قوس اك الى جيب قوس ك ل كنسبة جيب قوس اح الى جيب قوس ح ط و كـذلك ان كانـت زاوية ك قائمة و ركّبنا قوس اك على قوس اج تركيب قوس ال على قوس اب فالنسـبة تلـك النسبة

الباب الثاني في مقدمة اخرى هى من فروع المقدمة الاولى

كل مثلث من قسى دواير[14] عظام فى الكرة فيه زاوية قائمة فان نسبة جيب تمام احد الضلعين المحيطين بالزاوية القائمة الى جيب تمام وتر الزاوية القائمة[15] كنسبة الجيب الاعظم الى جيب تمام الضلع الثالث فليكن مثلث اب ج زاوية ب منه قائمة فاقول ان نسبة جيب تمام ب ج الى جيب تمام ج ا كنسبة الجيب[16] الاعظم الى جيب تمام اب

---

[9] V add. واحد

[10] F om. جيب

[11] In V this proof up to the end of IV.3.1 is written in red ink as a marginal note, and there is a reference to this marginal note at the end of IV.3.5 in that ms.

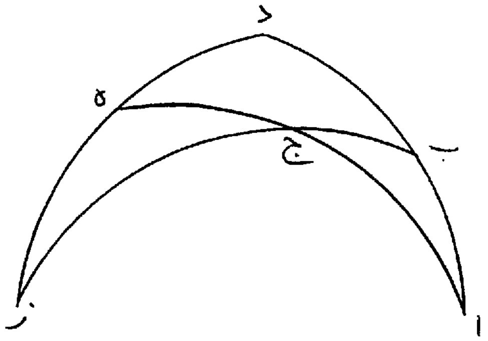
[12] V om. منه

[13] F repeats قوس

[14] V erroneously داوير instead of دواير

[15] F om. القائمة

[16] F om. الجيب



برهانه انا نجعل **ا** قطباً و ندير ببعد ضلع المربع دايرة **د ه ز** و نتمم ربـــع دوايـــر **د ه ز**[17] **اج ه**[18] **اب د**[19] **ب ج ز** فمثلث **زج ه** زاوية **ه** منه قائمة فعلى ما تبين فى المقدمـــة الاولـــى نسبة جيب **زج** الى جيب **ج ه** كنسبة الجيب الاعظم الى جيب زاوية **ز** و **ز ج** تمـــام **ب ج** و **ج ه** تمام **اج** و **ب د** قوس زاوية **ز** و هو تمام **اب** فنسبة جيب تمام **ب ج** الى جيب تمـــام **اج** كنسبة الجيب الاعظم الى جيب تمام **اب** و ذلك ما اردنا ان نبين

الباب الثالث في تذكرة من خواص المقادير المتناسبة

اذا كانت اربعة مقادير متناسبة و اربعة اخرى على نسبة اخرى و هما[٢٠] على غير التـــوالي فانه ان كان الاوسطان[٢١] من الاول مساويين[٢٢] للاوسطين من الآخر[٢٣] كان بالمســـاواة نســـبة المقدم الى المقدم كنسبة التالي الى التالي على التكافي[٢٤] و نسبة المقدم الى التالي كنسبة المقدم الى التالي على التكافي ايضاً[٢٥] و ان كان المقدمان[٢٦] من الاول مســـاويين للمقـــدمين[٢٧] مـــن الآخر[٢٨] كانت نسبة التالي الى التالي من الاول كنسبة التالي الى التالي من الآخر و ان كان

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[17] V om. ده ز
[18] V add. و
[19] V add. و
[20] F نسبتها instead of هما
[21] V الاوسطين instead of الاوسطان
[22] V مساويتين instead of مساويين
[23] V الاخرى instead of الآخر
[24] V نسبة المقدم الاول [الى المقدم من المقادير] من المقادير الاول الى المقدم الاول من المقادير الآخر كنسبة التالي الاخير من المقادير الآخر الى التالي الاخير من المقادير الاول instead of نسبة المقدم الى المقدم كنسبة التالي الى التالي على التكافي
[25] V نسبة المقدم الاول من الاولى الى التالي الاخير من الاخرى كنسبة المقدم الاول من الآخر الى التالي الاخير من الاول instead of نسبة المقدم الى التالي كنسبة المقدم الى التالي على التكافي ايضاً
[26] V المقدمين instead of المقدمان
[27] V الى المقدمين instead of للمقدمين
[28] V الاخرى

التاليان[29] من الاول مساويين للتاليان من الآخر كانت نسبة المقدم الى المقدم من[30] الاول كنسبة المقدم الى المقدم من الآخر و ذلك ما اردنا ان نذكر[31]

المثال		الاول	
ا	ب	ج	د
٢	٤	٣	٦
ه	و	ز	ح[32]
١	٤	٣	١٢

المثال		الثاني	
ا	ب	ج	د
٢	٤	٣	٦
ه	و	ز	ح
٢	٨	٣	١٢

المثال		الثالث	
ا	ب	ج	د
٢	٤	٣	٦
ه	و	ز	ح
١	٤	١* ٣٠	٦

[33]**برهانه** نسبة ا الى ب كنسبة ج الى د و نسبة ه الى و كنسبة ز الى ح و ب مثل و و ج مثل ز و ضرب ب في ج مثل ضرب ا في د و ضرب و في ز مثل ضرب ه في ح نلقي المتساوية يبقى ضرب ا في د مثل ضرب ه في ح فنسبة ا الى ح كنسبة ه الى د

و المثال الثاني ضرب ب في ج مثل ضرب ا في د و ضرب و في ز مثل ضرب ه في ح و ا مثل ه وج مثل ز فضرب ب في ز مثل ضرب ه في د و ضرب ج في و مثل ضرب ا في ح نلقي المتساوية يبقى ضرب ب في ح مثل ضرب و في د فنسبة ب الى و كنسبة د الى ح

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[29] V التاليين instead of التاليان

[30] V om. من

[31] F نبين instead of نذكر

[32] F uses the same letters ا ب ج د for the second proportion; V gives both series of letters for the second proportion; here we have followed A by using ه و ز ح which is more consistent with the text.

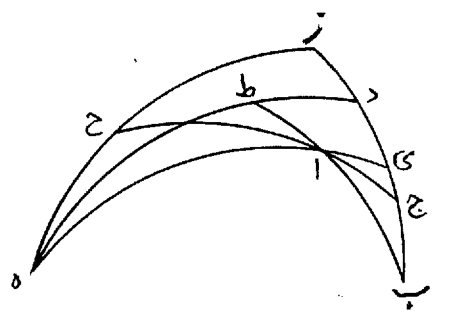
* This means 1; 30 in sexagesimal numeral system. This number is written as ٣ in F, erroneously.

[33] This proof up to the end of IV.3.3 is missing in F and Y. The ms. V includes it, and L has erroneously transferred the proof (but not the summary) to Chapter 5 (see the commentary on IV.3.3).

و المثال الثالث ضرب **ب** في **ج** مثل ضرب **ا** في **د** و ضرب **و** في **ز** مثل ضرب **ه** فـــي **ح** و **ب** مثل **و** و **د** مثل **ح** فضرب **ب** في **ز** مثل ضرب **ه** في **د** و ضرب **ج** في **و** مثل ضرب **ا** في **ح** نلقي المتساوية يبقى ضرب **ا** في **ز** مثل ضرب **ه** في **ج** فنسبة **ا** الى **ه** كنسبة **ج** الى **ز** و ذلك ما اردنا ان نبين جملة ما في هذا التذكرة من النسبة ان كـــان الثـــانيين متســـاويين و الثالثين متساويين كانت نسبة الاول الى الاول كنسبة الرابع الى الرابع بـــالعكس و ان كـــان الاولين متساويين و الثالثين متساويين كانت نسبة الثاني الى الثاني كنسبة الرابع الى الرابـــع بالنظم و ان كانت الثانيين متساويين و الرابعين متساويين كانت نسبة الاول الى الاول كنســـبة الثالث الى الثالث بالنظم و نسبة الاول الى الثالث كنسبة الاول الى الثالث

الباب الرابع في مقدمة اخرى هى ايضاً من فروع المقدمة الاولى

كل مثلث من قسى دواير عظام فان نسبة جيب زاوية منه الى جيب زاوية اخرى كنسبة جيب وتر الزاوية الاولى الى جيب وتر الزاوية الاخرى فليكن مثلث **اب ج** مختلـــف الاضـــلاع و الزوايا فاقول ان نسبة جيب زاوية **ب** الى جيب زاوية **ج** كنسبة جيب قوس **اج** الى جيب قوس **اب**



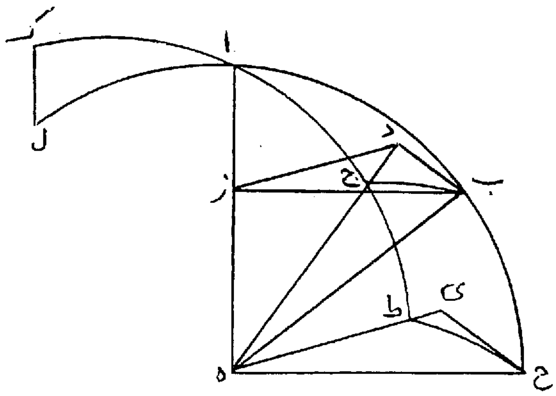
برهانه انا نجعل **ب** قطباً و ندير ببعد ضلع المربع **دط ه** و نجعل **ج** قطباً و ندير **زح ه** و نتمم كل واحد من **ب ج ز ب اط ج اح** و نخرج **ه ى** فلان **ب** قطـــب **ه ط د** يكـــون **ب ط ب د** ربعى[34] دايرة و لان **ه** قطب **ب دز** يكون **ه ز ه د ه ى** ربع دايـــرة و لان **ج** قطـــب **زح ه** يكون كل واحد من **ج ه ج ز** ربع دايرة فمثلث **ب اى** زاوية **ى** منه قائمة فنسبة جيـــب **ب ا** الى جيب **اى** كنسبة الجيب الاعظم و هو جيب **ب ط** الى جيب **ط د** و ايضاً مثلث **ج اى** زاوية

[34] V ربع instead of ربعى

ى منه قائمة فنسبة جيب **ج ا** الى جيب اى كنسبة الجيب الاعظم و هو جيب **ج ح**[35] الى جيب **ح ز** فلان الاوسطين من المقــادير الاول و همـــا <جيـــب> اى <جيـــب> **ب ط** مســـاويان للاوسطين من المقادير الآخر و هما <جيب> اى <جيب> **ج ح** يكون نسبة جيب **ب ا** و هـــو وتر زاوية **ج** الى جيب **اج** و هو وتر زاوية **ب** كنسبة جيب **ح ز** و هو مساوٍ لجيب زاوية **ج** الى <جيب> **ط د** و هو جيب زاوية **ب** فنسبة جيب الزاوية الى جيب الزاوية كنسبة جيب وتر الزاوية الى جيب وتر الزاوية و ذلك ما اردنا ان نبين

الباب الخامس في مقدمة تتعلق بالظل تنوب عـــن المقدمـــة الاولـــى فـــي اكثـــر البراهين[36]

كل مثلث من قسى دواير عظام فيه زاوية قائمة و فرضت زاوية اخرى فان نسبة جيب الضلع الذي يلي الزاويتين القائمة و المفروضة الى ظل وتر الزاوية المفروضة كنسبة الجيب الاعظم الى الظل الزاوية المفروضة فليكن المثلث **اب ج** و زاوية **ب** منه قائمة و المفروضـــة **ب اج** فاقول ان نسبة جيب قوس **اب** الى ظل قوس **ب ج** كنسبة الجيب الاعظم الى ظل زاوية **ب اج**



برهانه ان مركز الكرة **ه** و نصل اه و[37] نتمم كل واحد من **اب اج** ربع دايرة و هما **اح اط** و نخرج **ح ه ب ز** عمودين على اه و نجعل **ا** قطباً و ندير ببعد ضلع المربع قـــوس **ح ط** و نخرج **ه ج ه ط** نصف قطر دايرة **اج ط** و ننفدهما الى **د ى** و نخرج **ه ب** نصف قطر دايره **اب ح**[38] و نخرج **ب د ح ى** عمودين على **ب ح** من قطرى **ه ب ه ح** و نصـــل **دز** فـــب ز

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[35] V الجيب الاعظم و هو ج ح instead of ج ح و هو الجيب الاعظم

[36] Marginal note in V: هذا هو الشكل الظلي

[37] V add. نخرج و

[38] F اح instead of اب ح

في سطح **اب ح** فهو جيب قوس **اب** و **ح ه** ايضاً في سطحه فهو الجيب الاعظم و **ز ب ح ه** يحيطان مع عمودى **ب د ح ى** بزاويتين قائمتين فسطحا **ب زد ح ه ط** متوازيان و **ب د** عمود[39] على قطر **ه ب** فهو عمود[40] على سطح **اب ح** و كل[41] خط يخرج في سطح **اب ح** يحيط مع عمود **ب د** بزاوية قائمة فزاوية **دب ز** قائمة و زاويتا **ح ه ى ب زد**[42] متساويتان فمثلثا **ح ه ى زب د** متشابهان فنسبة **زب** الى **ب د** كنسبة **ه ح** الى **ح ى** و **زب** جيب قوس **اب** و **ب د** ظل قوس **ب ج** و **ه ح** الجيب الاعظم و **ح ى** ظل زاوية **ح اط** فنسبة جيب قوس **اب** الى ظل قوس **ب ج** كنسبة الجيب الاعظم الى ظل زاوية[43] **ب اج** و ذلك ما اردنا ان نبين و هنا لك استبان ان كل مثلثين في الكرة على زاويتين متساويين و فيهما زاويتان قائمتان فان نسبة جيب الضلع الذي يلي القائمة و المتساوية الى ظل الضلع الآخر من المحيطين من مثلث كنسبة جيب النظير الى ظل النظير من المثلث الآخر[44] و على هذا القانون لان مثلث **ال ك** ان كانت زاوية **ك** قائمة او كانت زاوية **ل** قائمة كان البرهان عليه البرهان الذي في المقدمة الاولى[45]

[46]المثلثات القائمة الزوايا في الكرة تكون معلومة بهذه المقدمات من الوجوه ثلثة[47]

الاول زاوية مع احد الضلعين اما وتر الزاوية القائمة و اما وتر الزاوية المعلومة نسبة جيب[48] وتر الزاوية القائمة الى جيب وتر الزاوية المعلومة كنسبة الجيب الاعظم الى جيب الزاوية المعلومة

الثاني كل ضلعين من اضلاعه اى ضلعين كانا نسبة جيب تمام احد الضلعين المحيطين بالزاوية القائمة الى جيب تمام وتر الزاوية القائمة كنسبة الجيب الاعظم الى جيب تمام ضلع الثالث و نسبة جيب وتر الزاوية القائمة الى جيب الضلع الآخر كنسبة الجيب الاعظم الى جيب زاوية الضلع الآخر المعلوم

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[39] V عموداً instead of عمود

[40] F om. فهو عمود

[41] V فكل instead of و كل

[42] F دزب instead of ب زد

[43] F and V قوس instead of زاوية found in Y

[44] F om. from here to the end of this chapter

[45] From و على هذا القانون up to here is only found as a marginal note in V that adds علقناه على الحاشية بالحمرة (see the commentary to IV.3.5).

[46] From here to the end of this section is given in V on an additional folio and in L at this position.

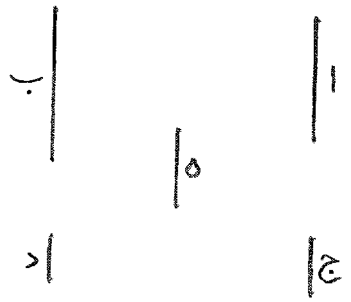
[47] V add. تكون معلومة

[48] V om. جيب

الثالث زاوية مع الضلع الذي يليها من المحيطين بالزاوية القائمة نسبة جيب هذا الضلع الى ظل الضلع الآخر من المحيطين بالزاوية القائمة كنسبة الجيب الاعظم الى ظل[49] الزاوية المعلومة

الباب السادس في تذكرة هى من خواص الظل

كل قوسين مختلفين فان ظلهما الاول مكاف[50] لظلهما الثاني فليكن كل واحد من **ا ب** ظل اول للقوسين المختلفين[51] و كل واحد من **ج د** ظل ثاني لهما و المقياس **ه** و ليكن القوس التي ظلها الاول **ا** ظلها الثاني **ج** و القوس التي ظلها الاول **ب** ظلها الثاني **د** فاقول ان نسبة **ا** الى **ب** كنسبة **د** الى **ج**



برهانه[52] ان نسبة **ا** الى **ه** كنسبة **ه** الى **ج** و نسبة **ب** الى **ه** كنسبة **ه** الى **د** فضرب **ا** في **د** مثل ضرب **ه** في نفسه و ضرب **ب** في **د** ايضاً مثل ضرب **ه** في نفسه فضرب **ا** في **ج** مثل ضرب **ب** في **د** فنسبة **ا** الى **ب** كنسبة **د** الى **ج** و ذلك ما اردنا ان نبين

الباب السابع في تذكرة اخرى هى ايضاً من خواص الظل

كل قوس فان ما يقسم على احد ظليه يكون مساوياً لما يضرب في ظله الآخر فليكن **ب** ظلاً مستوياً لقوس مفروض[53] و **ل** ظلاً معكوساً لها و المقياس **ا** و هو واحد و قد قسم مقدار و على **ب** فكان **ج** فاقول ان **ج** مساوٍ لما يكون من ضرب و في **ل**

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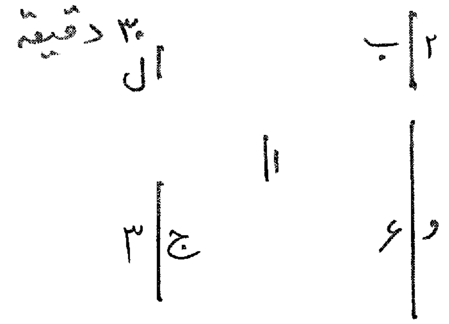
[49] L جيب instead of ظل

[50] V مكافية instead of مكاف

[51] V لقوسين مختلفين instead of للقوسين المختلفين

[52] V برهان ذلك instead of برهانه

[53] V مفروضة instead of مفروض



[54]**برهانه** ان و قسم على ب فكان ج فضرب ب في ج هو و وضرب ب في ل اعني في النصف

هو ا لان نسبة ب الى ا كنسبة ا الى ل فنسبة ل الى ج كنسبة ا الى و فضرب ل في و مساو لضرب ا في ج و ضرب ا في ج هو ج لان ا هو المقياس و قد فرض واحداً فضرب ل في و هوج و لان ظل كل قوس مستوياً هو ظل تمامه معكوساً صار كل قوس ما يقسم على ظله مساوياً لما يضرب في ظل تمامه و ذلك ما اردنا ان نبين[55]

[54] In the diagram, the numbers shown in digits and letters are as we find them in F. V only gives them in digits (see also the commentary on IV.3.7).

[55] V add. و نختم الفصل الثالث هذا الباب

الفصل الرابع في تقويم الكواكب و احوالها عشرة ابواب

الباب الاول في تعديل الايام بلياليها

قد تقدم القول في المقالة الثالثة ان هذا التعديل هو الفضل بين[1] اليوم الوسط و اليوم الحقيقي و ان اليوم الوسط هو دور معدل النهار من نصف النهار الى نصف النهار و زيادة قوس منه مساوية لوسط الشمس في اليوم و ان[2] اليوم الحقيقي هو دور معدل النهار من نصف النهار الى نصف النهار و زيادة ما يطلع منه مع مسير الشمس المختلف[3] و ان هذا التعديل يجتمع من ضعف الاختلاف بين درج السواء و مطالع خط الاستواء و ضعف الاختلاف بين وسط الشمس و مقومها و ذلك اما من اختلاف المطالع فخمس درج بالتقريب و اما من اختلاف الشمس فاربع[4] درجات[5] فيكون مجموع الاختلافين تسع[6] درجات بالتقريب و هو ثلثة اخماس ساعة مستوية الا شيئاً[7] يسيراً و لا يكاد[8] يستوفي هذا التعديل[9] كله لانه اذا كان احد الاختلافين في نهايته نقص الآخر شيئاً عن النهاية[10] الا اذا صار الاوج في العشر الاوسط او[11] الاخير من الاسد و لان في اليوم الواحد و اليومين لايظهر من هذا[12] التعديل شئ محسوس جاز ان يجعل اى موضع كان من فلك البروج اصلاً الا انه اذا جعل العشر الاوسط من الدلو صارت الايام الوسطى تفضل على الايام الحقيقية الى ان يصير من الاوج ما قلنا و ان جعل غيره اصلاً زادت الايام الوسطى على الايام الحقيقية مرةً و نقصت عنها مرةً فاقول ان ساعات فضل الايام[13] الوسطى على الحقيقية معلومة

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[1] F يسير instead of بين ; Kashino reads this بسير , but in III Kūshyār has the word بين in the same position.

[2] V فان instead of و ان

[3] V add. في اليوم

[4] V فاربعة instead of فاربع

[5] V add. بالتقريب

[6] V تسعة instead of تسع

[7] V شئ instead of شيئاً

[8] V يكا instead of يكاد

[9] V add. بل

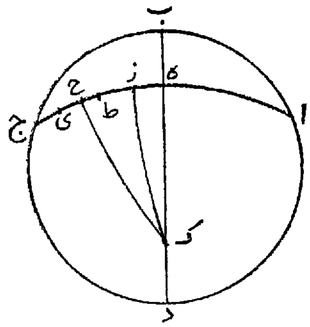
[10] V عن النهاية شيئاً instead of شيئاً عن النهاية

[11] F و instead of او

[12] F and Kashino om. هذا

[13] V فضل ساعات الايام instead of ساعات فضل الايام

برهانه اب ج د دايرة الافق و **دب** دايرة نصف النهار و **اه ج**[14] معدل النهـــار و **ك** قطبـــه و ليكن نقطة **ه** وسط الاصل و هي[15] احدى درجات العشر الاوسط من الدلو و **ز** مطالع مقومـــه و نخرج **زك** و ليكن نقطة **ط** وسط[16] آخر فقد يكون مطالع مقومه اقل منه او اكثر منه[17]



فليكن اولاً اكثر منه و هو **ح** و نخرج **ح ك** فعلى ما بينّا من الوضع يكون[18] ما بين الوسطين اعظم من مطالع ما بين المقومين فقوس **ه ط** اعظم من قـــوس **زح** و **زط** مشـــترك فـه **ز**[19] اعظم من **ط ح** فالزمان الذي يجوز فيه قوس **ه ط** نصف النهار اعظم من الزمان الذي يجوز فيه قوس **زح** بقدر زيادة **ه ز** على **ط ح** و كل واحد[20] من قسى **ه ط زح ه ز ط ح** معلومة ففضل **ه ط** على **زح** معلوم و كل خمسة عشر جزؤاً من اجزاء معدل النهار ساعةً فمقدار ذلك الفضل من الخمسة عشر معلوم فزيادة الايام الوسطى على الايام الحقيقيـــة معلومـــة و هـــي نقصان الحقيقية عن الوسطى اذا اردنا الايام الوسطى و ايضاً فليكن نقطة **ى** الوسط و نقطة **ح** مطالع مقومة اقل منه فـه **ى**[21] اعظـــم مـــن **زح** بقوســـى **ه ز ح ى** و قوســـا[22] **ه ى زح** معلومتان فقوسا **ه ز ح ى** معاً معلومة[23] و الزمان الذي يجوز فيه قوس **ه ى** نصف النهار اعظم من الزمان الذي يجوز فيه قوس **زح** بمقدار قوسى **ه ز ح ى** معاً فمقدارهما من خمسة عشر جزءاً معلوم و زيادة[24] الايام الوسطى على الايام الحقيقية معلومة و هي نقصان الحقيقية عن الوسطى اذا اردنا معرفة الوسطى من الحقيقية و ذلك ما اردنا ان نبين

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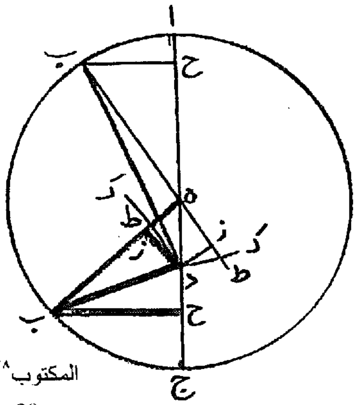
[14] F اه ح instead of اه ج
[15] V هو instead of هي
[16] F ه ط وسطا instead of ط وسط
[17] V و اكثر instead of او اكثر منه
[18] F and Kashino om. يكون
[19] F فهو instead of فـه ز
[20] V واحده instead of واحد
[21] F فهى instead of فـه ى
[22] V فقوسا instead of و قوسا
[23] F and Kashino معلومتان instead of معاً معلومة
[24] V فزيادة instead of و زيادة

الباب الثاني في تعديل الشمس

اب ج[25] دايرة الفلك الخارج المركز و[26] مركزها **ه** و قطرها **اج** و **د** مركز الفلك الممثل بفلك البروج فـ**ده** ما بين المركزين و هو على ما وجد درجتان و اربع دقايق و نصف و ربع على ان **ه ا** ستون جزءاً و **ا** موضع الاوج و **ب** جرم الشمس و[27] **اب** خاصة الشمس ونجعل **ب ح**



المكتوب[28] بالسواد على ان الخاصة اكثر من ص[29]

عموداً على **اه** فهو جيب قوس **اب** و[30] **دز**[31] عموداً على **ب ز** و زاوية[32] **زه د** مثل زاوية **ح ه ب** و زاويتا **ز ح** قائمتان فنسبة **ه ب** الى **ب ح** كنسبة **ه د** الى **دز** و **ه ب** ستون جزءاً و **ب ح** معلوم و **ه د** معلوم فـ**دز** معلوم و **زه** معلوم[33] لان **ح ه** جيب تمام الخاصة[34] فـ**ب ز** معلوم و مربعا[35] **ب ز زد** مثل مربع **ب د** فـ**ب د** معلوم و نسبة **ب د** الى **دز** المعلوم بمقدار نصف قطر **ب ه**[36] كنسبة الستين الى **دز** بالمقدار الذي هو المطلوب[37] فـ**دز** بمقدار نصف قطر **ب د**[38] معلوم و هو جيب زاوية **زب د** فزاوية **زب د** معلومة و هى زاوية التعديل و ذلك ما اردنا ان نبين و لان زاوية **اه ب** خارجة عن مثلث **ب ده** تكون زاوية **اه ب** و هى مقدار

---

[25] F اب ج د instead of اب ج

[26] V om. و

[27] V ف instead of و

[28] V الخطوط instead of المكتوب

[29] V تسعين instead of ص; A and M demonstrate this additional case in a separate figure.

[30] F om. ب ح عموداً على اه فهو جيب قوس اب و

[31] F زد instead of دز found in V

[32] V فزاوية instead of و زاوية

[33] V om. و ه د معلوم فـدز معلوم و زه معلوم

[34] A add. معلوم

[35] V مربعي instead of مربعا

[36] F, V and Kashino erroneously ب د instead of ب ه

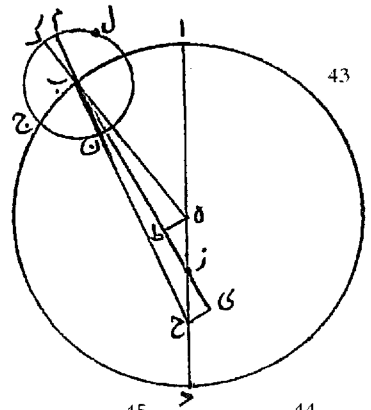
[37] A المطلوب على ان دب ستون جزءاً instead of الذي هو المطلوب

[38] F and Kashino om. كنسبة الستين الى دز بالمقدار الذي هو المطلوب فـدز بمقدار نصف قطر ب د; A على ان ب د ستون جزءاً instead of بمقدار نصف قطر ب د.

الخاصة اعظم من زاوية **ه د ب** و هى زاوية التقويم بمقدار زاوية **ه ب د** و هى زاوية التعديل فاذا نقص التعديل من الخاصة[39] او الوسط كان <الزاوية> التقويم و الخاصة اقل من مائة و ثمانين فاذا[40] كانت الخاصة اكثر من مائة و ثمانين فبالضد

الباب الثالث في التعديل الاول للقمر

ا ب ج د دايرة الفلك الخارج المركز و مركزه **ه** و قطره **اد** و **ز** مركز الفلك المايل و **ح** هي النقطة التي تتصوب نحوها الذروة و الحضيض من فلك التدوير و هما **م ن** و **ل ك ج**[41] فلك التدوير على مركز **ب**[42] و **ل** جرم القمر و زاوية **زب ح** زاوية التعديل فـ**ازب** زاوية البعد



المضاعف و **ه ز زح** متساويتان[44] و كل واحدة[45] منهما اثنا عشرة درجة و نصف على ان **اه** ستون جزءاً و **ه ط ح ى** عمودان[46] على **ب ى** فزاوية **ه زط** معلومة و زاوية **ط** قائمة فزاوية **ه** الباقية معلومة و اضلاع مثلث **ه زط** معلومة و **ه ب** ستون جزءاً و مربعه مثل مربعى **ب ط ط ه** فـ**ب ط** معلوم فجميع **ب ز** معلوم و زوايا مثلث **ه زط** مساوية لزوايا مثلث **زى ح** فنسبة **ه ز** الى **زح** كنسبة **زط** الى **زى**[47] و كنسبة **ه ط** الى **ح ى** و **ه ز زح** متساويان فـ**ى ز زط** متساويان و **ه ط ح ى** متساويان فجميع **ب ى** معلوم و مربعه مع مربع **ى ح** مثل مربع **ب ح**

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[39] A add. اعني زاوية ا ه ب ; A and M provide more details in this position using an additional figure.

[40] V و اذا instead of فاذا

[41] F and Kashino ل ك instead of ل ك ج

[42] F and Kashino om. على مركز ب

[43] V add. on the figure:
لو كان خط ب ز يمر بذروة فلك التدوير و حضيضه لا سعينا (؟) عن التعديل الاول و كانت الخاصة هى الخاصة المعدلة

[44] V متساويان instead of متساويتان

[45] V واحد instead of واحدة

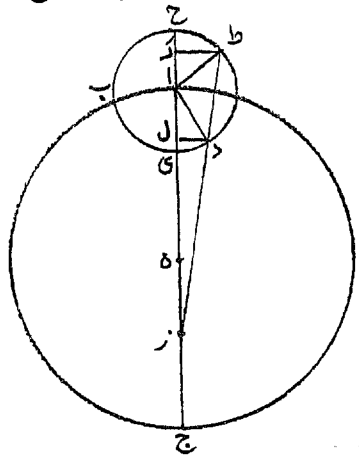
[46] V عمودين instead of عمودان

[47] F, V and Kashino erroneously زى الى زط

فـب ح معلوم فاذا جعلنا نقطة ب مركزاً[48] و ادرنا ببعد ب ح دايرةً كان ح ى جيب قوس زاوية ى ب ح على ان ب ح نصف القطر بالمقدار المعلوم فـح ى على ان ب ح[49] ســتون جزءاً معلوم فزاوية ى ب ح معلومة[50] و ايضاً زاوية ى ب ح مثل زاوية م ب ك فقوس م ك معلومة و م ل[51] خاصة القمر و ك ل الخاصة المعدلة على ان[52] اب و هو البعد المضـــاعف اقل من تسعين و بهذه الطريقة تبين لنا زاوية التعديل اذا كان المضاعف اكثر من تســعين و انه اذا كان اكثر من مائة و ثمانين نقص التعديل من الخاصة و ذلك ما اردنا ان نبين

الباب الرابع في التعديل الثاني للقمر و الكواكب

اب ج دايرة الفلك الخارج المركز[53] على مركز ه و ز مركز الفلك المايل و ح ط د[54] فلك التدوير على مركز ا و ليكن ط موضع القمر لان حركة القمر الى هذه الجهة و نصل ط ا ط ز و ط ك عمود على اح فزاوية ط زح زاوية التعديل فـط ك جيب الخاصة المعدلة اعني قوس ط ح و ك ا جيب تمامها و كل[55] واحد منهما على ان ط ا خمسة اجزاء و ربع معلوم لان نسبة اط الى ط ك كنسبة الجيب الاعظم الى جيب الخاصة و زا ستون جزءاً فجميع زك معلوم فمربعه مع مربع ك ط مثل مربع ط ز فـط ز معلوم فاذا جعلنا ز مركزاً و ادرنا ببعد ز ط دايرة كان ط ك[56] جيب قوس[57] زاوية ط زك على ان زط بالمقدار الذي هو معلوم



[48]V مركز instead of مركزاً

[49] F and Kashino om. نصف القطر بالمقدار المعلوم فـح ى على ان ب ح

[50] F om. معلومة

[51] L and Kashino م ب ل instead of م ل

[52] F om. ان

[53] V om. المركز

[54] V ح ط instead of ح ط د

[55] V فكل instead of و كل

[56] F and Kashino ك ط instead of ط ك

[57] F om قوس

فـ**ط ك** على ان **ط ز** ستون جزءاً معلوم و هو جيب قوس زاوية التعديل و كـذلك ان جعلنـا موضع القمر **د** كان **دى** معلوماً[58] و **دل** جيبه و **ال** جيب تمامه فبالطريقة الاولى يحصـل **دل** على ان **زد** ستون جزءاً و هو جيب قوس زاوية[59] **دزل**[60] زاوية التعديل و ذلك ما اردنـا ان نبين

فظاهر من هذه الصورة ان قوس[61] التعديل ينقص من وسط القمر اذا كانت الخاصة المعدلـة اقل من مائة و ثمانين و يزاد عليه ان كانت الخاصة اكثر و في ساير الكواكب يـنقص هـذا التعديل من المركز المعدل ان كانت الخاصة المعدلة اكثر من مائة و ثمانين و يزاد عليـه ان كانت الخاصة المعدلة[62] اقل لان حركة اجرامها في افلاك التداوير الى خلاف جهـة حركـة القمر و ذلك ما اردنا بيانه[63]

الباب الخامس في اختلاف نصف قطر فلك التدوير فيما بـين البعـد الابعـد و[64] الاقرب[65]

مركز فلك التدوير للقمر فرض عند البعد الابعد و ما بينه و بين مركز الفلك المايل[66] سـتون جزءاً و نصف قطر فلك التدوير بذلك المقدار خمسة اجزاء و ربع و غاية التعديل الثاني هـو بحسب نصف قطر فلك التدوير و يختلف مقداره فى الرؤية[67] فيما بين البعد الابعد الى البعـد الاقرب لان الزاوية التي عند مركز الفلك المايل و يوترها نصف قطر فلك التدوير تعظم كلما قرب مركز فلك التدوير من مركز الفلك المايل و هكذى[68] حال نصف قطر افـلاك تـداوير الكواكب الا ان مراكز افلاكها هذه مفروضة عند البعد الاوسط و ما بينها و بين مركز الفلـك المايل ستون جزءاً فنصف قطر افلاك تداويرها فيما بين البعد الاوسط و الابعد اصـغر مـن المقدار المفروض و فيما بين البعد الاوسط و الاقرب اعظم من المقدار المفروض[69] لان البعد

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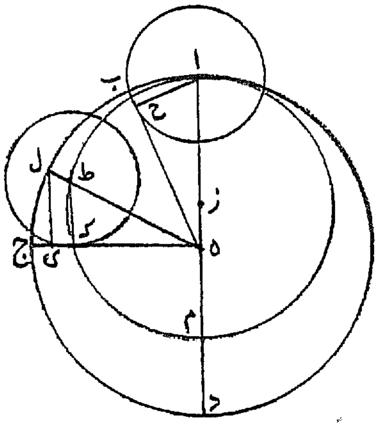
[58] V معلوم instead of معلوماً
[59] Only A has the word زاوية, missing in other mss. and Kashino
[60] F and M دل instead of دزل; Kashino دى
[61] F om. قوس
[62] F om. المعدلة
[63] V om. و ذلك ما اردنا بيانه
[64] V او instead of و
[65] A add. للقمر و الكواكب و معني دقايق النسب
[66] A add. اعني مركز فلك البروج
[67] F and Kashino بالرؤية instead of في الرؤية
[68] V and Kashino هكذا instead of هكذى
[69] F om. from here up to كذلك ساير الابعاد

الابعد لكل كوكب ستون و نصف ما بين المركزين و البعد[70] الاقرب ستون الا نصف ما بين المركزين و نسبة كل واحد من البعدين الى جيب التعديل الثاني عند البعد الاوسط كنسبة الستين الى جيب التعديل الثاني عند ذلك البعد و كذلك ساير الابعاد

فليكن **اب ج د** على مركز **ه** دايرة الفلك المايل و **اط م** على مركز **ز** دايرة الفلك الخارج المركز و **ا** مركز فلك التدوير عند البعد الابعد و **ط** مركزه عند بعد آخر و نخرج **ه ب** مماساً للدايرة على **ح** و نصل **اح** و نخرج **ه ج** مماساً للدايرة على **ك** و نصل **ط ك** و **ل ى** عمودين[71] على **ه ج** فزاوية **ل ه ج** اعظم من زاوية **اه ب** لان **ه ط** اصغر من **ه ا** فاذا



ركبناه[72] على **ه ا** وقع **ط ك** خارجاً عن خط **ه ح** فاح نصف قطر فلك التدوير عند البعد الابعد و قوسه **اب** و هى توتر زاوية **اه ب** فاب غاية التعديل عند البعد الابعد و **ط ك** نصف قطر فلك التدوير عند هذا البعد و زاوية التعديل **ل ه ج** و قوسها **ل ج** فل ج غاية التعديل عند هذا البعد و زاوية **ل ه ج** اعظم من زاوية **اه ب** فقوس **ل ج** اعظم من قوس **اب** و نسبة **ه ط** الى **ط ك** كنسبة **ه ل** الى **ل ى** لان مثلثى **ط ه ك** **ل ه ى** متشابهان و **ه ط** معلوم من شكل التعديل الاول و هو ما بين مركز فلك التدوير و مركز الفلك المايل و **ط ك** مثل **اح** بالمقدار و **ه ل** مثل **ه ا** فل ى معلوم و هو جيب قوس **ل ج** فل ج معلوم ففضله على **اب** معلوم و هو الاختلاف الكلي عند هذا البعد بحسب خط **ه ط** فالاختلاف الكلي عند ساير الابعاد بهذه الطريقة معلوم و ذلك ما اردنا ان نبين

و على هذا الحساب وضعنا الاختلاف للقمر و الكواكب فوقع لنا الاختلاف في النسخة التي لم نقصد فيها تقريب التعديل في جدول واحد اما للقمر فمن البعد الابعد الى البعد[73] الاقرب نسقاً واحداً زائداً و اما للكواكب فمن البعد الابعد الى البعد الاوسط ناقصاً و من البعد الاوسط الى

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70 V الابعد instead of البعد

71 V عمود instead of عمودين

72 F ركبنا ه instead of ركبناه; Y and Kashino ركبنا ه ط

73 F om. From here up to the next الى البعد

البعد الاقرب زائداً فظاهر ان هذا الاختلاف هو في القمر بحسب البعد المضاعف لانه من الصورة بحسب **ال** و في ساير الكواكب بحسب المركز المعدل ثم طلبنا دقايق نسبتها الى ستين[74] دقيقة كنسبة التعديل الثاني الجزئي[75] الى التعديل الكلي فاذا ضربنا تلك[76] الدقايق في الاختلاف للقمر[77] حصل منه بقسط التعديل في الموضع[78] لان عند ذروة فلك التدوير لايكون تعديل فلايلزمه اختلاف و عند غاية التعديل يلزم كل الاختلاف و ان من هذا الوجه ان تأخذ الاختلاف للقمر[79] بالبعد المضاعف و لساير الكواكب بالمركز المعدل و ان نأخذ دقايق النسب بالخاصة المعدلة و من هذا البرهان تبين لنا ان اختلاف نصف قطر فلك التدوير للمريخ[80] في البعد الابعد اقل مما هو موضوع في جداول المجسطي بدرجة و خمس و في البعد الاقرب بدرجتين و خمس و ذاك شئ وقع في حساب الجداول[81] فاما[82] الحساب في الرسالة فصحيح و مقدار هذا[83] الاختلاف بالحساب هو المقدار الواجب[84]

الباب السادس في التعديل الاول لعطارد

اب ج دايرة الفلك المعدل للمسير و مركزها **ه** و قطرها **اج** و **ز** مركز الفلك المايل و **ن** مركز الدايرة الصغيرة الحاملة لمركز الفلك الحامل لمركز[85] فلك التدوير و **م** مركز الفلك الحامل و نتوهم **م** تحركت فقطعت قوس **م د** مثل مسير الشمس الى خلاف التوالي و مركز فلك التدوير يحرك مع **م** الى التوالي حتى صار من **ح** الى **ط** فقطع من دايرة **اب ج** قوس **اب** شبيهة بقوس **دم** و نجعل **د** مركزاً و ندير دايرة الفلك الحامل بمقدار المعدل للمسير و هي **ح ط ك** و نصل **ه ط ب**[86] **زط دط دن ده** و **دل زى** عمودين على[87] **ب ى** و زاويةى **ط ز**

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[74] V ٦٠ instead of ستين

[75] F and Kashino om. الجزئي

[76] Kashino ذلك instead of تلك

[77] F has erroneously inserted here a fragment that should come after the next للقمر:
بالبعد المضاعف و لساير الكواكب بالمركز المعدل و ان نأخذ دقايق النسب بالخاصة المعدلة; Kashino has followed it.

[78] F and Kashino om. حصل منه بقسط التعديل في الموضع

[79] F and Kashino om. و ان من هذا الوجه ان نأخذ الاختلاف للقمر

[80] V تدوير المريخ instead of التدوير للمريخ

[81] F and Kashino يقع في الجداول و حسابها instead of وقع في حساب الجداول

[82] F add. في

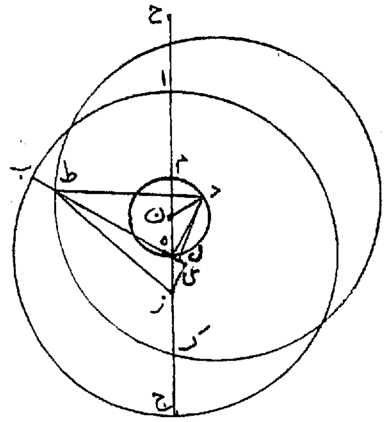
[83] V om. هذا

[84] A quotes some materials from the *Almagest* in this position where Kūshyār says that the difference between his results and those of the *Almagest* is more noticeable in the case of Mars, and that he does not know why it is so.

[85] V حامل مركز instead of الفلك الحامل لمركز

[86] F ه ط instead of ه ط ب

زاوية التعديل فزاويتا م ن د اه ب متساويتان لان قوسيهما متشابهتان و[88] كل واحدة منهما زاوية المركز فهما معلومتان و كل[89] واحدة من قوسى م د ده معلومة فوتر ده معلوم[90] من القطر الاعظم[91] و نسبته اليه[92] كنسبة وتر ده الى قطر ه م و ه م ستة اجزاء و ثلث فوتر ده معلوم و زاوية ده ن نصف زاوية دن م فزاوية ده ن نصف زاوية اه ب فجميع زاوية ده ب معلومة و زاوية دل ه قائمة فزاوية ل ده معلومة و ده معلوم فاضلاع مثلث ل ده معلومة و دط ستون جزءاً و مربعه مثل مربعى دل ل ط فل ط معلوم و ل ه معلوم فطه معلوم و ايضاً زاوية زه ى معلومة لانها مثل زاوية اه ب و زاوية ى قائمة فزاوية ى زه معلومة و زه



معلوم و هو ثلثة اجزاء و سدس فاضلاع مثلث زه ى معلومة و ط ه معلوم فط ى معلوم و مربعه مع مربع ى ز مثل مربع زط فزط معلوم فاذا جعلنا ط مركزاً و ادرنا ببعد ط ز دايرة كان زى جيب قوس زاوية ى ط ز[93] بمقدار نصف قطر زط فزى على ان زط ستون جزءاً معلوم و هو جيب زاوية التعديل و ذلك ما اردنا ان نبين

و بهذه الطريقة يحصل لنا التعديل من جميع جوانب الدايرة و يخرج بالحساب ان خط ط ز ان كان المركز صفراً فهو سط ل[94] و ان كان المركز سو[95] فهو س و ان كان المركز ص فهو نو ن[96] و ان كان المركز قك فهو نه ك و يطابق حينئذ خط دط خط ه ط و ان كان المركز قف فهو نو ن[97] ايضاً و اعظمه[98] عند البعد الابعد و اوسطه عند بعد سو[99] و اصغره عند

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[87] F om. عمودين على

[88] F om. و

[89] V فكل instead of و كل

[90] F and Kashino om. From here up to the next فوتر ده معلوم

[91] A add. معلوم

[92] A add. يعني الى القطر الاعظم

[93] F, V, L and A ى ب ز instead of ى ط ز found in Y and M

[94] M ٦٩ instead of سط ل

[95] A سه instead of سو

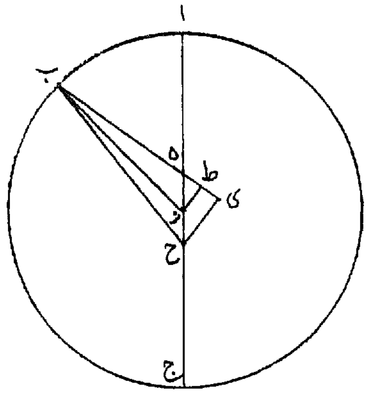
[96] F ن and M ه٠ instead of نو ن

[97] F ن and M ه٠ instead of نو ن

بعد **قك** و عند بعد **ص** و **قف** متساويان و لان زاوية **ازط** في هذه الصورة و هي زاوية المركز اصغر من زاوية **اه ط** و الفضل بينهما زاوية **ى ط ز** يجب ان ينقص التعديل من المركز و يزاد على الخاصة ان كان المركز اقل من مائة و ثمانين[100] و يزاد على المركز و ينقص من الخاصة ان كان المركز اكثر من مائة و ثمانين[101]

الباب السابع في التعديل الاول لباقي الكواكب

اب ج على مركز **ز** دايرة الفلك الحامل و **اج** قطرها و **ه** مركز الفلك المعدل و **ح** مركز الفلك المايل و **ه ز زح** متساويان و كل واحد منهما في زحل ثلثة اجزاء و ربع و سدس و في المشتري جزءان[102] و نصف و ربع و في المريخ ستة اجزاء و في الزهرة جزء واحد و دقيقتان[103] و نصف و **ب** مركز فلك التدوير و نصل خطوط **ه ب زب ح ب** و **زط ح ى** عمودان[104] على **ب ى** و زاوية **ه ب ح** زاوية التعديل فزاوية[105] **اه ب** المركز فزاوية **ط ه ز**



معلومة و زاوية **ط** قائمة فزاوية **ط زه** معلومة و **زه** معلوم و كل[106] واحد من **ه ط ط ز** معلوم و **ب ز** ستون جزءاً و مربعه مثل مربعى **زط ط ب** فـ**ط ب** معلوم و لان مثلثى **ى ح ه ط زه** متشابهان و **زه** نصف **ه ح** فـ**زط** نصف **ح ى** و **ه ط** نصف **ه ى** فـ**ط ى**[107] معلوم و

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[98] V فاعظمه instead of و اعظمه

[99] A سه instead of سو

[100] V ثمنين instead of ثمانين

[101] V ثمنين instead of ثمانين ; A provides 6 additional figures here for the values of the center (*markaz*) being 60°, more than 60°, 90°, more than 90°, 120°, and more than 120°.

[102] V جزءين instead of جزءان

[103] V دقيقتين instead of دقيقتان

[104] V and Kashino عمودين instead of عمودان

[105] V و زاوية instead of فزاوية

[106] V فكل instead of و كل

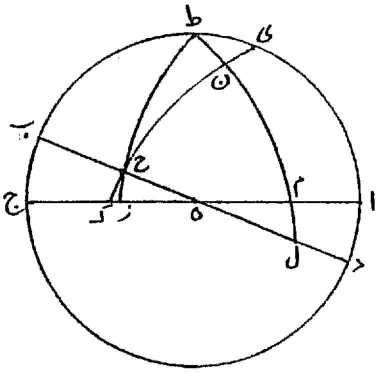
[107] V ىط instead of ط ى

ط ب معلوم[108] فجميع ىب[109] معلوم و مربعه مع مربع ىح مثل مربع ح ب فح ب معلوم فاذا جعلنا نقطة ب مركزاً و ادرنا ببعد ح ب[110] دايرة كان ح ى فيها جيب قوس زاوية ىب ح بمقدار نصف قطر ب ح فح ى[111] على ان ب ه ستون جزءاً معلوم و هو جيب قوس زاوية التعديل و ذلك ما اردنا ان نبين

و بهذه الطريقة يحصل لنا[112] التعديل من جميع جوانب الدايرة و لان زاوية ه ب ح هي الفضل بين زاويتى اه ب اح ب صار التعديل ينقص و يزاد كما تقدم القول[113] في عطارد[114] ان كان المركز اقل من مائة و ثمانين نقص من المركز و زيد على الخاصة و ان كان المركز اكثر من مائة و ثمانين[115] زيد على المركز و نقص من الخاصة

الباب الثامن في عرض القمر

اب ج د على مركز ه الدايرة المارة باقطاب الفلك المايل و فلك البروج و ليكن اه ج دايرة الفلك المايل و قطبه ط و ده ب دايرة فلك البروج و قطبه ى و نقطة ه عقدة الجوزهر و ح موضع القمر من فلك البروج و ك جرم القمر على الفلك المايل و لافرق بينه و بين موضعه على فلك التدوير لان سطح فلك التدوير في سطح الفلك المايل فه ح حصة العرض و نجيز على ح قوسى ط ح ز ىح ك فقوس ح ك عرض القمر و اهل الصناعة يأخذون على موجب حسابهم قوس ح ز و ليس ح ز بعرض القمر و انما[116] هى قوس قريبة من العرض فاقول ان ح ك معلوم



[108] F and Kashino om. و ط ب معلوم

[109] V فب ى instead of فجميع ىب

[110] V ح ب instead of ح ب

[111] F and Kashino add. معلوم

[112] V om. لنا

[113] F and Kashino om. القول

[114] A instead of here to the end of this chapter: في الزيادة و النقصان على المركز و الخاصة

[115] V ثمنين instead of ثمانين

[116] V فانما instead of و انما

برهانه اما على ما تبين في المقدمة الرابعة[117] مثلث **ه ح ك** زاوية **ح** منه قائمة و زاوية **ح ه ك** زاوية العرض كله اعنى قوس **ب ج** فنسبة جيب **ه ح** الى ظل **ح ك** كنسبة الجيب الاعظم الى ظل زاوية **ح ه ك** و **ه ح** حصة العرض و زاوية **ه** العرض كله و الجيب الاعظم معلوم فظل **ح ك** معلوم فـ**ح ك** معلوم

[118] و اما بالجيب المطلق فمثلث **ه ح ز** زاوية **ز** منه قائمة و زاوية **ه** العرض كله فعلى ما تبين في المقدمة الاولى نسبة جيب **ه ح** الى جيب **ح ز** كنسبة الجيب الاعظم الى جيب زاوية **ه** و **ه ح** حصة العرض و زاوية **ه** العرض كله و الجيب الاعظم معلوم فـ**ح ز** معلوم و نجعل نقطة **ك** قطباً و ندير ببعد ضلع المربع ربع دايرة **ن م ل** فـ**ل** قطب دايرة **ى ح ك** فكل واحد من **ن ك ح ل** ربع دايرة فـ**ه ل** تمام **ه ح** فمثلث **ه ل م** زاوية **م** منه قائمة و زاوية **ه** زاوية العرض كله فنسبة جيب **ه ل** الى جيب **ل م** كنسبة الجيب الاعظم الى جيب زاوية **ه** و **ه ل** تمام حصة العرض و زاوية **ه** معلومة فـ**ل م** معلوم فتمامه[119] **م ن** معلوم و هو مقدار زاوية **ح ك ز** فمثلث **ح ك ز** زاوية **ز** منه قائمة و زاوية **ك** معلومة[120] فنسبة جيب **ك ح** الى جيب **ح ز** كنسبة الجيب الاعظم الى جيب زاوية **ك** و **ح ز** معلوم و زاوية **ك** معلومة فـ**ح ك** معلوم و هو عرض القمر و ذلك ما اردنا ان نبين

الباب التاسع في عروض الكواكب

قد تقدم القول في المقالة الثالثة ان لكل واحد من الكواكب العلوية اختلافين[121] في العرض احدهما ميل الفلك المايل عن فلك البروج و الآخر ميل الذروة و الحضيض من الفلك التدوير عن الفلك المايل و ان ميل الذروة الى ما يلي فلك البروج و ميل الحضيض الى خلافه و ان الزهرة و عطارد لهما في العرض ثلث اختلافات احدها و ثانيها ما تقدم للكواكب العلوية و ثالثها ميل القطر الذي يمر بالبعدين الاوسطين من فلك التدوير و مقادير هذه الميول على ما وُجِدَت[122] بالرصد مذكورة عند صفاتها فلتكن دايرة **اب ج د** على مركز **ه** دايرة فلك البروج و **اح ج س** دايرة الفلك المايل على مركز **ز** و **ا** عقدة الرأس و **ج**[123] عقدة الذنب و[124] **اح ج** شمالي الا في عطارد و **ب ى ل** فلك التدوير على مركز **ح** و نتوهم **ح ب** الى[125] ما يلي فلك البروج و **ح ل** الى خلافه و نجعل نصف قطر فلك التدوير و هو **ح ى** مقدار[126]

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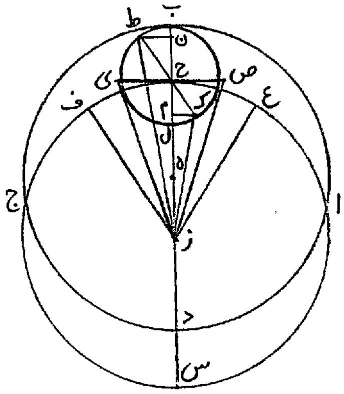
[117] A om. اما على ما تبين في المقدمة الرابعة

[118] A om. from here up to the end of this chapter

[119] F فتمام instead of فتمامه

[120] Marginal note in V: فعلى ما تبين في المقدمة

جيب غاية ميل الذروة او الحضيض من فلك التدوير و لتكن الدايرة قاطعة لسطح الفلك المايل على زوايا قائمة حتى يكون النصف الذي عليه **ب ط ل** الى ما يلي فلك البــروج و النصـــف الآخر الى ما يلي الفلك المايل و لتكن زاوية **ح ز ى** غاية ميل ذروة فلك التدوير عن الفلك



المايل الى ما يلي فلك البروج و زاوية **ىزف** فضل ميل الفلك المايل على ميــل ذروة فلــك التدوير و زاوية **ص زح** غاية ميل الحضيض عن الفلك المايل الى خــلاف ميــل الــذروة و زاوية **ع زص** فضل[127] ميل الفلك المايل و هذه الزوايــا معلومـــة بالرصـــد فلــيكن **ىط**[128] الخاصة المعدلة و ط ب[129] تمامها و **ط ن** جيب **ط ب** و **ح ن** مساوٍ لجيب **ىط** فكل واحــد من **ط ن ح ن** بمقدار **ح ط** معلوم و **زح** ستون جزءاً فــ**زن** معلوم و مربعه مع مربـــع **ن ط** مثل مربع **زط** فــ**زط** معلوم فــ**ط ن** على ان **ط ز**[130] ستون جزءاً معلوم و هو جيب زاويـــة **ط زن** فزاوية **ط زن** معلومة فزاوية **ط زى** معلومة فجميع زاوية **ط زف** معلومة و للزهرة[131] و عطارد ما يحصل[132] من زاوية **ط زن** ينقص من زاوية **ح زى** وهى زاويـــة غايـــة ميــل الذروة عند احدى عقدتى الرأس و الذنب فيهما و ايضاً قوس **ص ك** هي ما تفضل من الخاصة المعدلة على تسعين فــ**ك ل** الخاصة المعدلة و **ك م**[133] جيب **ك ل** و **م ح** مساوٍ لجيب تمامهـــا

[121] V احتلافان instead of احتلافين
[122] V وجد instead of وجدت
[123] V ح instead of ج
[124] V ف instead of و
[125] V على instead of الى
[126] V بمقدار instead of مقدار
[127] F and V om. فضل, found in A
[128] F and V erroneously ب ط instead of ىط
[129] F and V erroneously ط ى instead of ط ب
[130] V زط instead of ط ز
[131] V الزهرة instead of للزهرة
[132] V حصل instead of يحصل
[133] V ك instead of ك م

ص ك فكل واحد من **ك م م ح** بمقدار **ح ك** معلوم و **زح** ستون جزءاً[134] فزم معلوم و مربعه مع مربع **م ك**[135] مثل مربع **ك ز** فك **ز** معلوم فم **ك** على ان **ك ز** ستون جزءاً معلوم و هو جيب زاوية **م زك** فزاوية **م زك** معلومة فزاوية **ك زص** معلومة فجميع زاوية **ك زع** معلومة و للزهرة[136] و عطارد ما يحصل من زاوية **م زك** ينقص من زاوية **ح زص** كما قلنا في الاول و ذلك ما اردنا ان نبين

شرح حسابه اما دقايق حصص العرض فهى دقايق نسبتها الى ستين دقيقة كنسبة الجزء من ميل الفلك المايل الى كله و كنسبة الجزء من عرض القمر الى كله[137] فنقسم الجزء من عرض القمر[138] على كله منحطاً فيحصل الجزء من دقايق حصص العرض و اما ميل البعدين الاوسطين للزهرة وعطارد و هو الملقب بالانحراف فنسبة الجزء منه الى كله كنسبة الجزء من التعديل الثاني الى كله فنضرب الجزء من التعديل الثاني في كل الانحراف و هو درجتان[139] و نصف و نقسمه على كل[140] التعديل الثاني فيحصل الجزء من الانحراف و اما ميل الذروة و الحضيض من فلك التدوير فان[141] ذلك ايضاً بحسب الخاصة المعدلة على[142] ما يرشدك اليه الشكل و البرهان

شرح جداوله اوايل الجداول المرسومة بالشمال و الجنوب هى فضل ميل[143] الفلك المايل على ميل ذروة فلك التدوير اما الشمال فاذا كان مركز فلك التدوير في النصف[144] الشمالي[145] من الفلك المايل و اما الجنوب فاذا كان مركز فلك التدوير في النصف الجنوبي من الفلك المايل و اما ميل الزهرة و عطارد فهو غاية ميلهما عند احدى العقدتين اما للزهرة[146] فعند الرأس و اما لعطارد[147] فعند الذنب و كلاهما[148] جنوبي اعنى ميل ذروة فلك التدوير

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[134] V om. جزءاً

[135] V م ط instead of م ك

[136] V الزهرة instead of للزهرة

[137] A om. و كنسبة الجزء من عرض القمر الى كله

[138] A ميل الفلك المايل instead of عرض القمر

[139] V درجتين instead of درجتان

[140] V om. كل

[141] V الى instead of فان

[142] F الى instead of على

[143] F om. ميل

[144] F om. From here up to the next في النصف

[145] V الشمال instead of الشمالي found in A

[146] V الزهرة instead of للزهرة

[147] V عطارد instead of لعطارد

[148] V كليهما instead of كلاهما

شرح العمل بالجداول[149] نأخذ دقايق حصص العرض بالمركز المعدل لزحل بزيادة **ن** جزءاً و للمشتري بنقصان **ك** جزؤاً و للمريخ كما هو لان اوج زحل منتج[150] عن نقطة **ح** الى ما يلـــي **ج** و هو عقدة الذنب بخمسين جزءاً و اوج المشتري عن **ح** الى ما يلي **ا** بعشـــرين جـــزءاً و اوج[151] المريخ عند **ح** و هو نهاية ميل الفلك المايل و قد قلنا ان دقايق حصص العرض هـــي بدل من ميل فلك المايل بحسب بعد مركز فلك التدوير من العقدة ثم نأخذ العـــرض بالخاصـــة المعدلة اما اذا كان المركز المعدل في نصف **اح ج** فالعرض شمالي لان ميل فلك التدوير في هذا النصف الى[152] الشمال و اما اذا كان المركز المعدل في نصف **اس ج** فالعرض جنـــوبي لان ميل فلك التدوير في هذا النصف الى الجنوب ثم نضرب العرض فـــي دقـــايق حصـــص العرض لنأخذ منه بحسب بعد مركز فلك التدوير عن[153] عقدتى الجوزهر فاما[154] الزهـــرة و عطارد فاوج الزهرة عند **ح** و هو النهاية الشمالية و اوج عطارد عنـــد **س** و هـــو النهايـــة الجنوبية فنأخذ الميل و الانحراف بالخاصة المعدلة فاما انحراف عطارد فعند الاوج **ب يـــه** و عند مقابلة الاوج **ب مه** فاستثقل وضع جدولين لذلك فوضع جدول واحد على **ب ل** ثـــم فـــي ناحية الاوج نقص منه العشر و في ناحية مقابلة الاوج زيد عليه العشر و اكتفى بذلك ثم نزيد على المركز المعدل للزهرة ثلثة بروج و لعطارد تسعة بروج ليكون المبلغ هـــو البعـــد مـــن الرأس او الذنب اما ان كان المبلغ اقل من تسعين او اكثر من مأتى و سبعين فالبعد من الرأس و اما ان كان المبلغ اكثر من تسعين [ا]و اقل من مأتى و سبعين فالبعد من الذنب فنأخـــذ بـــه دقايق حصص العرض و نضربه في الميل لنأخذ منه بقسط بعد المركز من العقدة لان نهايـــة هذا الميل عند العقدتين فان وقع المركز المزيد[155] عليه و التدوير في نصف واحد من الفلـــك المايل فهذا العرض جنوبي و ان اختلف موقعهما فالعرض شمالي لان ميل الذروة مـــن فلـــك التدوير فيما بين **س ا اح** جنوبي و ميل الحضيض شمالي و فيمابين **ح ج**[156] **ج س** فبالضـــد فاذا وقع المبلغ فيما بين **اح ج** وقع المركز فيما بين **س ا اح** فان وقع التـــدوير ايضـــاً فـــي النصف الاعلى فالميل جنوبي و اذا وقع المبلغ فيما بين **ج س ا** و هو النصف الاســـفل وقـــع المركز فيما بين **ح ج س** فان وقع التدوير ايضاً في نصف الاسفل فالميل جنوبي فظاهر ممـــا قلنا انه ان اختلف موقع المبلغ و موقع التدوير كان هذا العرض شمالياً ثم نأخذ المركز المعدل

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[149] V بالجدول instead of بالجداول

[150] V منتجي instead of منتج

[151] V فاوج instead of و اوج

[152] V repeats from here up to the next هذا النصف الى

[153] V من instead of عن

[154] V و اما instead of فاما

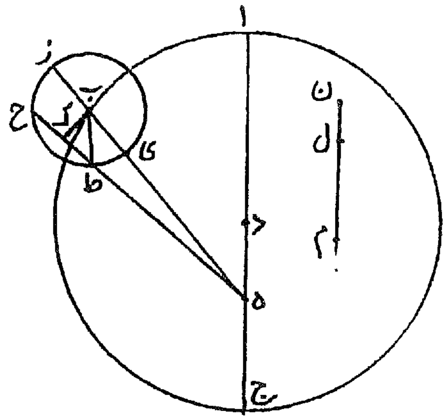
[155] V المزاد instead of المزيد

[156] F ج ح instead of ح ج

للزهرة كما هو و لعطارد بزيادة ستة بروج و نأخذ به دقايق حصص العرض و نضـــربها[157] في الانحراف لنأخذ منه بقسط بعد المركز من الاوج للزهرة و من مقابلة الاوج لعطـــارد لان نهاية الانحراف عند نهاية ميل الفلك المايل فان وقع المركز هذا[158] فيما بـــين **اح ج** و هـــو النصف الاعلى و التدوير فيما بين **ب ىل** من فلك التدوير فهذا العرض شـــمالي و ان كـــان التدوير في النصف الآخر فالعرض جنوبي لان قطر **ح ى** طرف **ى** منه فيما بين **اح ج** الـــى الشمال و الطرف الآخر الى الجنوب و ان وقع المركز فيما بين **ج س ا** و التدوير اقل مـــن مائة و ثمانين[159] فهذا العرض جنوبي و ان كان التدوير اكثر فالعرض شمالي لان طـــرف **ى** من قطر **ح ى** فيما بين **ج س ا** الى الجنوب و الطرف الآخر الى الشمال و في هذه الجهـــات اغفل البتاني في رسالة زيجه ان لم يكن السهو وقع من الوراق ثم نضرب دقـــايق حصـــص العرض هذه التي اخذناها اخيراً للزهرة في سدس درجة و لعطارد في نصف و ربع درجـــة لنأخذ من ميل الفلك المايل بحسب بعد المركز من العقدة و هذا الميل للزهرة شمالي و لعطارد جنوبي ابداً فاما زيادة ستة بروج على مركز عطارد في اخذ دقايق حصص العـــرض اولاً و ثانياً فلان ينتقل من ناحية الاوج الى مقابلته فيكون الحكم على عروضه و جهاته[160] كـــالحكم على عروض الزهرة وجهاتها فيطرد الكلام عليها بعبارة واحدة و ذلك ما اردنا ان نبين

الباب العاشر في رجوع الكواكب

اب ج دايرة الفلك الحامل على مركز **د** و **اج** قطره و **ه** مركز الفلك المايل و **زح ط ى** دايرة فلك التدوير على مركز **ب** و خط **ب ه** بعد مركز فلك التدوير من مركـــز **ه** و قـــد تقـــدمت



[157] V نضربه instead of نضربها
[158] F om. هذا
[159] V ثمنين instead of ثمانين
[160] V جهاتها instead of جهاته

معرفته في باب التعديل الاول و نخرج **ه ط ح** يمر بالوقفة الاولى و نصل **ب ك** عموداً على **ح ط** فعلى ما بين بطلميوس و من قبله من المتقدمين نسبة **ك ط** الى **ط ه** كنسبة مسير[161] مركز فلك التدوير الى مسير الكوكب في فلك التدوير و **ب ز** نصف قطر فلك التدوير المعدل بحسب بعد مركزه من البعد الاوسط و هو معلوم و نصل **ب ط** فقوس **ىط** نصف قوس الرجوع من فلك التدوير و زاوية **ب ه ك** نصف زاوية الرجوع فـ**ب ه** معلوم و **ب ى** معلوم فـ**ى ه** الباقي ايضاً[162] معلوم و جميع **زه** معلوم فضرب **زه** في **ه ى** معلوم و هو على ما تبين في الاصول[163] مساوٍ لضرب **ح ه** في **ه ط** فضرب **ح ه** في **ه ط** معلوم و نسبة **ك ط** الى **ط ه** معلومة[164] و **ح ط** ضعف **ك ط** فنسبة **ح ط** الى **ط ه** معلومة[165] و لتكن كنسبة **ن ل** الى **ل م** فمسطح **ح ه** في **ه ط** شبيه بمسطح **ن م** في **ل م**[166] لان زواياهما متساوية واضلاعهما متناسبة فعلى ماتبين في الاصول نسبة مسطح **ح ه** في **ه ط** الى مسطح **ن م** في **م ل** كنسبة مربع **ح ه** الى مربع **ن م**[167] و مسطح **ح ه** في **ه ط** معلوم و مسطح **ن م** في **م ل** معلوم و مربع **ن م** معلوم فمربع **ح ه** معلوم فـ**ح ه** معلوم و نسبة **زه** الى **ه ط** كنسبة **ح ه** الى **ه ى** لان ضرب **زه** في **ه ى** مثل ضرب **ح ه** في **ه ط** و **زه** معلوم و **ح ه** معلوم و **ه ى** معلوم فـ**ه ط** معلوم فكل واحد من **ه ط ط ح** معلوم فـ**ط ك** معلوم و **ك ه** معلوم فـ**ك ه** على ان **ب ه** ستون جزواً معلوم فقوسه معلومة و هي زاوية **ه ب ك** فزاوية **ه ب ك** معلومة و ايضاً **ك ط** على ان **ب ط** ستون جزءاً معلوم فقوسه معلومة و هي زاوية **ط ب ك** فزاوية **ط ب ك** معلومة فاذا نقصناها من زاوية **ه ب ك** بقيت زاوية **ىب ط** و هي زاوية قوس **ط ى**[168] فقوس **ط ى** معلومة وهي نصف قوس الرجوع من فلك التدوير و اذا نقصنا زاوية **ه ب ك** من زاوية **ب ك ه** القائمة بقيت زاوية **ب ه ك** و هي زاوية نصف قوس الرجوع من فلك البروج فلو لم يكن لمركز فلك التدوير حركة الى جهت المشرق لكانت زاوية **ب ه ك** و قوس **ى ط** معدله لكن لما كانت له حركة عمدنا الى وجود عدد نسبته الى قوس **ىط** كنسبة مسير مركز فلك التدوير الى مسير الكوكب في فلك التدوير و ننقص العدد الموجود من زاوية **ب ه ك** و قوس **ىط** فيبقى زاوية **ب ه ك** و قوس **ىط** معدله و اذا قسمنا اجزاء زاوية **ب ه ك** المعدلة على

[161] V repeats مسير

[162] V om. ايضاً

[163] A add. كز من ج in red; as a reference to the *Elements*, it should be لو من ج (III.26)

[164] V معلوم instead of معلومة

[165] V معلوم instead of معلومة

[166] V م ل instead of ل م

[167] A add. يد من ز in red; as a reference to the *Elements*, it should be كج من و (VI.23)

[168] F om. و هي زاوية قوس ط ى

وسط يوم الكوكب[169] حصل نصف ايام الرجوع و ضعفها[170] ايام الرجوع كلها و ذلك ما اردنا ان نبين[171]

[169] F للكوكب instead of الكوكب

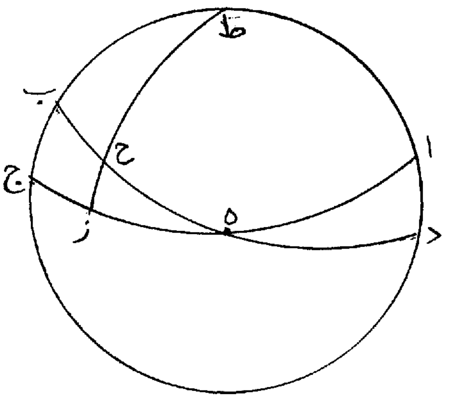
[170] V ضعف ذلك instead of ضعفها

[171] V add. و نختم الفصل الرابع بهذا الباب و الله محمود

الفصل الخامس في اعمال طوالع النهار و الليل ستة عشر[1] باباً

الباب الاول في الميل الاول

ليكن **اب ج د** الدايرة الماره بقطبى معدل النهار و فلك البروج و **اه ج**[2] معدل النهار على قطب **ط** و **ب د** فلك البروج و **ه** احد الاعتدالين و نفرض **ه ح** من فلك البروج نريد ميلها الاول و ندير قوس **ط ح ز** فـ**ح ز** هو الميل الاول لقوس **ه ح** فاقول انه معلوم



برهانه مثلث **ه ح ز**[3] زاويه ز منه قائمة و زاوية **ه** الميل الاعظم فنسبة جيب **ه ح** الى جيب **ح ز** كنسبة الجيب الاعظم الى جيب زاوية **ه** و **ه ح** معلوم و الميل الاعظم معلوم بالرصد فـ**ح ز** معلوم و ذلك ما اردنا ان نبين[4]

الباب الثاني في مطالع البروج بخط الاستواء

ليكن **اب ج د** الدايرة المارة بالاقطاب و **اه ج** معدل النهار على قطب **ط** و **ب ه د** فلك البروج و **ه** احد الاعتدالين و نفرض **ه ح** من فلك البروج نريد مطالعها بخط الاستواء و ندير قوس **ط ح ز** فـ**ه ز** مطالع قوس **ه ح** فاقول انه معلوم

برهانه مثلث **ه زح**[5] زاوية **ز** منه قائمة و **ه** زاوية الميل الاعظم و **ح ز** ميل **ح ه**[6] فعلى ما تبين في المقدمة الرابعة نسبة جيب **ه ز** الى ظل **زح** كنسبة الجيب الاعظم الى ظل زاوية **ه** و

[1] V erroneously خمسة عشر instead of ستة عشر

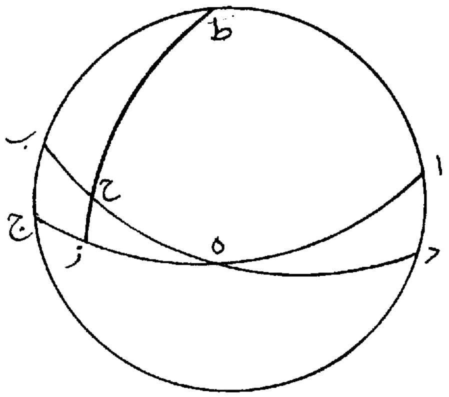
[2] V اج instead of اه ج

[3] F ه ح instead of ه ح ز

[4] F add. و هذه الدايرة مثالها

[5] V ه ح ز instead of ه زح

[6] V ه ح instead of ح ه

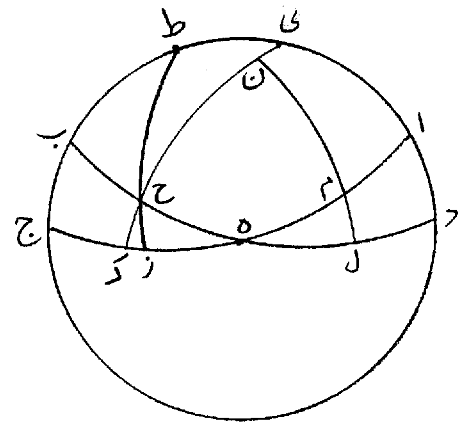


زح معلوم و الميل الاعظم معلوم فـه **ز**[7] معلوم و ذلك ما اردنا ان نبين

وجه آخر و ايضاً مثلث **ه زح**[8] زاوية **ز** منه قائمة فعلى ما تبين في المقدمة الثانية نسبة جيب تمام **ه ز** الى جيب تمام **ه ح** كنسبة الجيب الاعظم الى جيب تمام **زح** و تمــام **ه ح** معلــوم و تمام **زح** معلوم فتمام **ه ز**[9] معلوم فـه **ز** معلوم و ذلك ما اردناه[10]

الباب الثالث[11] في الميل الثاني

اب ج د على مركز **ه** الدايرة المارة بالاقطاب و **اه ج** معدل النهار علــى قطــب **ط** و **ب ه د** فلك البروج على قطب **ى** و **ه** احد الاعتدالين و نفرض **ه ح** من فلك البروج نريد ميلها الثاني و ندير قوس **ى ح ك** فـ**ك ح** هو الميل الثاني لقوس **ه ح** فاقول انه معلوم



[7] F فهو instead of فـه ز

[8] V ه ح ز instead of ه زح

[9] F ه ز ح instead of ه ز

[10] V اردنا ان نبين instead of اردناه

[11] F الثاني instead of الثالث

برهانه مثلث **ه ح ك** زاوية **ح** منه قائمة و زاوية **ه** زاوية الميل الاعظم فنسبة[12] جيب **ه ح** الى ظل **ح ك** كنسبة الجيب الاعظم الى ظل زاوية **ه** و **ه ح** معلوم فظل **ح ك** معلوم فهو اذن معلوم و ذلك ما اردناه[13]

وجه آخر و ايضاً ندير قوس **ط ح ز** و نجعل نقطة **ك** قطباً و ندير ببعد ضلع المربع قوس **ن م ل** فـ**ل** قطب دايرة **ى ح ك** و كل[14] واحد من **ك ن ن ل** ربع دايرة و **ح ز** هو الميل الاول لقوس **ه ح** و **ه ل** تمام **ه ح** و **م ل** هو الميل الاول[15] لقوس **ه ل** و **م ن** تمام **م ل** و هو مقدار زاوية **ن ك م** فمثلث **ح ك ز** زاوية **ز** منه قائمة و زاوية **ك** معلومة فنسبة جيب **ك ح** الى جيب **ح ز** كنسبة الجيب الاعظم الى جيب زاوية **ك** و **ح ز** معلوم و زاوية **ك** معلومة فـ**ح ك** معلوم و ذلك ما اردناه[16]

وجه آخر و[17] اذا جعلنا **ه ح** من معدل النهار و هو معلوم و **ه ك** من فلك البروج كان **ك ح** الميل الاول لقوس **ه ك** فاذا قوسنا **ه ح** في مطالع خط الاستواء[18] حصل **ه ك** معلوماً و يسمى عكس المطالع فاذا اخذنا ميله الاول كان **ح ك** و هو الميل الثاني لقوس **ه ح** فـ**ح ك** معلوم و ذلك ما اردنا ان نبين

المطالع من الميلين **ه ح** من فلك البروج و **ه ز** من معدل النهار و **ح ز** الميل الاول لقوس **ه ح** و الميل الثاني لقوس **ه ز** فاذا قوسنا **ح ز** في جدول الميل الثاني حصل **ه ز** معلوماً و هو مطالع **ه ح** بخط الاستواء[19] فالمطالع من الميلين معلوم و ذلك ما اردنا ان نبين

الباب الرابع في بعد الكواكب عن معدل النهار

اب ج د الدايرة المارة بالاقطاب و **اه ج** معدل النهار على قطبى **ل م** و **ب ه د** فلك البروج على قطبى **ك ن** و نفرض الكوكب اولا نقطة **ز** ليكون العرض و الميل الثاني في جهة[20] و نجيز قوسى **ك ط ز ل ى ز** فـ**ح ز** عرض الكوكب و **ح ط** ميله الثاني و **ز ى** بعده عن معدل النهار فاقول انه معلوم

---

[12] V نسبة instead of فنسبة

[13] V و الميل الاعظم معلوم فـح ك معلوم و ذلك ما اردنا ان نبين instead of فظل ح ك معلوم ... اردناه

[14] V فكل instead of و كل

[15] From الاول لقوس ه ح ... up to here is missing in F, restored from V

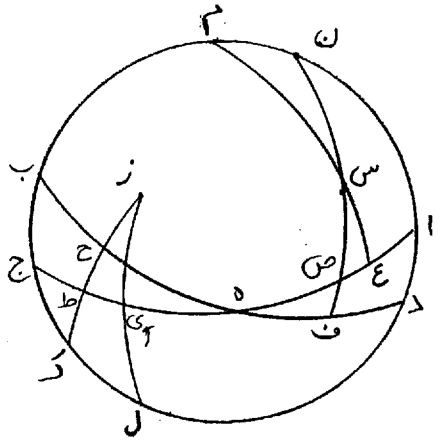
[16] V اردنا ان نبين instead of اردناه

[17] V add. ايضاً

[18] V الاستوى instead of الاستواء

[19] V الاستوى instead of الاستواء

[20] F وجهة instead of جهة



برهانه ان مثلثی[21] **زط ی ك ط ج** متشابهان لان زاويتی **ط** منهما متساويتان و زاويتا **ی ج** قائمتان فنسبة جيب **ط ز** الی جيب **ز ی** كنسبة جيب **ط ك** الی جيب **ك ج** و **ط ز** معلوم و هـــو العرض و الميل الثاني و **ط ك** تمام الميل الثاني و **ك ج** تمام الميل الاعظم لان **ك ل** هو الميل الاعظم فـ**ز ی**معلوم

و ايضاً نفرض الكوكب نقطة **س** لتكون العرض و الميل الثاني في جهتين و نجيز قوسی **م س ع ن س ف** فـ**ف س** عرضه و **ف ص** ميله الثاني و **س ع** بعده عن معـــدل النهـــار فاقول انه معلوم

برهانه ان مثلثی[22] **ص س ع ص ن ا** متشابهان لان زاويـــة **ص** مشـــتركة و زاويتـــا **ا ع** قائمتان فنسبة جيب **ص س** الی جيب **س ع** كنسبة جيـــب **ص ن** الـــی جيـــب **ن ا** و **ص س** معلوم و **ص ن** تمام الميل الثاني و **ن ا** تمام الميل الاعظم لان **ن م** هو الميل الاعظم فـ**س ع** معلوم و ذلك ما اردنا ان نبين

الباب الخامس في عرض البلد

اب ج د دايرة الافق و **ه**[23] سمت الرأس و **اه د** نصف النهار و **ج ی ك** معـــدل النهـــار و **ط** قطبه و **ب ی ز** فلك البروج و[24] **ه ح** عرض البلد فاقول انه معلوم

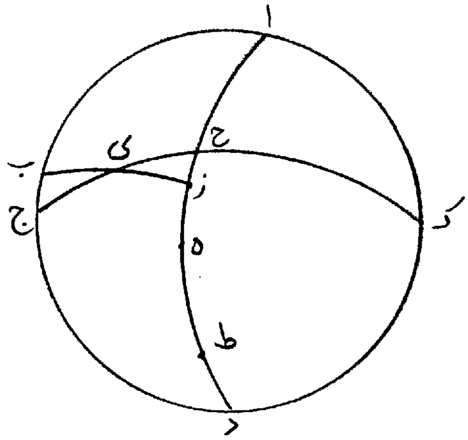
---

[21] V فمثلثا instead of ان مثلثی

[22] V فمثلثا instead of ان مثلثی

[23] F om. ه

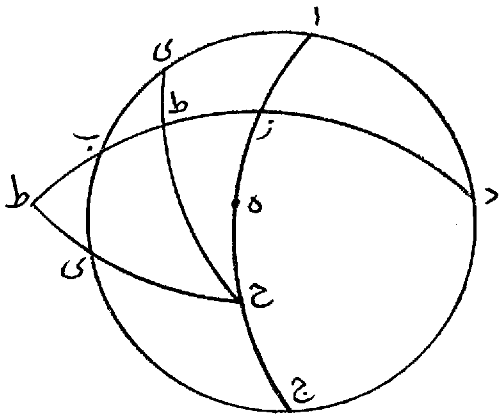
[24] V ف instead of و



برهانه ان[25] **ا ز** غاية ارتفاع الشمس موجود[26] بآلة من آلات الارتفاع و **زح** ميــل الشــمس فـ**اح** معلوم و هو تمام **ه ح** فـ**ه ح** معلوم و هو عرض البلد[27] و اذا كان نقطه **ح** مــن فلــك البروج و **ز** من معدل النهار و **ه ز** عرض البلد كان **اح** غاية الارتفاع و **زح** ميــل الشــمس فجميع **ا ز** معلوم و هو تمام **ه ز** فـ**ه ز** عرض البلد معلوم و ذلك ما اردنا ان نبين

الباب السادس في سعة مشرق الشمس و الكوكب

اب ج د دايرة الافق و **ه** سمت الرأس و **اه ج** نصف النهار و **ب زد** معدل النهار و **ح** قطبه و لتكن نقطة **ى** مطلع الشمس او الكوكب يومئذ فقوس **ب ى** سعة المشرق فاقول انه[28] معلوم



برهانه ندير قوس **ح ط ى** فكل واحد[29] من قوسى **ب ز ب ا** ربع دايرة و **ا ز** تمام عــرض البلد و هو مقدار زاوية **زب ا** فمثلث **ب ى ط** زاوية **ط** منه قائمة و زاوية **ب** معلومة و **ى ط**

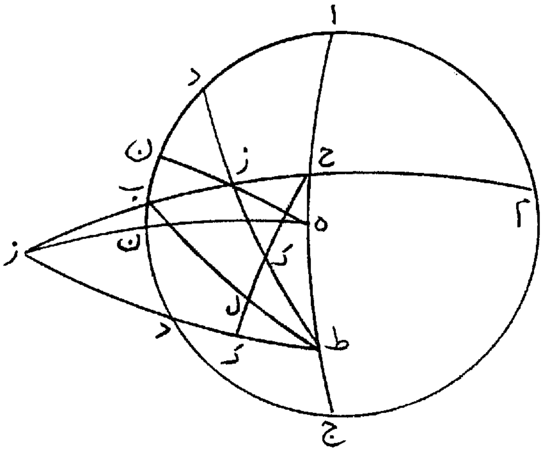
[25] V om. ان
[26] V موجوداً instead of موجود
[27] V عرض البلد معلوم instead of معلوم و هو عرض البلد
[28] F فانه اته instead of فاقول انه
[29] V واحدة instead of واحد

ميل الشمس او[30] بعد الكوكب عن معدل النهار فنسبة جيب **ب ى** الى جيب **ى ط** كنسبة الجيب الاعظم الى جيب زاوية **ب** فـ**ب ى** معلوم و ذلك ما اردناه[31]

وجه آخر ط ز نصف قوس النهار على ما نبينه في الباب العاشر فمثلثا **ح ط ز ح ى ا** زاويتا **ز ا** منهما قائمتان و **ط ز** مقدار زاوية **ح** لان **ط ح**[32] ربع دايرة و نسبة[33] جيب **ح ى** الى جيب **ى ا** كنسبة الجيب الاعظم و هو جيب **ح ط** الى جيب زاوية **ح** و جيب **ح ى**[34] مساوٍ لجيب تمام **ى ط** و **ى ط** ميل الشمس او بعد الكوكب عن معدل النهار و زاوية **ح** معلومة و هى قوس **ط ز** فـ**ى ا** معلوم و هو تمام **ى ب** و **ى ب** سعة المشرق فسعة المشرق معلومة[35] و اذا اقمنا نقطة **ج** مقام نقطة **ا** كان البرهان على وجود[36] تمام سعة المشرق من جهتى الشمال و الجنوب سواء و ذلك ما اردنا ان نبين

الباب السابع في تعديل نهار الشمس و الكوكب

ا ب ج د دايرة الافق و **ه** سمت الرأس و **اه ج** نصف النهار و **ب ح م** معدل النهار على قطب **ط** و لتكن نقطة **د** الدرجة التي نريد تعديل نهارها و نجيز عليها قوس **ط زد** فيكون **د ز** ميل نقطة **د** و **ب ز** تعديل نهارها و **ب د** سعة مشرقها فاقول ان **ب ز** معلوم



[30] F repeats او

[31] V اردنا ان نبين instead of اردناه

[32] V ح ط instead of ط ح

[33] V فنسبة instead of و نسبة

[34] F and M زاوية ح instead of ح ى

[35] V معلوم instead of معلومة

[36] F الوجود على and M البرهان على instead of البرهان على وجود found in V

برهانه مثلث **ب دز** قائم الزاوية و هى زاوية **ز** فنسبة[37] جيب تمام **دز** الى جيب تمام **دب** كنسبة الجيب الاعظم الى جيب تمام **ب ز** و **دز** معلوم و **دب** معلوم فـ**ب ز** معلوم[38] و ذلك ما اردنا ان نبين

وجه آخر نخرج قوس **ه ز ن**[39] فمثلثا **دزن**[40] **دط ج** زاويتا **د** منهما متساويتان و زاويتا **ن ج** قائمتان فعلى ما تبين في المقدمة الاولى نسبة جيب **دز** الى جيب **زن** كنسبة جيب **دط** الى جيب **ط ج** و **دز** هو الميل او البعد و **دط** تمام الميل او البعد و **ط ج** عرض البلد فـ**زن** معلوم و ايضاً نسبة جيب **ب ز** الى جيب **زن** كنسبة جيب **ب ح** الى جيب **ح ا** و **زن** معلوم و **ب ح** ربع دايرة و **ح ا** تمام عرض البلد فـ**ب ز** معلوم و ذلك ما اردناه[41]

وجه آخر مثلث **ب زد** زاوية **ز** منه قائمة و زاوية **ب** مثل تمام عرض البلد و هو **ح ا** لان كل واحدة من قوسى **ب ح ب ا** ربع دايرة فعلى ما تبين في المقدمة الرابعة نسبة جيب **ب ز** الى ظل **زد** كنسبة الجيب الاعظم الى ظل زاوية **ب** فينبغي ان نقسم ظل الميل و هو **زد** على ظل[42] تمام عرض البلد منحطاً الا ان ذلك كما تبين في التدكره الثالثة و هو الباب السابع من الفصل الثالث[43] في المقدمات مساوٍ لضرب[44] ظل الميل في ظل[45] عرض البلد منحطاً فما حصل فهو جيب **ب ز** فـ**ب ز** معلوم و ذلك ما اردناه[46]

وجه آخر اذا كان تعديل نهار المنقلبين معلوماً فنخرج قوس **ب ط** و هى ربع دايرة و دايرة[47] **دزط** تقوم مقام معدل النهار لانا اذا اثبتنا نقطة **ز** و ادرنا قوس **ط زد** طابقت قوس **ح ب ز** من معدل النهار فنفرض منها مطالع قوس نريد تعديل نهارها و ليكن **ط ك** و نجيز عليه قوس **ح ك ل** فهى ربع دايرة لان قوس **ب ط** مخطوطة على قطب **ح** فزاوية **ل** قائمة فمثلث **ط ك ل** زاوية **ل** منه قائمة و زاوية **ط** مثل التعديل الكلي فعلى ما تبين في المقدمة الاولى نسبة جيب **ط ك** الى جيب **ك ل** كنسبة الجيب الاعظم الى جيب زاوية **ط** و **ط ك** هو المطالع المفروضة و زاوية **ط** معلومة فـ**ك ل** معلوم و هو تعديل النهار الجزؤي و ذلك ما اردناه[48]

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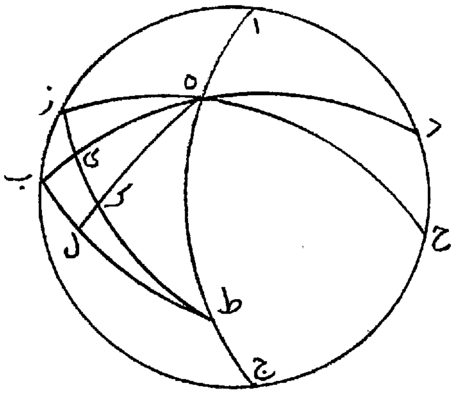
[37] V نسبة instead of فنسبة
[38] F om. فـب ز معلوم
[39] V ه ن ز
[40] F دن ز instead of دزن
[41] V اردنا ان نبين instead of اردناه
[42] V om. ظل
[43] V om. الثالث
[44] V يضرب instead of لضرب
[45] F om. ظل
[46] V اردنا ان نبين instead of اردناه
[47] V om. و دايرة
[48] V اردنا ان نبين instead of اردناه

وجه آخر[49] **لتعديل النهار** اذا كان التعديل الكلي معلوماً **ا ب ج د** دايرة الافق و **اه ج** نصف النهار و **ب ه د** معدل النهار على قطب **ط** و **زه ح** فلك البروج و لتكن نقطة **ز** اول الجدى و لنخرج[50] قوس **ط ى ز** فـ**ب ى** تعديل النهار الكلي و نخرج قوس **ط ب** فكل واحدة من قوسى



ط ى ط ب تقوم مقام معدل النهار لانه اذا اثبت نقطة[51] التقاطع اعني **ى**[52] و اديرت[53] القوس طابقت معدل النهار فنفرض من قوس **ط ى** مقداراً نريد تعديل نهاره و ليكن **ط ك** و نخرج **ه ك ل** يقطع قوس **ط ب** على زاوية قائمة لان **ط ب** مخطوطة على قطب **ه** فيكون **ه ل** ربع دايرة فتصير نسبة جيب **ط ك** الى جيب **ك ل** كنسبة جيب **ط ى** و هو الجيب الاعظم الى جيب **ى ب** و **ط ك** مفروض من معدل النهار و **ط ى** ربع دايرة و **ى ب** التعديل الكلي فـ**ك ل** معلوم و ذلك ما اردنا ان نبين

الباب الثامن في مطالع البلد

لتكن[54] **ا ب ج د** دايرة الافق و **ب ه د** نصف النهار[55] و **اه ج** معدل النهار و **ز ه ى**[56] فلك

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[49] V om. آخر

[50] V نخرج instead of لنخرج

[51] F نقطتا instead of نقطة founding V

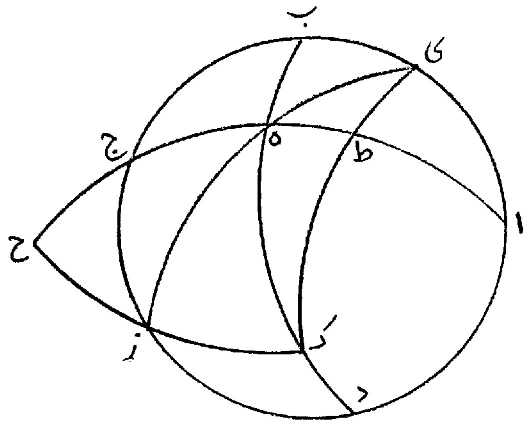
[52] F add. و ب and V add. او ب but both are superfluous

[53] V ادير instead of اديرت

[54] V om. لتكن

[55] The phrase و ب ه د نصف النهار is abundant, but found in all mss.; see commentary

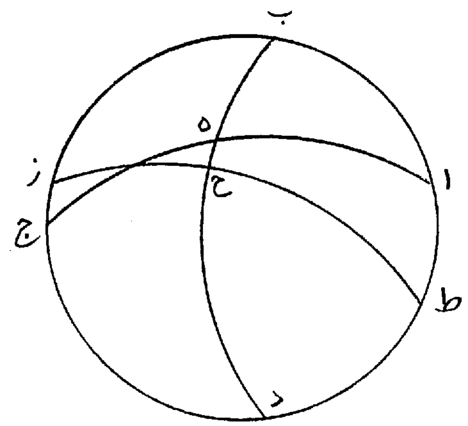
[56] V ى ه ز instead of ز ه ى



البروج و **ك** قطب معدل النهار و نخرج قوسى **ك ط ى**[57] **ك زح** فقوسا **ط ا ج ح** تعديل النهار لنقطتى ى ز و قوسا **ه ط ه ح** مطا لع قوسى **ه ى ه ز** بخط الاستواء و ليكن ه ز شمالياً و ه ى جنوبياً فاذا زدنا **ط ا** على **ه ط** حصل ه ا مطالع ه ى في البلد و اذا نقصنا **ج ح** من **ه ح** حصل **ه ج** مطالع **ه ز** في البلد و ذلك ما اردنا ان نبين

الباب التاسع في غاية ارتفاع الشمس والكوكب

اب ج د دايرة الافق و **ب ه د** نصف النهار و **اه ج** معدل النهار و **زح ط** فلك البروج و نفرض الشمس او الكوكب نقطة **ح** فقوس **ب ح** غاية ارتفاعه فب **ه** تمام عرض البلد و **ه ح**

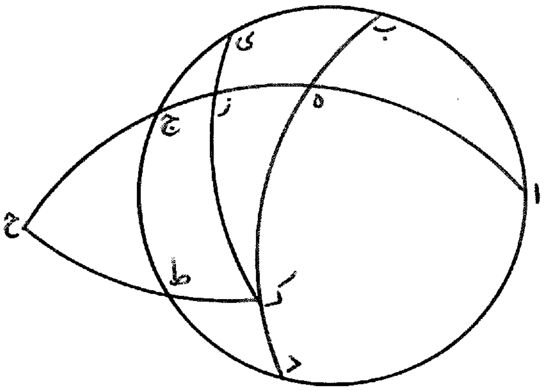


ميل الشمس او بعد الكوكب عن معدل النهار[58] فب **ح** معلوم و ايضاً فليكن **زح ط** معدل النهار و **اه ج** فلك البروج و **ه** موضع الشمس او الكوكب فب **ح** تمام عرض البلد و **ه ح** الميل او البعد فب **ه** معلوم و ذلك ما اردنا ان نبين

[57] F ط ك ى instead of ك ط ى
[58] V om. عن معدل النهار

الباب العاشر في نصف قوس نهار الشمس و الكوكب

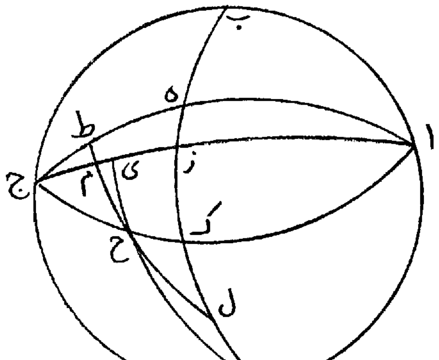
اب ج د دايرة الافق و ب ه د نصف النهار و اه ج معدل النهار و نفرض نقطتى ى ط مطلع فلك البروج و ك قطب معدل النهار و نجيز قوسى ك زى ك ط ح فـج ز تعديل نهار نقطة ى و هي جنوبية و ز ه نصف قوس نهارها و ه ج ربع دايرة فـزه معلوم و ج ح تعديل نهار



نقطة ط و هى شمالية و ه ح نصف قوس نهارها و ه ج ربع دايرة فـه ح معلوم و ذلك ما اردنا ان نبين

الباب الحادي عشر في درجة ممر الكوكب بنصف النهار

اب ج د دايرة الافق و[59] ب ه د مار بالاقطاب و اه ج معدل النهار على قطب ل و ا ز ج فلك البروج على قطب د و ح جرم الكوكب و نجيز[60] ل ح ط د ح ى اح ج[61] فى درجة



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[59] The phrase ا ب ج د دايرة الافق و is abundant, although it is found in all mss.; see commentary

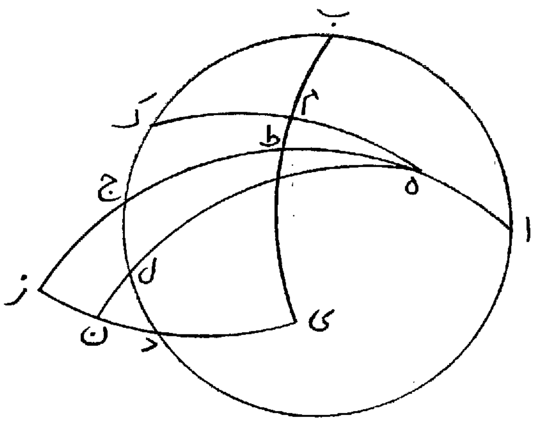
[60] F نج يز instead of نجيز

[61] F om. اح ج

الكوكب و ى ح عرضه و ح ط بعده عن معدل النهار و م درجة ممره و ز نقطة احد المنقلبين فمثلث ك دح زاوية ك منه قائمة و زاوية د معلومة و هى قوس زى لان دح ى ربع دايرة و زى بعد درجة الكوكب من المنقلب و دح تمام العرض و نسبة جيب دح الى جيب ح ك كنسبة الجيب الاعظم الى جيب زاوية د فـح ك معلوم و ايضاً مثلث ل ح ك زاوية ك منه قائمة و ل ح تمام بعد الكوكب عن معدل النهار و ح ك معلوم و نسبة جيب ل ح الى جيب ح ك كنسبة الجيب الاعظم الى جيب زاوية ل و هى جيب قوس ط ه و قوس ط ه مطالع زم بخط الاستواء[62] من اول المنقلب فعلى ما تبين في التذكرة الاولى الاوسطان[63] من المقادير الاول مساويان للاوسطين من المقادير الآخر فبالمساواة نسبة جيب دح تمام العرض الى جيب ط ه مطالع درجة الممر من اول المنقلب بمطالع خط الاستواء[64] كنسبة جيب ل ح تمام البعد عن معدل النهار الى جيب ى ز بعد درجة الكوكب من الانقلاب فـط ه معلوم فـزم معلوم[65] [و ى ز معلوم] فنقطة م معلومة و ذلك ما اردنا ان نبين

الباب الثاني عشر في[66] درجة طلوع الكوكب و غروبه

اب ج د دايرة الافق و اه ج معدل النهار على قطب ى و ه احد الاعتدالين و ه ك من فلك البروج جنوبياً و ه ل منه شمالياً و نفرض ب[67] جرم الكوكب جنوبياً و د جرمه شمالياً و كل



[62] V الاستوى instead of الاستواء

[63] V الاوسطين instead of الاوسطان

[64] V الاستوى instead of الاستواء

[65] F om. معلوم

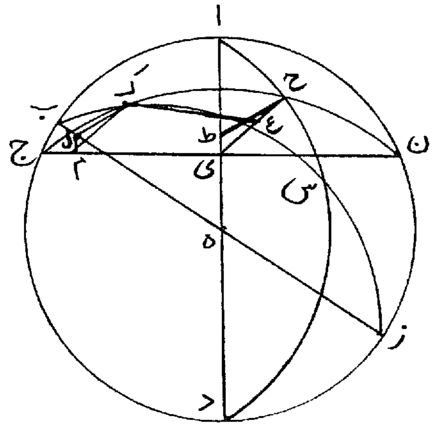
[66] V add. معرفة

[67] F ج instead of نفرض ب

واحد من م ن درجة ممر الكوكب ونخرج ى ب ط ى د ز فـ ج ط[68] تعديل نهار ب فاذا زيد على ه ط و هو مطالع درجة الممر بخط الاستواء لكوكب ب حصل ه ج مطالع ه ك في البلد و ك الدرجة التي تطلع مع الكوكب و ايضاً ه ز مطالع درجة الممر بخط الاستواء[69] لكوكب د و ج ز تعديل نهاره فاذا نقص من ه ز بقي ه ج مطالع ه ل في البلد و ل الدرجة التي تطلـع مع الكوكب فـه ج مطالع الدرجة التي تطلع مع كوكب ب و كوكب د و اذا توهمنا نقطـة ج تحركت بحركة الكل فصارت على الافق الغربي اعني على[70] نقطة ا تكون قد تحركت بقـدر قوس نهار الكوكب و صارت على الافق الشرقي نقطة من معدل النهار تكون مطالع نظيـر الدرجة التي تغيب مع الكوكب و ذلك ما اردنا ان نبين

الباب الثالث عشر في الداير من الفلك لطلوع الشمس و الكوكب من ارتفاع الوقت و الارتفاع من الداير

اب ج د دايرة الافق و اس د نصف النهار و اه د قطره و ب س ز دايرة الارتفاع و ب ه ز قطرها فـس سمت الرأس و قوس ج ح ن من دايرة المـدار فـوق الارض و ج ن وترهـا فنقطة ح تقاطع دايرة المدار و نصف النهار و ك تقاطع دايرة المدار[71] و الارتفاع و نخـرج ح ط عموداً على اه فهو جيب قوس اح و اح ارتفاع نصف النهار لنقطة ح من دايرة المدار و نصل ح ى فهو سهم قوس ج ك ح و ج ك ح نصف قوس النهار و نخرج ك ل عموداً على



[68] F ح ط instead of ج ط
[69] V الاستوى instead of الاستواء
[70] F om. على
[71] F om. و نصف النهار و ك تقاطع دايرة المدار

ب ه فهو جيب قوس ب ك و ب ك ارتفاع الوقت و نخرج ك م عموداً على وتر ج ن فهو جيب ترتيب الداير فاذا توهمنا اس د ب س ز قائمتين في[72] الكرة على قطرى اد ب ز تبين انه كما قلنا ح ط جيب قوس اح و ح ى سهم قوس ج ك ح و ك ل جيب الارتفاع و ك م مواز لح ى فمثلثا ى ح ط م ك ل متشابهان لان زاويتى ط ل قائمتان و زاويتى[73] ح ك متساويتان لان خطى ى ح[74] ح ط موازيان لخطى م ك ك ل فنسبة ح ط جيب ارتفاع نصف النهار الى ك ل جيب ارتفاع الوقت كنسبة ح ى سهم نصف قوس النهار الى ك م جيب ترتيب الداير ف ك م معلوم و نصل ك ع عموداً على ح ى فـ ك م في الكرة مساو لع ى و يبقى ح ع سهم قوس ح ك و ح ك فضل الداير و ك ج معلوم و هو الداير من الفلك و ذلك ما اردنا ان نبين[75]

الارتفاع من الداير و ايضاً فانه اذا كان ك ج معلوماً و هو الداير من الفلك كان ب ك و هو الارتفاع معلوماً و ذلك[76] ان قوس ج ك ح وهو نصف قوس النهار معلوم و ج ك معلوم و هو الداير من الفلك[77] فـ ك ح فضل الداير معلوم فسهمه معلوم و هو فضل ى ح على م ك فـ م ك معلوم و نسبة م ك[78] جيب ترتيب الداير الى ك ل جيب الارتفاع كنسبة ى ح[79] سهم نصف قوس النهار الى ح ط جيب ارتفاع نصف النهار فـ ك ل معلوم فالارتفاع معلوم[80] و ذلك ما اردناه[81]

الباب الرابع عشر في الطالع من الداير و الداير من الطالع

اب ج د دايرة الافق و اه ج نصف النهار و ب ل د معدل النهار و ه قطبه و ح ل ط فلك البروج و نقطة ح منها على الافق و هى المطلوبة فاقول انها معلومة فنفرض ز موضع الشمس او الكوكب من فلك البروج و قوس ز ع مدارها و نرسم قوسين تمران بقطب معدل النهار و بنقطتى ز ع و يقطعان معدل النهار على ك م فقوس م ك شبيهة[82] بقوس ز ع و ز ع[83]

[72] F add. [؟] ن

[73] V زاويتا instead of زاويتى

[74] F ى ح instead of ح ى

[75] V om. و ذلك ما اردنا ان نبين

[76] V ذاك instead of ذلك

[77] V الداير من الفلك معلوم instead of معلوم وهو الداير من الفلك

[78] V add. الى

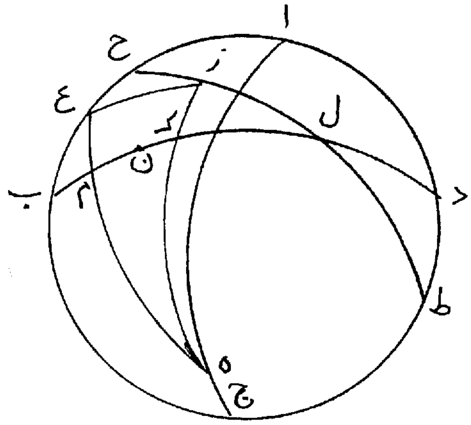
[79] F ى ح instead of ح ى

[80] V om. فالارتفاع معلوم

[81] V اردنا ان نبين instead of اردناه

[82] F شبيه instead of شبيهة

هو الداير من الفلك فـم ك هو الداير من الفلك[84] و ب م تعديل النهار لنقطة[85] ز و ل ك مطالع ل ز بخط الاستواء[86] و نفصل ك ن مساوياً[87] لب م فـل ن مطالع ل ز في البلد و ب م مساوٍ



لك ن و م ن مشترك فـم ك مساوٍ[88] لب ن و م ك الداير من الفلك فـب ن مساوٍ للداير من الفلك[89] فاذا زيد ب ن على ل ن حصل ل ب معلوماً و هو مطالع ل ح في البلد فنقطـة ح و هى درجة الطالع معلومة و ذلك ما اردنا ان نبين[90]

الداير من الطالع و ايضاً فاذا كانت نقطة ح معلومة و ز درجة الشمس او الكوكـب و كـل واحدة[91] من مطالع ل ح ل ز معلومة و هما ل ب ل ن فـب ن معلوم و هو مساوٍ للداير من الفلك و ذلك ما اردناه[92]

الباب الخامس عشر في البرهان على[93] اصل يعم الداير و ما يتعلق به

قد علم[94] من الشكل الحادي و الاربعين[95] في البرهان على الداير من الارتفاع ان نسبة جيـب كل ارتفاع الى جيب ترتيب دايره كنسبة جيب ارتفاع آخر الى جيب ترتيب دايره و معلوم ان

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[83] V ع ز و ع ز instead of زع و زع

[84] V om. فـم ك هو الداير من الفلك

[85] V نهار نقطة instead of النهار لنقطة

[86] V الاستوى instead of الاستواء

[87] V مساوٍ instead of مساوياً

[88] F om. مساوٍ

[89] F om. فـب ن مساوٍ للداير من الفلك

[90] V om. و ذلك ما اردنا ان نبين

[91] V فكل واحد instead of و كل واحدة

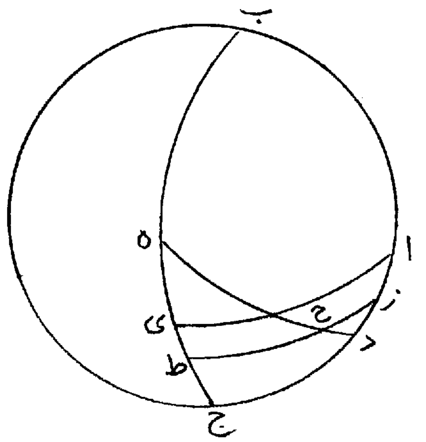
[92] V اردنا ان نبين instead of اردناه

[93] V om. البرهان على

[94] V يعلم instead of علم

[95] A الرابع و الاربعين instead of الحادي و الاربعين; see commentary

كل نقطة نفرض من فلك البروج على الافق تمر بها دايرة من المارة بقطبى معدل النهار فان ما بين النقطة المفروضة و بين معدل النهار من الدايرة المارة بقطبى معدل النهار هو ميل النقطة المفروضة و الخط الخارج من النقطة المفروضة عموداً على قطر معدل النهار[96] هو جيب ميل النقطة و القطر هو الخارج من تقاطع معدل النهار و الدايرة التي تمر بقطبيه و ما بين موقع العمود من هذا القطر و بين تمام نصف القطر جيب تمام ميل[97] النقطة و هو مساو لنصف قطر الدايرة الموازية المارة بالنقطة المفروضة و القطر هو الخارج من النقطة المفروضة فنصف قطر كل دايرة موازية مساو لجيب تمام ميلها[98] و من بعد ما تقدم ذلك فلتكن **اب ج د**[99] دايرة الافق و **ب ه ج** نصف النهار و **ه ح د** من دايرة الارتفاع و **اى** معدل النهار و **زح ط** من[100] الموازية فنسبة جيب **دح** الارتفاع الى جيب ترتيب **ح ز** كنسبة جيب **ىج** ارتفاع نقطة **ى** الى جيب ترتيب **اى** و **اى** ربع دايرة و **ى ج** تمام عرض البلد فضرب جيب الارتفاع في الجيب الاعظم مثل ضرب جيب تمام عرض البلد في جيب ترتيب **ح ز** فجيب ترتيب **ح ز** معلوم بالمقدار الذي يكون به[101] نصف قطر دايرة **اى** ستين جزؤاً[102] و



نريد ان نعلم ذلك بمقدار نصف قطر دايرة **زط** و نصف قطر دايرة **زط** مثل جيب تمام الميل فنسبة جيب ترتيب **ح ز** الى جيب تمام الميل كنسبة الاصل المطلوب الى الجيب الاعظم فضرب جيب ترتيب **ح ز** المعلوم في الجيب الاعظم مثل ضرب الاصل بالمقدار المطلوب في جيب تمام ميل الدرجة[103] فجيب ترتيب **ح ز** بمقدار نصف قطر دايرة **زح ط** معلوم فيصير

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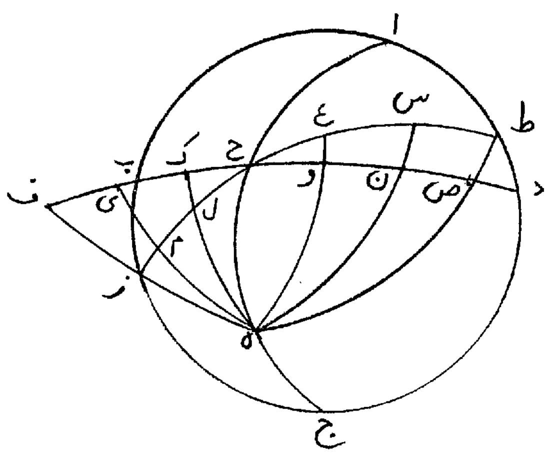
[96] V add. و
[97] F om. ميل
[98] F مثلها instead of ميلها
[99] V فليكن اب ج instead of فلتكن اب ج د
[100] F om. من
[101] F به يكون instead of يكون به
[102] V جزءاً instead of جزؤاً
[103] All mss. ستين instead of جيب تمام ميل الدرجة found in a marginal note of M

الضرب بان[104] نضرب جيب الارتفاع في الجيب الاعظم و نقسمه على جيب[105] تمام عرض البلد ثم نضرب الحاصل في الجيب الاعظم و نقسمه على جيب تمام ميل الدرجة فكانّا ضربنا جيب الارتفاع في الجيب الاعظم مرتين و قسمناه على جيب تمام عرض البلد ثم على جيب تمام الميل و ذلك مساوٍ لما يكون من ضربه في الجيب الاعظم مرتين ثم قسمته[106] على مضروب جيب[107] تمام عرض البلد في جيب تمام ميل الدرجة فاذا ضربنا جيب تمام عرض البلد في جيب تمام ميل الدرجة منحطاً مرتين لانه يحتاج ان يضرب في الجيب الاعظم مرتين كان ما حصل الاصل الذي يخرج منه الداير و ما يتعلق به و ذلك ما اردنا ان نبين[108]

الباب السادس عشر في تسوية البيوت

اب ج د دايرة الافق و **اه ج** نصف النهار و **ب ح د** معدل النهار و **ه** قطبه و **زح ط** فلك البروج و نقطة **ز** الطالع و **ح** وسط السماء و **ط** الغارب و نجيز **ه زف ه ص ط فح ف**[109] نصف قوس نهار درجة الطالع و **ح ص** نصف قوس ليلها فاذا قسمنا **ح ف** بثلثة اقسام **ف ى ى ك ك ح** كان كل قسم منه مثل اجزاء ساعات الطالع مضاعفة و اذا قسمنا **ح ص** بثلثة اقسام **ح و ون ن ص** كان كل قسم منه مثل اجزاء ساعات الغارب مضاعفة لان ازمان كل



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[104] V ان instead of بان

[105] F om. جيب

[106] V نقسمه instead of قسمته

[107] F om. جيب

[108] F om. و ذلك ما اردنا ان نبين

[109] F فح ى instead of فح ف

واحد من **ح ف ح ص** ست ساعات زمانية و اذا اخرجنا من قطب معدل النهار دواير تمـــر بهذه الاقسام قطعت فلك البروج على اقسام هى درجات السواء للاقســـام الاول مـــن معـــدل النهار و هى اقسام **ح ل ل م م ز** و اقسام **ح ع ع س س ط** فاذا نقصنا من مطالع الطالع و هى[110] **ح ب** تسعين درجة[111] بقي مطالع العاشر بمطالع خط الاستواء[112] فاذا وضعنا مطالع العاشر بمطالع خط الاستواء[113] في موضعين و زدنا عليه اجزاء ساعات الطالع مضاعفة مرة بعد مرة و نقصنا منه اجزاء ساعات الغارب مضاعفة[114] مرة بعد مرة حصل مـــن الزايـــد مطالع الحادي عشر و الثاني عشر و الطالع و من الناقص مطالع التاسع و الثامن و الغـــارب بمطالع الاستواء[115] و ان وضعنا مطالع الطالع بمطالع الاستواء[116] في موضعين و نقصنا منها اجزاء ساعات الطالع مضاعفة مرة بعد مرة و زدنا عليه اجزاء ساعات الغارب مضاعفة مرة بعد مرة حصل من الناقص مطالع الثاني عشر و الحادي عشر و العاشر بمطالع الاســـتواء[117] و من الزايد مطالع الثاني و الثالث و الرابع و ذلك ما اردنا ان نبين

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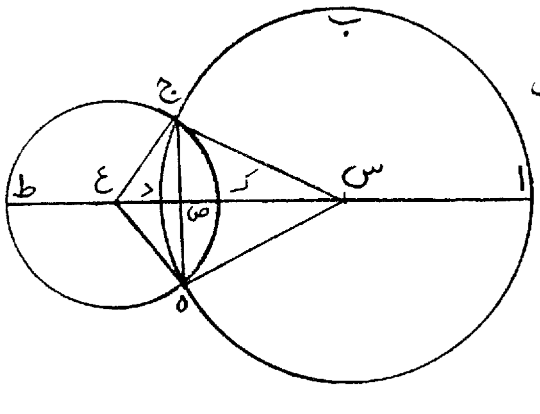
[110] V هو instead of هى
[111] V add. و هو ح ب
[112] V الاستوى instead of الاستواء
[113] V الاستوى instead of الاستواء
[114] F om. مضاعفة
[115] V الاستوى instead of الاستواء
[116] V الاستوى instead of الاستواء
[117] V الاستوى instead of الاستواء

الفصل السادس في الكسوفات و ما يتعلق بها[1] اربعة عشر باباً

الباب الاول في اصابع الخسوف مطلقة و معدلة

اب ج د[2] دايرة الظل في موضع ممر القمر و ج ط ه ك[3] دايرة صفحة القمر و هما على بسيط واحد عند الحس و اد قطر دايرة الظل و ك ط قطر دايرة القمر و س د ك ع نصف القطرين و س ع عرض القمر فك د هو فضل س د ك ع على س ع فك د دقايق الخسوف[4] معلوم و ك ط معلوم فك د على ان ك ط اثناعشر[5] اصبعاً معلوم و هو اصابع الخسوف مطلقة و سطح دج ه ك من صفحة دايرة القمر دقايق الخسوف[6] المعدلة <و هو اصابع الخسوف المعدلة> على ان تكسير صفحة دايرة القمر اثناعشر[7] اصبعاً و هي المطلوبة فنصل ه ج[8] و نخرج خطوط س ج س ه ع ج ع ه فلان اد ج ه يقاطعا في دايرة صار ضرب اص في ص د مثل ضرب ج ص في ص ه و ضرب ط ص في ص ك مثل ضرب ج ص في ص ه ايضاً فضرب اص في ص د مثل ضرب ط ص في ص ك فنسبة اص الى ص ط كنسبة ك ص الى ص د فاذا نقصنا ك د من كل واحد من قطرى اد ك ط بقيت نسبة ا ك الى د ط كنسبة ك ص الى ص د فاذا ركبنا فنسبة ا ك د ط جميعاً الى ط د كنسبة ك د الى د ص و مجموع ا ك د ط معلوم و

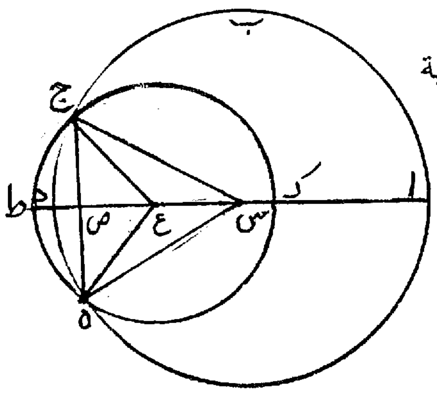
الاولى



[1] F om. و ما يتعلق بها ; V و ما يليق بها instead of و ما يتعلق بها
[2] V اج ده instead of اب ج د
[3] F ح ط ه ك instead of ج ط ه ك
[4] V الكسوف instead of الخسوف
[5] F اثناعشرة instead of اثنا عشر
[6] V الكسوف instead of الخسوف
[7] F اثنتا عشر instead of اثنا عشر
[8] V ه ج instead of ج ه

ط د معلوم و ك د معلوم فـد ص معلوم و هو سهم دايرة الظل فـك ص معلوم[9] و هو سهم دايرة القمر فكل واحد من ط ص ص ك معلوم و ضرب ط ص في ص ك مثل مربع ج ص لان ج ص مساوٍ لـص ه فـج ص معلوم و هو جيب قوس ج ك على ان ع ج نصف قطر القمر فهو على ان ع ج ستون جزءاً معلوم فقوس[10] ج ك من دايرة عظيمة معلومة و هي من الصورة الثانية تمام قوس ج ط من مائة و ثمانين و نسبتها الى ثلثمائة و ستين كنسبة قوس ج ك من محيط الدايرة القمر الى محيط دايرة القمر كله فقوس ج ك من دايرة القمر معلومة و ع ك معلوم فمساحة قطاع ع ج ك ه معلومة و ع ص معلوم و ج ص معلوم فمساحة مثلث ع ج ه معلومة فمساحة قطعة[11] ج ك ه ص من دايرة القمر معلومة و ايضاً ج ص جيب قوس ج د على ان س ج نصف قطر الظل فهو على ان س ج ستون جزءاً معلوم فقوس ج د من دايرة عظيمة معلومة و نسبتها الى ثلثمائة و ستين كنسبة قوس ج د من محيط دايرة الظل الى محيط الدايرة كله فقوس ج د من دايرة الظل معلومة و س د معلوم فمساحة قطاع س ج د ه معلومة و س ص معلوم و ج ص معلوم فمساحة[12] مثلث س ج ه معلومة فمساحة قطعة ج د ه ص من دايرة الظل معلومة فمجموع ج ك ه ص ج د ه ص معلوم و هو دقايق الخسوف[13] معدلة و نسبتها الى مساحة[14] بسيط دايرة القمر كنسبة اصابع الخسوف[15] المعدلة الى اثنى عشر و ذلك ما اردناه[16]

الثانية



[9] F om. و ط د معلوم و ك د معلوم فـد ص معلوم و هو سهم دايرة الظل فـك ص معلوم

[10] F و قوس instead of فقوس

[11] F فقطعة instead of فمساحة قطعة

[12] V om. قطاع س ج د ه معلومة و س ص معلوم و ج ص معلوم فمساحة

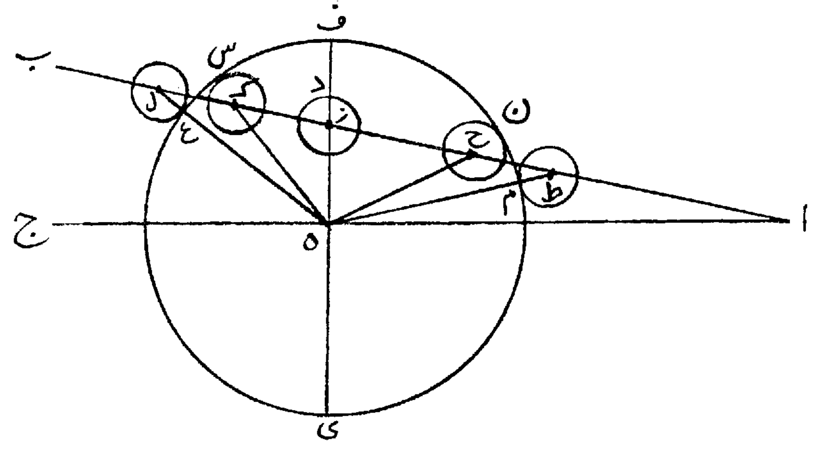
[13] V الكسوف instead of الخسوف

[14] F om. مساحة

[15] V الكسوف instead of الخسوف

[16] V اردنا ان نبين instead of اردناه

لتكن **اب** قطعة من الفلك المايل و **اج** قطعة من فلك البروج و نقطة **ه** مركز دايــرة الظـــل و **ه د**[١٧] قطعة من دايرة **ى ه ف** تمر بقطبى فلك البروج و **ه ز**[١٨] منها عــرض القمــر لوســط الكسوف و **ط** مركز دايرة القمر لبدو الكسوف مماساً لدايرة الظل على نقطة **م** تزيد الــدخول في الكسوف و **ح** مركز دايرة القمر لتمام الكسوف و بدو[١٩] المكث مماساً لدايرة الظل على **ن** تزيد التوغل في الكسوف و **ز** مركز دايرة القمر لوسط الكسوف و هو اقرب ما يكــون مــن مركز الظل و **ك** مركز دايرة القمر لتمام المكث و بدو الانجلاء[٢٠] مماساً لدايرة الظل على **س** تزيد الخروج منها و **ل** مركز دايرة القمر لتمام الانجلاء[٢١] مماساً لدايرة الظل علـــى **ع** تزيــد مفارقة الظل و كل واحد من خطى **ه ط ه ل** نصف القطرين و كل واحد من خطى **ه ح ه ك** نصف قطر الظل منقوصاً منه نصف قطر القمر[٢٢] و **زط** دقايق السقوط من ابتداء[٢٣] الكســوف الى وسطه و **زح** دقايق المكث من بدو المكث الى وسط الكسوف و **زك** دقايق المكــث مــن وسط الكسوف الى بدو الانجلاء[٢٤] و **زل** دقايق السقوط من وسط الكسوف الى تمام الانجلاء[٢٥] فهذه الخطوط هي



[١٧] V ده instead of ه د
[١٨] F هو instead of ه ز
[١٩] V فبدو instead of و بدو
[٢٠] V انجلى instead of الانجلاء
[٢١] V انجلى instead of الانجلاء
[٢٢] F, V, L and A erroneously خطوط ه ط ه ن ه س ه ل نصف القطرين instead of
خطى ه ط ه ل نصف القطرين و كل واحد من خطى ه ح ه ك نصف قطر الظل منقوصاً منه نصف قطر القمر found in Y
[٢٣] V ابتدى instead of ابتداء
[٢٤] V الانجلى instead of الانجلاء
[٢٥] V الانجلى instead of الانجلاء

المطلوبة لان كل واحد منها اذا قسم على سبق القمر حصلت ساعات[٢٦] هذه الدقايق فنأخذ لكل واحد من **اب اج ه د** خطوط مستقيمة اذ لا فرق في الكسوفات بين ان تكون قسياً و بين ان تكون خطوطاً مستقيمة لصغرها فـ**ه ط** نصف القطرين و **ه ز** عرض القمر لوسط الكسوف و زاوية **ز** قائمة بالتقريب فاذا نقص مربع **ه ز** من مربع **ه ط** حصل مربع **ط ز** فـ**ط ز** معلوم و هو دقايق السقوط فاذا نقصت ساعاتها من ساعات وسط الخسوف[٢٧] حصلت ساعات بدو الخسوف[٢٨] و اذا زيدت عليها حصلت ساعات تمام الانجلاء[٢٩] لان **ه ط** مثل **ه ل** و ايضاً فان **ه ح** نصف قطر الظل نقص منه نصف قطر القمر فاذا نقص من مربعه مربع **ه ز** حصل مربع **ح ز** فـ**ح ز** معلوم فاذا نقصت ساعاتها من ساعات وسط الخسوف[٣٠] حصلت ساعات[٣١] بدؤ المكث و اذا زيدت عليها حصلت ساعات بدو الانجلاء[٣٢] لان **ه ح** مثل **ه ك** فهذه خمسة ازمان و اذا لم يكن للكسوف مكث سقطت ساعات بدو المكث و بدو الانجلاء[٣٣] و ذلك ما اردناه[٣٤]

الباب الثالث في تعديل الازمان

لتكن **اب** قطعة من الفلك المايل و **اج** قطعة من فلك البروج و **ه** مركز دايرة الظل و **ه د** مار بقطبي الفلك البروج و **ه ل** منه عرض القمر لوسط الكسوف و نقط **ط س ل ع ى** مراكز القمر لبدو الخسوف[٣٥] و بدو المكث و وسط الخسوف[٣٦] و بدو الانجلاء و تمام الانجلاء و نخرج من[٣٧] هذه النقط[٣٨] خطوط **ط ف س ص ع ق ى ر** موازية لخط **ل ه** فكل واحد منها عرض القمر بحسب هذه المراكز و نخرج خطوط **ط ن س م ع ك ى ش** موازية لخط **اج** و زاوية **ل ه ج**[٣٩] قائمة و نصل خطوط **ط ه س ه ع ه ى ه** فلان **ط ن**[٤٠] موازل**ف ه**

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[٢٦] V om. ساعات

[٢٧] V الكسوف instead of الخسوف

[٢٨] V الكسوف instead of الخسوف

[٢٩] V الانجلى instead of الانجلاء

[٣٠] V الكسوف instead of الخسوف

[٣١] V om. ساعات

[٣٢] V الانجلى instead of الانجلاء

[٣٣] V الانجلى instead of الانجلاء

[٣٤] V اردنا ان نبين instead of اردناه

[٣٥] V الكسوف instead of الخسوف

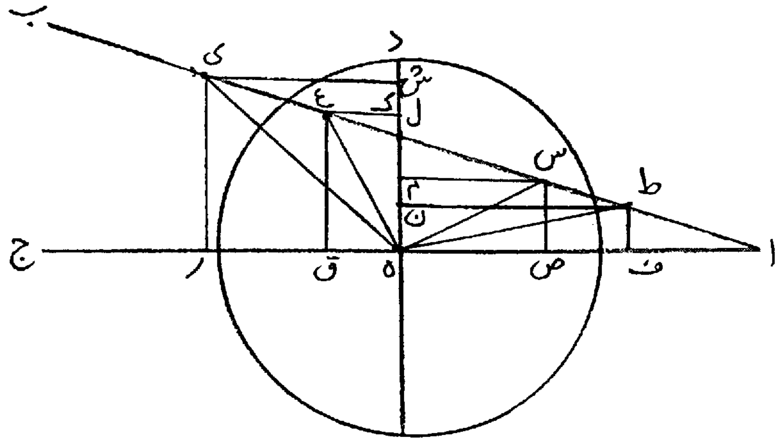
[٣٦] V الكسوف instead of الخسوف

[٣٧] F من instead of الى found in V

[٣٨] V النقطة instead of النقط

[٣٩] V ل ه ا instead of ل ه ج , being still correct

يكون ه ن مساوياً لـ**ط ف** و كلاهما عرض القمر لبدو الخسوف[٤١] و **ط** ه نصف القطـــرين و زاوية ه ن **ط** قائمة فـ**ط ن** معلوم و **ن ل** ما بين عرض البـــدو[٤٢] و عـــرض الوســـط فـ**ل ط** معلوم و هي دقايق السقوط معدلة و ايضاً ه **س** هو ما يبقى من نصف قطر الظل اذا نقص



منه نصف قطر القمر و **س ص** عرضه لبدو المكث و زاوية ه **ص س** قائمة فـه **ص** معلوم و هو مساوٍ لـ**س م** فـ**س م** معلوم و **ل م** ما بين عرض بدو المكث و وسط الكسوف و زاوية **ل م س** قائمة فـ**ل س** معلوم و هو دقايق المكث[٤٣] و ايضاً ه **ع** هو ما يبقى من نصف قطـــر الظل اذا نقص منه نصف قطر القمر و **ع ق** عرض القمر لبدو الانجلاء[٤٤] فـ**ق** ه[٤٥] معلوم و هو مساوٍ لـ**ع ك** و **ك ل** ما بين عرض الوسط و بدو الانجلاء[٤٦] و زاوية **ع ك ل** قائمة فـ**ل ع** معلوم و هو دقايق المكث الى بدو الانجلاء[٤٧] و ايضاً ه ى نصف القطرين و ى ر عرض

---

[٤٠] F ه ط instead of ط ن

[٤١] V الكسوف instead of الخسوف

[٤٢] V البدء instead of البدو

[٤٣] F om. المكث

[٤٤] V الانجلى instead of الانجلاء

[٤٥] V زه instead of ق ه

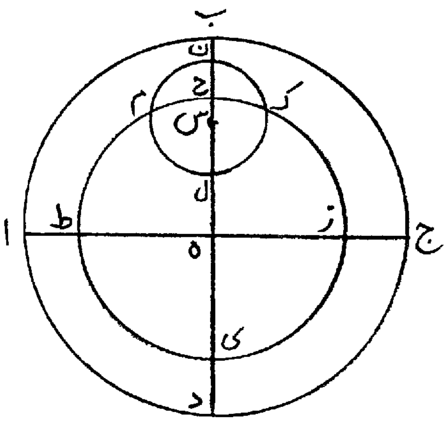
[٤٦] V الانجلى instead of الانجلاء

[٤٧] V الانجلى instead of الانجلاء

تمام الانجلاء[48] فـره معلوم و هو مساو ل ى ش[49] فـى ش معلوم و ل ش ما بـين عـرض الوسط و تمام الانجلاء[50] فـل ى معلوم و هو دقايق السقوط من الوسط الى تمام الانجـلاء[51] فالازمان الخمسة المعدلة معلومة و ذلك ما اردناه[52]

الباب الرابع في تصوير الخسوف

لتكن اب ج د دايرة نصف قطرها مساو لدقايق نصف القطرين و مركزهـا ه و علـى هـذا المركز دايرة زح ط ى نصف قطرها مساو لنصف قطر الظل و خطا اج ب د يتقاطعان عند نقطة ه على زوايا قائمة و ليكن ه ب خط الجنوب و ه د خط الشمال و ه ا خـط المشـرق و ه ج خط المغرب و ه س عرض القمر لوسط الخسوف و ك ل م ن دايرة القمر على مركـز س فقوس ك ل م منها هي الواقعة في دايرة الظل و هو[53] مقدار ما وقع من صـفحة دايـرة القمر في الخسوف على ان جميع صفحته اثنتا عشرة[54] اصبعاً و ل ح اصابع الخسوف غيـر معدلة[55] فـه ب نصف قطر الظل مع نصف قطر القمر و ه ح منه نصف قطر الظل فيبقـى ح ب مساوياً لنصف قطر [56] القمر و ل ح اصابع الخسوف و ذلك ما اردنا ان نبين



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[48] F om. و ايضاً ه ى نصف القطرين و ى ر عرض تمام الانجلاء

[49] F add. و

[50] V الانجلى instead of الانجلاء

[51] V الانجلى instead of الانجلاء

[52] V اردنا ان نبين instead of اردناه

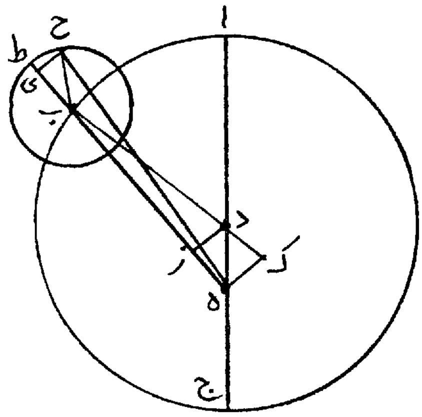
[53] V فهو instead of و هو

[54] V اثنا عشر instead of اثنتا عشرة

[55] V om. غير معدلة

[56] V add. in the margin الظل فيبقى ح ب مساو لنصف قطر القمر

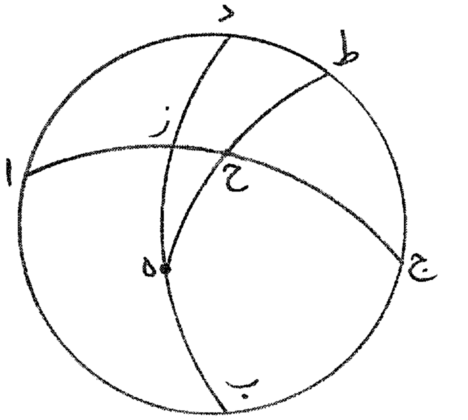
لتكن دايرة **اب ج**[57] الفلك الخارج المركز و مركزها **د** و **اج** قطرها و **ه** مركز فلك البروج و **ب** مركز فلك تدوير القمر و **ط** ذروة فلك التدوير و **ح** جرم القمر و نصـــل الخطـــوط فـ**ه ح** بعد القمر من الارض و هو المطلوب فخطا **دز ح ى** عمودان على **ه ط** فزاوية **اه ب** معلومة و هي البعد المضاعف و زاوية **ز** قائمة فزاوية **ه د ز** معلومة و **ده** عشرة اجزاء و ثلث على ان **ه ا**[58] ستون جزءاً فكل واحد من **دز زه** معلوم و **د ب** تسعة و اربعون جزءاً و ثلثان[59] و مربعه مثل مربعى **ب ز زد** فـ**ب ز** معلوم و **زه** معلوم فـ**ه ب** معلوم و هو بعد مركز فلـــك تدوير القمر[60] من الارض و ايضاً زاوية **ط ب ح** خاصة القمر المعدلـــة و زاويـــة **ى** قائمـــة فزاوية **ب ح ى** معلومة و **ب ح** نصف قطر فلك التدوير بحسب بعد مركزه مـــن نقطـــة **ا** و كل[61] واحد من **ح ى ى ب** معلوم و **ه ب** معلوم فـ**ه ى** معلوم و مربعه مع مربع **ح ى**[62] مثل مربع **ه ح** فـ**ه ح** معلوم و هو بعد القمر من الارض و ذلك ما اردنا ان نبين



[57] V دايرة اب ج د instead of لتكن دايرة اب ج
[58] V اه instead of ه ا
[59] V ثلثى instead of ثلثان
[60] V التدوير instead of تدوير القمر
[61] V فكل instead of و كل
[62] V ى ح instead of ح ى

الباب السادس في ارتفاع قطب فلك البروج

اب ج د[٦٣] دايرة الافق و **ب ه د** نصف النهار و **اح ج**[٦٤] فلك البروج و **ه ط** من دايرة الارتفاع مخطوطة على قطب **ا** و ببعد ضلع المربع فـ**ح ط** مقدار زاوية **ح ا ط** لان كل واحد من **ا ط** **اح** ربع دايرة و المطلوب قوس **ه ح** لانها مثل ارتفاع[٦٥] قطب فلك البروج فمثلث **ادز** زاوية **د** منه قائمة و ضلع **از** ما بين الطالع و وسط السماء من فلك البروج و **زد** ارتفاع درجة وسط السماء و نسبة جيب **از** الى جيب **زد** كنسبة الجيب الاعظم و هو جيب **اح** الى جيب **ح ط** فـ**ح ط** معلوم فتمامه **ه ح** معلوم و ذلك ما اردناه[٦٦]



الباب السابع في ارتفاع اية[٦٧] درجة نريد من درجات فلك البروج

ا ب ج د دايرة الافق و **ب ه د** نصف النهار و **ازج** فلك البروج و نقطتا **ا ز** الطالع و العاشر و **ه ط** من دايرة الارتفاع و **ح** الجزء الذي نريد ارتفاعه و المطلوب قوس **ح ط** فمثلثا **اح ط** **ازد** زاويتا **د ط** منهما قائمتان و **اح** ما بين درجة الطالع و الجزء الذي نريد ارتفاعه و **زد** مقدار ارتفاع درجة وسط السماء و نسبة جيب **اح** الى جيب **ح ط** كنسبة جيب **از** الى جيب **زد** فـ**ح ط** معلوم و ذلك ما اردنا ان نبين

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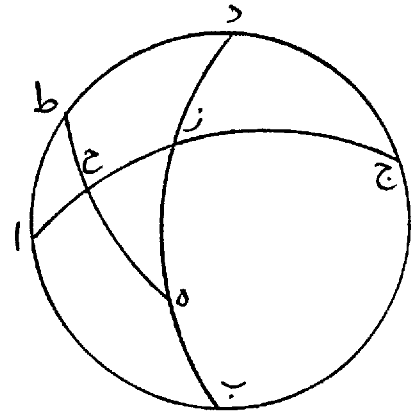
[٦٣] V اب ج instead of اب ج د

[٦٤] V ازج instead of اح ج

[٦٥] V om. ارتفاع

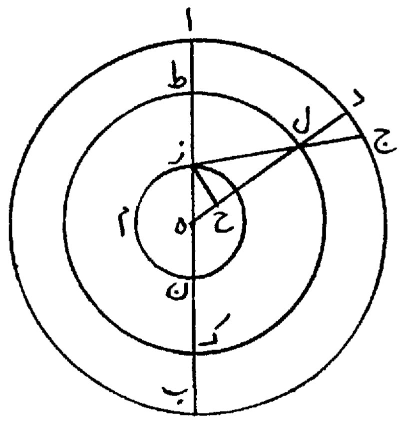
[66] V اردنا ان نبين instead of اردناه

[٦٧] V اي instead of اية



الباب الثامن في اختلاف منظر النيرين من دايرة الارتفاع

اب ج د دايرة الارتفاع على سطح كرة الكل و **ط ك ل** دايرة الارتفاع على سطح[٦٨] كرة القمر و القمر عليها نقطة **ل** و دايرة **ز م ن** على بسيط الارض و الدوائر الثلث في سطح واحـــد و مركزها[٦٩] **ه** و خطوط **اه ط ه زه**[٧٠] نصف قطر هذه الدواير ماراً بسمت الرأس و نخرج من نقطتى **ه ز** خطين يتقاطعان على نقطة **ل** و ينتهيان الى نقطتى **ج د**[٧١] فزاوية **ه ل ز** اخـــتلاف المنظر لانها فضل زاوية **ل ز ط** اعني قوس **اج** المرئية[٧٢] من ظهر[٧٣] الارض علـــى زاويـــة **ل ه ط** اعني قوس **اد** المرئية[٧٤] من مركز الارض فنخرج من نقطة **ز** عموداً على **ه ل** و هو



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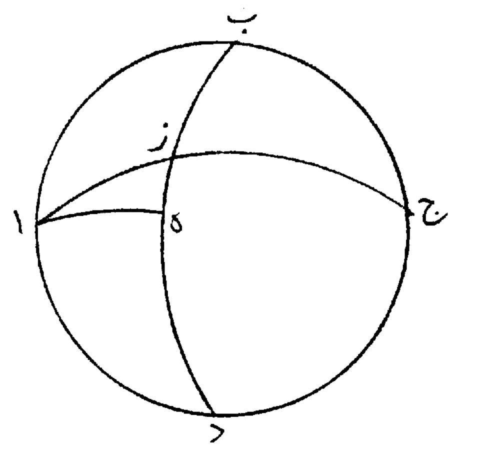
[٦٨] F om. سطح
[٦٩] V مراكزها instead of مركزها
[٧٠] F ز ه ط ه instead of ط ه ز ه
[٧١] V د ج instead of ج د
[٧٢] V المرية instead of المرئية
[٧٣] F ظاهر instead of ظهر
[٧٤] V المرية instead of المرئية

زح فقوس **اد** تمام الارتفاع المرئي[٧٥] من مركـــز الارض فزاويـــة **زه ح** معلومـــة و زاويـــة **زح ه**[٧٦] قائمة و **زه** نصف قطر الارض و هو درجة واحدة فكل واحد من **ه ح ح ز** معلوم و **ه ل** بعد القمر من مركز الارض فـ**ح ل** معلوم فـ**ل ز** معلوم فـ**ز ح** علـــى ان **ل ز** ســـتون جزءا معلوم فقوسه معلومة فزاوية **ه ل ز** معلومة و مقدارها قوس **ج د** و ذلك مـــا اردنـــا ان نبين

الباب التاسع في الزوايا الست التي يحتاج اليها في الكسوفات الشمسية

الاولى و هي ان يكون موضع القمر اول الحمل او الميزان و هو درجة طالع الوقت فلـــيكن **اب ج د** من الصورة الاولى[٧٧] دايرة الافق و **ب ه د** نصف النهار و **ه** سمت الرأس[٧٨] و **ازج**[٧٩] فلك البروج و **ه ا** من دايرة الارتفاع و نقطة **ا** مطلع الاعتدال و المطلـــوب زاويـــة **ه ا ز** و قدرها قوس ه ز فلان[٨٠] كل واحد من **از ه ا**[٨١] ربع دايـــرة و **ه ز** معلـــوم لان نقطـــة **ز** اول السرطان او اول[٨٢] الجدى فزاوية **ه ا ز** معلومة



[٧٥] V المري instead of المرئي

[٧٦] V زه ح instead of زح ه

[٧٧] The mss. Y and A use one figure for each case, but F, V, L and M use the first figure for the first four cases. Here we have followed A and Y for the figures.

[٧٨] F add. و ازط معدل النهار

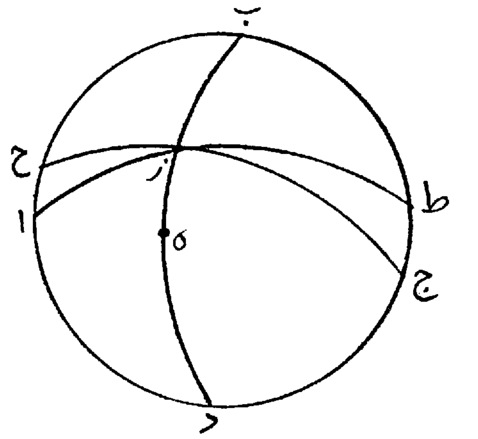
[٧٩] F ج ز ح instead of ازج

[٨٠] V لان instead of فلان

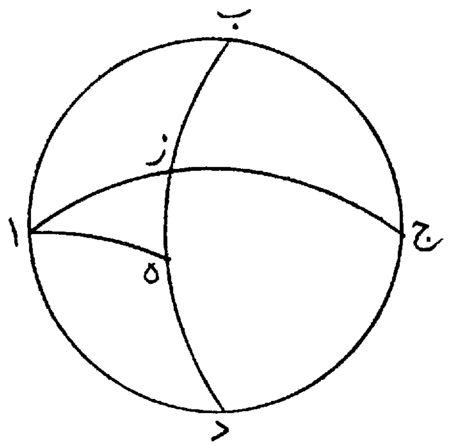
[٨١] V اه instead of ه ا

[٨٢] V om. اول

الثانية و هي ان يكون موضع القمر اول الحمل او الميزان و هو درجة عاشر الوقت و هـــي من الصورة الثانية زاوية **ج زد** على ان **ازط** معدل النهار و **ح زج** فلك البـــروج و نقطـــة **ز** الاعتدال و **ب ه د** نصف النهار فزاوية **ط زد** قائمة و زاوية **ط زج** جملة الميل فزاوية[٨٣] **ج زد** الباقية[٨٤] تمام الميل



الثالثة و هي ان يكون موضع القمر غير اول الحمل و الميزان و هو درجة طالع الوقـــت و هي من الصورة الثالثة زاوية **زاه** على ان **ا** غير مطلع الاعتدال و دايـــرة **ب ه د** مخطوطـــة على قطب **ا** و ببعد وتر ربع دايرة فيكون قوس **ه ز** مثل ارتفاع قطب فلك البروج و قـــدرها زاوية[٨٥] **ه از**

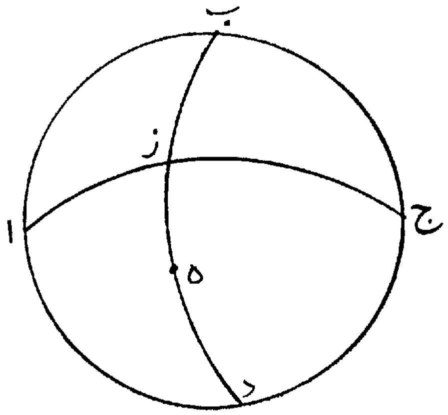


[٨٣] V فيبقى زاوية instead of فزاوية

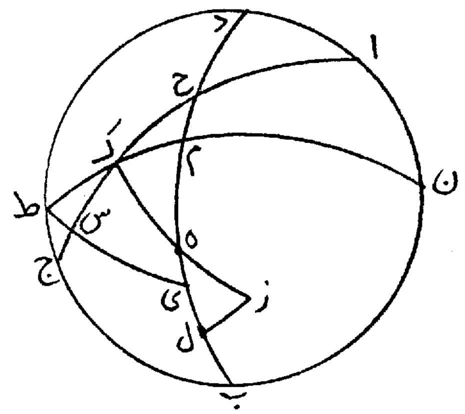
[٨٤] V om. الباقية

[٨٥] F om. زاوية

الرابعة و هي ان يكون موضع القمر اول السرطان او اول[٨٦] الجدى و هو درجة عاشر الوقت و هي[٨٧] من الصورة الرابعة زاوية **ازد** على ان **ازج**[٨٨] فلك البروج و **ب ه د** نصف النهار و هي قائمة لان **از** ربع دايرة



الخامسة و هي ان يكون موضع القمر غير نقطة الاعتدال و الانقلاب و هو درجة عاشر الوقت فليكن **اب ج د** من الصورة الخامسة دايرة الافق و **ب ه د** نصف النهار و **ل**[٨٩] قطب معدل النهار و **ج ك ا** فلك البروج و **ز** قطبه و المطلوب زاوية **ك ح ه** فمثلث **ك ح ه**[٩٠] زاوية **ك** منه قائمة و ضلع **ه ح** ما بين سمت الرأس و فلك البروج من نصف النهار و **ه ك** مثل ارتفاع قطب فلك البروج و نسبة جيب **ح ه** الى جيب[٩١] **ه ك** كنسبة الجيب الاعظم الى جيب زاوية **ح** فزاوية **ح** معلومة[٩٢]



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[٨٦] V om. اول

[٨٧] V هو instead of هي

[٨٨] F and V ازط instead of ازج

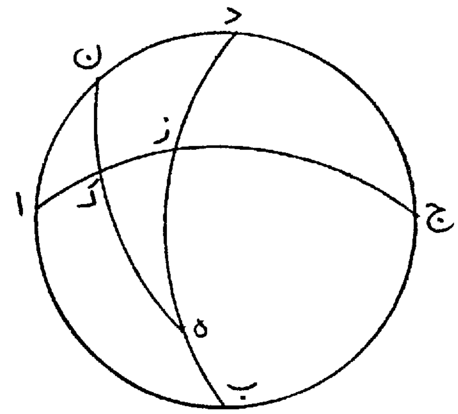
[٨٩] F ب instead of ل

[٩٠] F و instead of فمثلث ك ح ه

[٩١] V om. جيب

[٩٢] Marginal note in F, V and L providing another method (A only contains this method):
و اذا كان **ط م ن** معدل النهار و **ك** نقطة الاعتدال و **ط ى** محطوطة على قطب ح و ببعد ضلع المربع فنسبة جيب ح ك من فلك البروج الى جيب **ك م** مطالع بعد **ك** من نصف النهار كنسبة جيب ح س الى جيب **س ى** فس ى معلوم فزاوية م ح ك معلومة و هو المطلوب

السادسة و هي ان يكون موضع القمر اية درجة كانت و هو فيما بين الطـــالع و الغـــارب[93] فليكن **اب ج د** من الصورة السادسة دايرة الافق و **ازج** فلك البروج و نقطة **ك** منهـــا درجـــة القمر و **ب ه د** مار بقطبيه و **ه ك ن** من دايرة الارتفاع و المطلـــوب زاويـــة **ه ك ز** فمثلـــث **ه ك ز** زاوية **ز** منه قائمة و ضلع **ك ه** تمام ارتفاع درجة القمر و ضلع **ه ز** مثل ارتفاع قطب فلك البروج و نسبة جيب **ك ه** الى جيب **ه ز** كنسبة الجيب الاعظم الى جيب زاوية **ك** فزاوية **ك** معلومة و ذلك ما اردناه[94]



الباب العاشر في اختلاف منظر القمر[95] طولاً و عرضاً من هذه الزوايا

ليكن **اح**[96] قوساً من فلك البروج و **ى ل** قوساً من دايرة العرض و **ه ط** عرض القمر شـــمالياً فنقطة **ه** درجة القمر و **ط** جرم القمر و **س** سمت الرأس و نخرج قوسين من دايرة الارتفـــاع تمران بنقطتى **ط ه** و هما قوسا **س ا س ه** و ليكن **ط ع** اختلاف المنظر من دايرة الارتفـــاع و نخرج **ع ك** موازياً لـ**ا ح** و **ع ج** موازياً لـ**ى ه** و خطوط هذا الشكل قسى لكن ليس بين ان يكون قسياً و بين ان يكون خطوطاً مستقيمةً فرق لصغرها في اوقات الكسوفات الشمســـية و المطلوب خطا **ه ج ع ج** اما **ه ج** فهو اختلاف الطول و اما **ع ج** فهو العرض المرئي فزاوية

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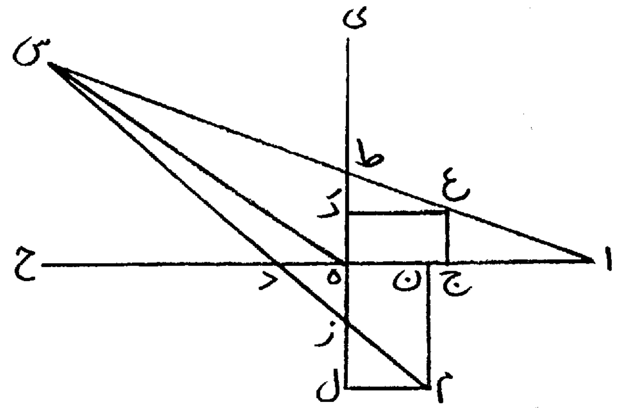
[93] Y add. اما الزوايا الخمس المتقدم فلا يكاد يتفق ان نحتاج اليها الا في الندرة و اما هذه نحتاج اليها دائماً

[94] F add. و هذه الدواير الثلاث صورها; V اردنا ان نبين instead of اردناه

[95] F المنظر instead of منظر القمر

[96] F اج instead of اح

س ه ح[٩٧] زاوية العرض و ليس بينها و بين زاوية **س ا ح** ما يحس و زاوية **س ا ح** مساوية لزاوية **ط ع ك** لان **ع ك** مواز لـ**ا ه** فزاويتا **س اح ط ع ك** كل واحدة منهما مساوية[٩٨] لزاوية **س ه ح** فهما معلومتان و زاوية **ع ك ط** قائمة فزاوية **ع ط ك** معلومة لان **ع ط** وتر الزاويـــة القائمة معلوم فكل واحد من **ع ك ك ط** معلوم و **ك ط** اختلاف العرض و **ط ه** معلوم فـ**ك ه**[٩٩] معلوم و هو[١٠٠] مساوٍ لـ**ع ج** فـ**ع ج** معلوم و هو العرض المرئي و **ع ك** مساوٍ لـ**ج ه** فـ**ج ه** معلوم و هو اختلاف الطول فالقمر يُرى بحسب عرض **ه ط** في نقطة **ج** من فلك البـــروج و ايضاً فليكن[١٠١] **ه ز** عرض القمر في الجنوب و **زم** اختلاف المنظر في دايرة الارتفاع و نصل



م ل موازياً[١٠٢] لـ**ا ح** و **م ن** موازياً[١٠٣] لـ**ه ل** و المطلوب خطا **م ن ن ه** فزاوية **س دح** مثل زاوية **س ه ح** بالتقريب و زاوية **زم ل** مثل زاوية **س دح** لان **م ل** موازية[١٠٤] لـ**ه ح** فزاويـــة **زم ل** مثل زاوية **س ه ح** و زاوية **ل** قائمة و **زم** معلوم فزاوية **م زل** معلومة فاضلاع مثلـــث **م ل ز** معلومة و **ه ز** معلوم فـ**ه ل** معلوم[١٠٥] و هو مساوٍ لـ**م ن** فـ**م ن** معلوم و هو العرض المرئي و **م ل** معلوم فـ**ه ن** معلوم و هو اختلاف الطول فالقمر يرى بحسب عرض **ه ز** فـــي نقطة **ن** من فلك البروج و ذلك ما اردنا ان نبين

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[٩٧] V س ح instead of س ه ح

[٩٨] F om. from لزاوية up to here

[٩٩] F و ك ه instead of فـك ه

[١٠٠] V om. هو

[١٠١] V ليكن instead of فليكن

[١٠٢] V موازٍ instead of موازياً

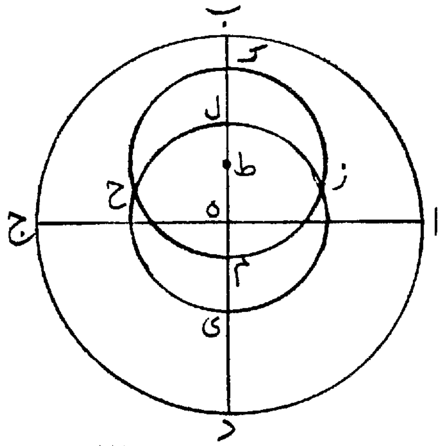
[١٠٣] V موازٍ instead of موازياً

[١٠٤] V موازٍ instead of موازية

[١٠٥] V repeats فـه ل معلوم

الباب الحادي عشر في تصوير الكسوف

اب ج د دايرة نصف القطرين على مركز ه و ه ب[106] مثل نصف قطر القمر مع نصف قطر الشمس[107] و ه ل نصف قطر الشمس و زل ح ى دايرة صفحته[108] و ه ط عـــرض القمـــر و ط ك نصف قطره و دايرة صفحته زك ح م فـم ل من قطر الشمس اصابع الكسوف و خط اج خط المشرق و المغرب و خط[109] ب د خط الشمال و الجنوب و ذلك ما اردناه[110]



الباب الثاني عشر في ارتفاع القمر بحسب عرضه[111]

اب ج د دايرة الافق و ب ح د فلك البروج على قطب ل و اه ج يمر بقطبيه و ى جرم القمر و نجيز به ل ى ك ب ى د ه ى ط فالمطلوب قوس ى ط فـى ك عـــرض القمـــر فمثلثـــا ل ى ز ل ك ح زاوية ل مشتركة و زاويتا ز ح قائمتان فنسبة جيب ل ى الـــى جيـــب ى ز كنسبة جيب ل ك الى جيب ك ح و ل ى تمام عرض القمر و ل ك ربع دايرة و ح ك تمام بعد درجة القمر من الطالع فـى ز معلوم فتمامه ى ب معلوم و ايضاً مثلثا ب ى ك ب زح زاوية ب مشتركة و زاويتا ك ح قائمتان فنسبة جيب ب ى الى جيب ى ك كنسبة جيـــب ب ز الـــى

[106] V فـه ب instead of و ه ب

[107] V القمر مع نصف قطر الشمس instead of الشمس مع نصف قطر القمر

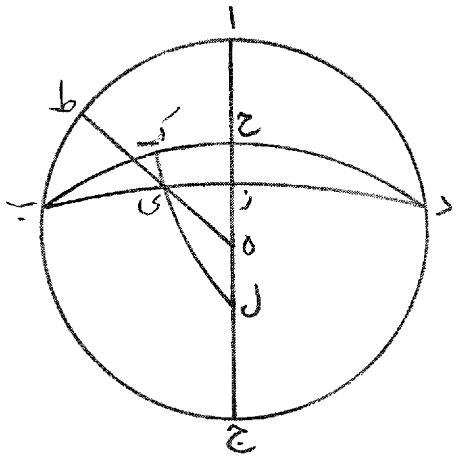
[108] V دايرة صفحته زل ح ى instead of زل ح ى دايرة صفحته

[109] V om. خط

[110] V اردنا ان نصور instead of اردناه

[111] This chapter has been taken from the ms. A. Other mss. Contain inconsistent versions that do not seem to be authentic. However, they provide a brief account of the method found in A, as "another method". Only the version in the ms. Y is similar to that of A, but using different letters in the figure and thence in the text.

جيب **زح** و **ب ى** معلوم و **ى ك** عرض القمر و **ب ز** ربع دايرة فـ**زح** معلوم و **ح ا** تمـام ارتفاع قطب فلك البروج معلوم فجميع **ا ز** معلوم



و ايضاً مثلثا **ب ى ط ب زا** زاوية **ى ب ط** مشتركة و زاويتا **ط ا** قائمتان فنسبة جيب **ب ى** الى جيب **ى ط** كنسبة جيب **ب ز** الى جيب **زا** و **ب ى** معلوم و **ب ز** ربع دايرة و **زا** معلـوم فـ**ى ط** معلوم و ذلك ما اردنا ان نبين

الباب الثالث عشر في اختلاف منظر القمر طولاً و عرضاً بطريقة مبرهنة

[١١٢]قد تقدم في المقالة الاولى في الباب الحادي عشر من الفصل السادس ان الارتفاع الحاصل من الحساب هو الارتفاع الحقيقي فالمنقوص منه اختلاف المنظر هو الارتفاع المرئي و مـن بعد ما تقدم ذلك فان هذا الباب يقع على خمسة اوجه

الاول ان يكون ارتفاع عاشر الوقت تسعين جزءاً و ليس للقمر عرض فاختلاف المنظر فـي دايرة الارتفاع هو اختلاف المنظر في الطول وحدة

الثاني ان يكون بعد درجة القمر من طالع الوقت تسعين جزءاً كان للقمر عرض او لم يكـن فاختلاف المنظر في دايرة الارتفاع هو العرض المرئي وحدة

[١١٢] The beginning of this chapter up to the "third case" is found only in F and M. Mss. V, L, and A start with the "third case". Ms. A first mentions that:

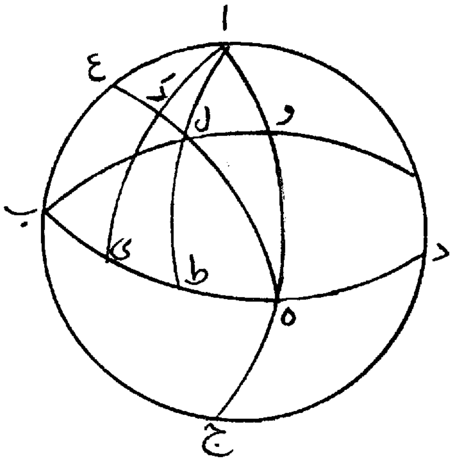
قد تقدم القول في المقالة الاولى ان هذا يقع على خمسة اوجه اما الاول و الثاني فهما ظاهران لايحتاجان الى البرهان

Y starts with the "fifth case" and first mentions that:

قد تقدم في المقالة الاولى ان هذا الباب يقع على خمسة اوجه الا ان الوجوه الاربعة لا يكاد يتفق وقوعه و لانحتاج اليها فاما الوجه الخامس فربما نحتاج اليه في الندرة فلذلك اقتصرنا على برهان هذا الوجه فلا فايدة في التطويل فقد تقدم في المقالة الاولى ايضاً ان الارتفاع الحاصل من الحساب هو الارتفاع الحقيقي و المنقوص منه اختلاف المنظر في دايرة الارتفاع هو الارتفاع المرئي

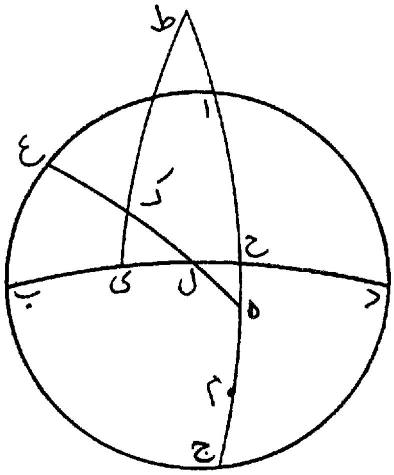
[113]**الثالث** اذا كان ارتفاع عاشر الوقت تسعين جزءاً و للقمر عرض فليكن **اب ج د** من الصورة الاولى دايرة الافق و **ب ه د** فلك البروج و نقطتا **ا ج** قطباه[114] و **اه ج** تمر بهذين القطبين و **ه ع** من دايرة الارتفاع و **ل** جرم القمر و **ل ك** اختلاف المنظر[115] في دايرة الارتفاع و نجيز على نقطتى **ل ك** قسى **ال ط اك ى ب ل** و **[ب ك]**[116] فل **ط**[117] عرض القمر جنوبياً و **ك ى** العرض المرئي و **ط ى** اختلاف الطول فمثلثا **ه ل ط ه ك ى** زاوية **ه** مشتركة و زاويتا **ط ى** قائمتان فنسبة جيب **ه ل** الى جيب **ل ط** كنسبة جيب **ه ك** الى جيب **ك ى** و **ه ل** تمام الارتفاع الحقيقي و **ل ط** عرض القمر و **ه ك** تمام الارتفاع المرئي ف **ك ى** معلوم و هو العرض المرئي و ايضاً مثلثا **اك ع ا ى ب**[118] زاوية **ا** مشتركة و زاويتا **ع ب** قائمتان فنسبة جيب **اك**[119] تمام العرض المرئي الى جيب **ك ع** الارتفاع المرئي كنسبة جيب **اى** الجيب الاعظم الى جيب **ى ب** ف **ى ب** معلوم و **ط ب** بعد درجة القمر من الطالع ف **ط ى** معلوم وهو اختلاف الطول

الاولى



[113] V starts from here and adds الوجه
[114] V قطبيه instead of قطباه
[115] V منظر القمر instead of المنظر
[116] V ب ك ز instead of ب ك but the arc is not drawn in the first figure.
[117] F ك ط instead of ل ط
[118] V اك ك ع اى ى ب instead of اك ع اى ب
[119] F ل ك instead of اك

[120]**الرابع** اذا كان ارتفاع عاشر الوقت اقل من تسعين جزءاً و ليس للقمرعرض فليكن **اب ج د** من الصورة الثانية دايرة الافق و **ب ج د** فلك البروج و نقطتا **ط م** قطبـــاه[121] و دايـــرة **اه ج** مارة بهما و **ه ع** من دايرة الارتفاع و **ل** جرم القمر و **ل ك** اختلاف المنظر في دايرة الارتفاع و نجيز على نقطتى **ل ك** قوس[122] **[ط ل] ط ك ى [ب ل د]** فــك **ى** العرض المرئـــي و **ل ى** اختلاف منظر الطول فمثلثا **ك ل ى ح ل ه** زاويتا **ل** منهما[123] متســاويتان و زاويتـــا **ى ح** قائمتان فنسبة جيب **ل ه** الى جيب **ه ح**[124] كنسبة جيب **ل ك** الى جيب **ك ى**[125] و **ل ك** اختلاف



الثانية

المنظر من[126] دايرة الارتفاع و **ل ه** تمام الارتفاع الحقيقي و **ه ح** ارتفاع قطب فلك البـــروج فــك **ى** معلوم و هو العرض المرئي

اختلاف الطول:[127] مثلث **ل ك ى** زاوية **ى** منه قائمة فنسبة جيب تمام **ى ك** العرض المرئـــي الى جيب تمام **ك ل** اختلاف المنظر من دايرة الارتفاع كنسبة الجيب الاعظم الى جيب تمـــام **ل ى** اختلاف المنظر في الطول فــل **ى** معلوم

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[120] V add. الوجه

[121] V قطبيه instead of قطباه

[122] F and V قسى instead of قوس that we read in A, which is more consistent in this position; F and V provide the superfluous Arabic letters shown in brackets.

[123] F om. منهما; V add. منه

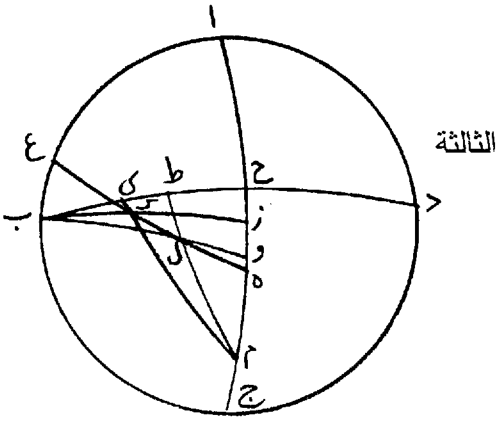
[124] V ل ك الى جيب ك ى instead of ل ه الى جيب ه ح

[125] V ل ه الى جيب ه ح instead of ل ك الى جيب ك ى

[126] F في instead of من

[127] F و instead of اختلاف الطول:

[128] **الخامس** اذا كان ارتفاع عاشر [طالع] الوقت اقل من تسعين جزءاً و للقمر عرض فليكن **ا ب ج د** من الصورة الثالثة دايرة الافق و **ب ح د** فلك البروج و **م** قطبه و **اه ج**[129] دايرة تمر به و **ه ع** من دايرة الارتفاع و **ل** جرم القمر و **ل ك** اختلاف المنظر في دايرة الارتفاع و نجيز على نقطتي **ل ك** قسي **ب ك ز ب ل و م ل ط م ك ى** فـ**ط ل** عرض القمر شمالاً و **ك ى** العرض المرئي و **ط ى** اختلاف منظر الطول فمثلثا **م ل و م ط ح** زاوية **م** مشتركة و زاويتا **و ح** قائمتان فنسبة جيب **م ل**[130] الى جيب **ل و** كنسبة جيب **م ط** الى جيب **ط ح** و **م ل** تمام عرض القمر و **م ط** ربع دايرة و **ط ح** تمام بعد درجة القمر من الطالع فـ**ل و** معلوم و ايضاً مثلثا **ه ل و ه ك ز** زاوية **ه** مشتركة و زاويتا **و ز** قائمتان فنسبة جيب **ه ل** الى جيب **ل و** كنسبة جيب **ه ك** الى جيب **ك ز** و **ه ل** تمام الارتفاع الحقيقي و **ل و** معلوم و **ه ك** تمام الارتفاع المرئي فـ**ك ز** معلوم فتمامه **ك ب** معلوم و ايضاً مثلثا **ب ك ع ب ز ا** زاوية **ب** مشتركة و زاويتا **ع ا** قائمتان فنسبة جيب **ب ك** الى جيب **ك ع** كنسبة جيب **ب ز** الى جيب **ز ا**



و **ب ك** معلوم و **ك ع** الارتفاع المرئي و **ب ز** ربع دايرة فـ**زا** معلوم و **اح** تمام ارتفاع قطب فلك البروج فـ**ح ز** معلوم و ايضاً مثلثا **ب ك ى ب زح** زاوية **ب** مشتركة و زاويتا **ح ى**[131] قائمتان فنسبة جيب **ب ك** الى جيب **ك ى** كنسبة جيب **ب ز** الى جيب **زح** و **ب ك** معلوم و **ب ز** ربع دايرة و **زح** معلوم فـ**ك ى** معلوم و هو العرض المرئي

اختلاف الطول:[132] و مثلثا **م ك ز م ىح**[133] زاوية[134] **م** مشتركة و زاويتا **ز ح** قائمتان فنسبة

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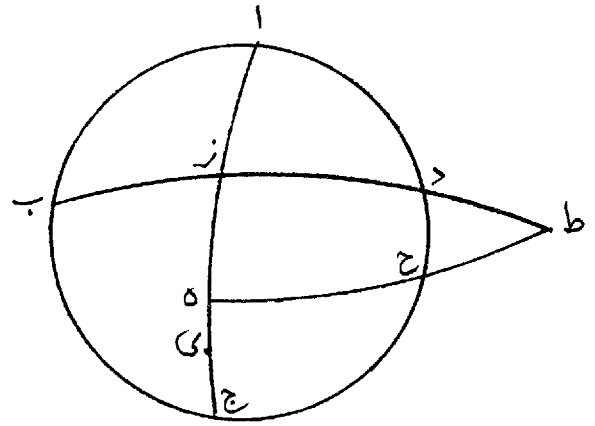
[128] V add. الوجه
[129] F اه instead of اه ج
[130] F م instead of م ل
[131] V ح ى instead of ح ى
[132] F om. اختلاف الطول:
[133] F م ح ى instead of م ى ح
[134] F زاويتا instead of زاوية

جيب **م ك** الى جيب[١٣٥] **ك ز** كنسبة جيب **م ى** الى جيب **ى ح** و **م ك** تمام العرض المرئي و **ك ز** معلوم و **م ى** ربع دايرة فى **ح** معلوم و **ط ح** معلوم فـ**ط ى** معلوم و هو اختلاف المنظر في الطول فالعرض المرئي و اختلاف الطول على اختلاف الوقوع معلومان و ذلك ما اردنا ان نبين

الباب الرابع عشر في قوس الرؤية[١٣٦]

اب ج د دايرة الافق و **ب زد** فلك البروج و **ى** قطبه و **اه ج** يمر بقطبى فلك البروج و **ه ح ط** من دايرة الارتفاع و **د** الجزء الذي يغيب معه القمر[١٣٧] و **زا** تمام ارتفاع قطب فلك البروج و هو مقدار زاوية **ادز** المساوية لزاوية **ط دح** و المطلوب قوس **ط ح** فمثلثا **دح ط**[١٣٨] **دزا**[١٣٩] زاويتا **د** منهما متساويتان و زاويتا **ح ا** قائمتان فنسبة جيب **د ط** الى جيب **ط ح** كنسبة جيب **د ز** الى جيب **زا** و **د ط** ما بين الشمس و هي نقطة **ط** و بين الجزء الذي يغيب معه القمر و هي نقطة **د** و **د ز** ربع دايرة و **زا** تمام ارتفاع قطب فلك البروج فـ**ط ح** معلوم و هو المطلوب و اقل ما وجد[١٤٠] من مقداره الى هذه الغاية ست درجات و نصف[١٤١] الى سبع درجات و ذلك ما اردنا ان نبين



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[١٣٥] F om. جيب

[١٣٦] Marginal note in V: هذا الباب وجدته زيادة في ورقة صغيرة في الكتاب

[١٣٧] F القمر معه instead of معه القمر

[١٣٨] V دط ح instead of دح ط

[١٣٩] V repeats مثلثا دط ح دزا

[١٤٠] V وجدت instead of وجد

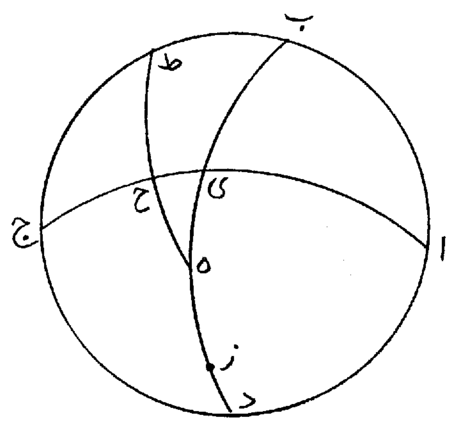
[١٤١] V add. و

الفصل السابع فيما يتعلق بالاحكام و هو باب واحد

في[1] مطرح الشعاع بحسب عرض الكوكب

المتقدمون[2] من الاحكاميين الذين عرفوا شيئاً من علم الهيئة قالوا اذا كان للكوكب عرض فلايؤخذ شعاعاته من منطقة فلك البروج لكن من دايرة تمر بالكوكب و تقطع فلك البروج على الشعاع المفروض و البتاني لما اراد ان يستخرج ذلك ركب مركباً عظيماً في طول الحساب و بُعد البيان فان كان لذلك في الاحكام تأثير و للاحكام اليه احتياج فان الامر[3] فيه قريب جداً و حسابه على ما اثبته[4] في المقالة الاولى

و[5] **برهانه** فلتكن **اب ج د** دايرة فلك البروج و **ه** قطبه و نقطة **ح** جرم الكوكب و **ه ح ط**[6] يمر بقطبى فلك البروج فيكون **ط** درجة الكوكب و **ط ح** عرضه و **ه ح** تمام العرض و ليكن **اى ج** يمر بالكوكب و قطبه **ز** و **ب ه د** يمر بالقطبين و ليكن **ج ح** ستين جزءاً بالفرض و هو قوس التسديس من هذه الدايرة و تمامه **ح ى** ثلثين جزءاً فمثلثا **ه ح ى** **ه ط ب** زاوية **ه** مشتركة و زاويتا **ى ب** قائمتان فنسبة جيب **ه ح** الى جيب **ح ى** كنسبة جيب **ه ط** و هو الجيب الاعظم الى جيب **ب ط** فب **ط** معلوم و هو المقدار[7] المطلوب من فلك البروج فاذا نقص من **ب ج** كان ما بقي **ط ج** قوس التسديس و اذا زيد على **ب ج** كان ما بلغ قوس التثليث فان كان



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[1] V add. الباب الاول; L add. الباب الواحد

[2] V المدققون instead of المتقدمون

[3] V الاحكام احتياج اليه فالامر instead of للاحكام اليه احتياج فان الامر

[4] V اثبت instead of اثبته

[5] V فاما instead of و

[6] V ه ح instead of ه ح ط

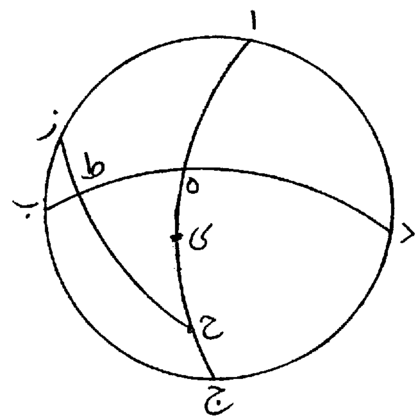
[7] F om. المقدار

المفروض قوس **ج ط** ستين جزءاً و المطلوب قوس **ج ح** فالنسبة تلك النسبة و يحصل المعلوم قوس **ح ى** و تمامه **ح ج** قوس التسديس و زيادته على **ج ى** قوس التثليث فامـــا **ج ى ج ب** فهما قوس التربيع من ايهما اخذ و ذلك ما اردنا ان نبين

الفصل الثامن في اعمال يقل الاحتياج اليها ثمانية ابواب

الباب الاول في عرض البلد من ساعات النهار الاطول و[1] الاقصر

اب ج د دايرة الافق و **اه ج** نصف النهار و **ب ه د** معدل النهار و **ح** قطبه و **ز** مطلع الجدى او السرطان و نخرج **ح ط ز** فـ**ب ز** سعة المشرق و **زط** الميل كله و **ط ه** نصف قوس النهار و هو معلوم من ضرب نصف ساعات النهار في خمسة عشر و **ه ا** تمام عرض البلد و **ه ى** عرض البلد و هو المطلوب فمثلثا **ح ط ه ح زا** زاوية **ح** منهمـا[2] مشـتركة و زاويتـا **ه ا** قائمتان فنسبة جيب **ح ز** الى جيب **ز ا** كنسبة جيب **ط ح** الى جيب **ط ه** و جيـب **ح ز** مثـل جيب تمام الميل و **ح ط** ربع دايرة و **ط ه** معلوم فـ**زا** معلوم و هو تمام سعة المشرق فـ**ز ب** سعة المشرق معلوم و[3] مثلثا **ب ط ز ب ه ا** زاوية **ب**[4] مشتركة و زاويتا **ط ه** قائمتان فنسـبة جيب **ب ط** تعديل النهار الى ظل **ط ز** الميل كله كنسبة جيب **ب ه** الجيب الاعظم الى ظل **ه ا**[5] تمام عرض البلد



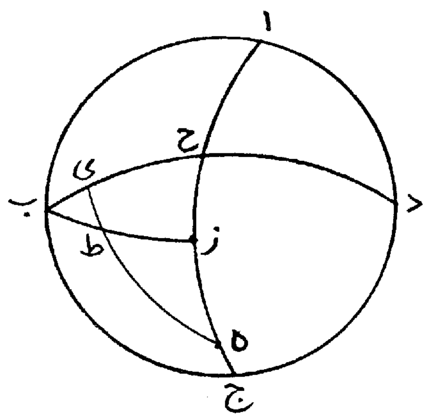
<وجه آخر> و مثلثا **ب زط ب ا ه** زاوية **ب** منهما[6] مشتركة و زاويتا **ط ه** قائمتـان فنسـبة جيب **ب ز** الى جيب **زط** كنسبة جيب **ب ا** الى جيب **اه** و **ب ز** سعة المشرق و **زط** الميـل و

[1] V او instead of و
[2] V om. منهما
[3] V om. و
[4] V add. منه
[5] F om. ه ا
[6] V om. منهما

ب ا ربع[٧] دايرة فـ**اه** معلوم و هو تمام عرض البلد فـ**ى ه** معلوم و هو عرض البلد و ذلك ما اردنا ان نبين و هنا لك استبان ان هذا البرهان يطرد في ساعات ايام السنة كلها اذا اخذ ميــل الشمس بحسب درجتها من فلك البروج

الباب الثاني في الارتفاع الذي لا سمت له

اب ج د دايرة الافق و **ز** سمت الرأس و **ازج** نصف النهار و **ب ح د** معدل النهــار و **زط ب** من دايرة الارتفاع ماراً بمطلع الاعتدال و **ط** جرم الشمس او الكوكب فـ**ى ط** ميل الشمس او بعد الكوكب عن معدل النهار و مثلثا **ب ط ى ب زح** زاوية **ب** منهما[٨] مشــتركة و زاويتــا **ى ح**[٩] قائمتان فنسبة جيب **ب ط** الى جيب **ط ى** كنسبة جيب **ب ز** الى جيب **زح** و **ط ى** الميل او البعد و **ب ز** ربع دايرة و **زح** عرض البلد فـ**ب ط** معلوم و هو الارتفاع الذي لا سمت له و ذلك ما اردنا ان نبين



الباب الثالث في سمت اى ارتفاع يفرض[١٠]

اب ج د دايرة الافق و **اه ج** الفصل المشترك بين نصف النهار و[١١] الافــق و **ب ه د** الفصــل بين معدل النهار و الافق و **ل ك** الفصل بين دايرة المدار و الافق و **ه ح** <نصف> قطر دايرة الارتفاع فـع **ز** العمود الخارج من تقاطع دايرة الارتفاع و دايرة المدار على سطح الافق فهو جيب الارتفاع و نخرج **زط** عموداً على **ه ب** فهو عمــود علـــى **ل ك** ايضــاً لان **ب ه ك ل**

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[٧] F مربع instead of ربع

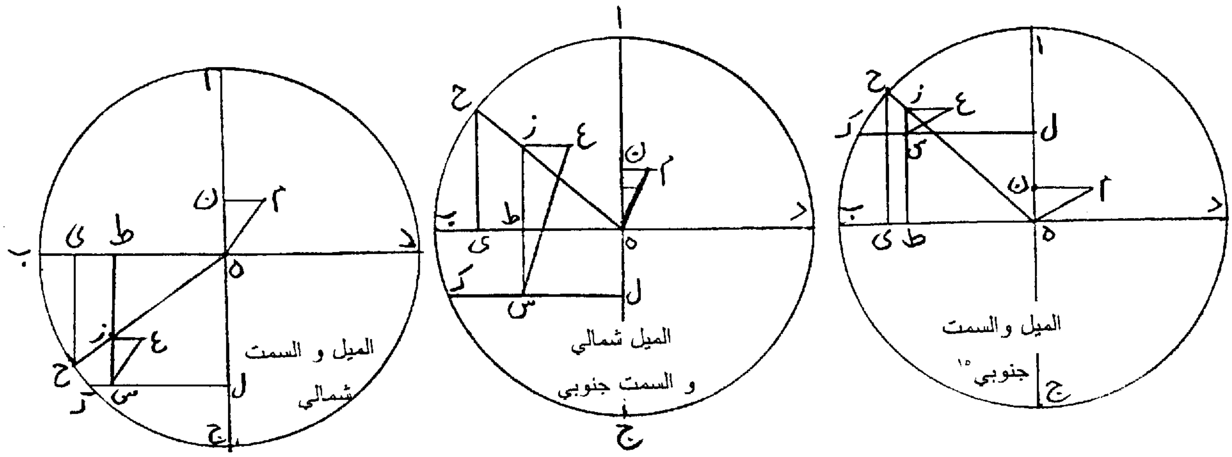
[٨] V om. منهما

[٩] F ط ز instead of ى ح

[١٠] V add. و الطالع و العاشر غير معلومين

[١١] V او instead of و

متوازيان و نخرج **ح ى** عموداً على **ب ه** فهو جيب السمت و **م ن** العمود الخارج من تقـــاطع نصف النهار و معدل النهار على سطح الافق فهو جيب تمام عرض البلد و نصل **م ه ع س** فمثلثا **م ن ه ع زس** متوازيا الاضلاع فنسبة **م ن** جيب تمام عرض البلد الـــى **ن ه** جيـــب عرض البلد كنسبة[12] **ع ز** جيب الارتفاع الى **زس** حصة السمت فـ**زس** معلوم و **س ط** جيب مثل[13] قوس **ب ك** و هو سعة المشرق فـ**زط** معلوم و هو تعديل السمت و ايضاً مثلثا **ه زط ه ح ى** قاعدتا **زط ح ى** منهما متوازيتان[14] فنسبة **ه ز** جيب تمام الارتفاع الى **زط** تعديل السمت كنسبة **ه ح** الجيب الاعظم الى **ح ى** جيب السمت فـ**ح ى** معلوم فـ**ح ب** معلوم و هو السمت المطلوب و ذلك ما اردنا ان نبين .

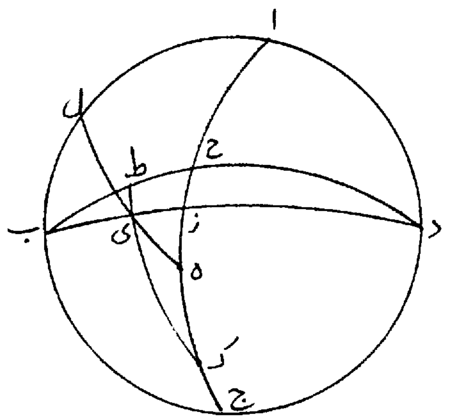


و[16] **زس** في الصورة الثانية[17] حصة السمت و هو اكثر من **ط س** المســـاوي لجيـــب ســـعة المشرق فاذا نقص **س ط** من **س ز** بقي **زط** تعديل السمت و السمت **ب ح** جنـــوبي[18] و[19] **زس** في الصورة الثالثة[20] حصة السمت و هو اقل من **ط س** المساوي لجيب سعة المشـــرق فـــاذا نقص **زس** من **س ط** بقي **زط** تعديل السمت و السمت **ب ح** شمالي[21] و ذلك ما اردنا ان نبين

[12] F add. جيب
[13] V ميل instead of مثل
[14] V متوازيان instead of متوازيتان
[15] F جنوبيان instead of جنوبي
[16] V om. و
[17] V om. في الصورة الثانية
[18] V جنوب instead of جنوبي
[19] V om. و
[20] V om. في الصورة الثالثة
[21] V شمال instead of شمالي

وجه آخر اذا كان الطالع و العاشر معلومين **اب ج د** دايرة الافق و **اه ج** نصف النهـــار و **ك** قطب معدل النهار و **ب ح د** معدل النهار[٢٢] و **ى** جزء الشمس و **ب ى د** يمر به و **ه ى ل** من دايرة الارتفاع و **ك ى ط** يمر بقطب معدل النهار و بجزء الشمس و المطلـــوب قـــوس **ب ل** فاقول انه معلومة

برهانه مثلثا **ك ى ز ك ط ح**[٢٣] فزاوية **ح ك ط**[٢٤] مشتركة و زاويتا **ز ح** قائمتان فنسبة جيـــب **ك ى** الى جيب **ى ز** كنسبة جيب **ك ط** الى جيب **ط ح** فـ**ى ز** معلوم لان **ك ى** تمام الميـــل و **ط ح**[٢٥] مطالع بعد **ى** من نصف النهار و ايضاً مثلثا **ه ى ز ه ل ا**[٢٦] زاوية **زه ى**[٢٧] مشـــتركة و زاويتا **ز ا** قائمتان فنسبة جيب **ه ى** الى جيب **ى ز** كنسبة جيب **ه ل** الى جيـــب **ل ا** و[٢٨] **ل ا** و هو تمام **ب ل** معلوم فـ**ب ل** معلوم لان **ه ى** تمام الارتفاع و **ى ز** معلوم[٢٩] فلان الاوسطين



من المقادير الاول مساويان[٣٠] للاوسطين من المقادير الآخر صارت نسبة جيـــب **ك ى** الـــى جيب **ه ى**[٣١] كنسبة جيب **ا ل** الى جيب **ح ط** فـ**ا ل** تمام السمت معلوم فـ**ب ل** معلوم و ذلك ما اردناه[٣٢]

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[٢٢] V om. و ب ح د معدل النهار

[٢٣] F om. from فاقول up to here

[٢٤] V ط ح ك instead of ح ك ط

[٢٥] V ط ح instead of ح ط

[٢٦] F om. مثلثا ه ى ز ه ل ا

[٢٧] V ه instead of زه ى

[٢٨] V ف instead of و

[٢٩] V om. from فب ل معلوم up to here

[٣٠] V مساويين instead of مساويان

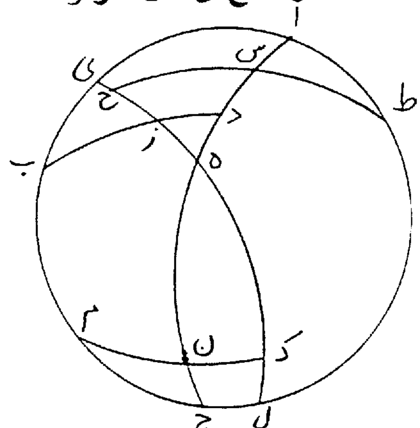
[٣١] F ى ز instead of ه ى

[٣٢] V اردنا ان نبين instead of اردناه

الباب الرابع في الارتفاع من السمت

مقدمة اذا تقاطع دايرة معدل النهار و دايرة الارتفاع و فرض من عند الافق قوس من نصف النهار مثل عرض البلد[33] فان ما بين سمت الرأس و معدل النهار[34] من دايرة الارتفاع كما بين الافق و الدايرة المارة بقطب دايرة[35] الارتفاع و بمثل عرض البلد[36]

<**برهانه**> فليكن **اب ج** دايرة الافق و **اه ج** نصف النهار و **ب زد** معدل النهار و **ن** قطبه و **ى ه ل** دايرة الارتفاع و كل واحد من **ج ن ه د س ا** عرض البلد و **ط**[37] **م** قطبا دايرة الارتفاع و نجيز **م ن ك ط س ح**[38] فاقول ان **ه ز** مثل **ح ى** فنقطة **ز** قطب دايرة **م ن ك** و **ك ز** ربع دايرة و **ل ه** ربع دايرة فنلقى **ك ه** المشترك فيبقى **ل ك** مثل **ه ز** و نسبة جيوب **م ن** الى **ن ج** كنسبة **م ك** الى **ك ل** و **م ن** مثل **ط س** و **ن ج** مثل **س ا** و **م ك** مثل **ط ح** فيبقى **ح ى** مثل **ك ل** و قد تبين ان **ك ل** مثل **ه ز** فـ**ح ى** مثل **ه ز** و ذلك ما اردنا ان نبين



السمت اذا كان شمالياً[39] **اب ج د** دايرة الافق و **اه ج** نصف النهار و **ى** قطب معدل النهار و **ب ح ز** من معدل النهار و **ه د** من دايرة الارتفاع و **ك** جزء الشمس فـ**ب د** السمت المفروض و **دج** تمامه و **ى ج** عرض البلد و نجعل **ج ل** مثل **ب د** و نجيز **ل ى ط** و **ح ك ى**[40] و المطلوب قوس **ك د** الذي هو ارتفاع الوقت فـ**ك ح** ميل الشمس شمالي ابداً و **ك ى** تمامه فنسبة جيب **ه ى** الى جيب **ى ط** كنسبة جيب **ه ج** الى جيب **ج د** و **ه ى** تمام عرض البلد و

---

[33] From و فرض up to here is found only in A

[34] F om. و معدل النهار

[35] F om. دايرة

[36] All mss. except A have بعرض البلد او بمثله instead of بمثل عرض البلد found in A

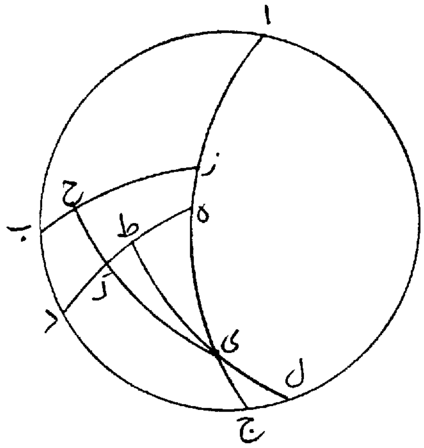
[37] V add. و

[38] F س ط ح instead of ط س ح

[39] Marginal note in V: اذا كان السمت و عرض البلد معلومين فان ارتفاع السمت معلوم

[40] V ى ك ح instead of ح ك ى

دج[41] تمام السمت فـ ى ط معلوم فتمامه ل ى معلوم و ذلك لان مثلثى ه ى ط ه ج د زاوية ه مشتركة و زاويتا ط د قائمتان و ايضاً مثلثا[42] ل ى ج ل ط د زاوية ل مشتركة و زاويتا ج د قائمتان فنسبة جيب ل ى الى جيب ى ج كنسبة جيب ل ط الى جيب ط د و ل ى معلوم و ى ج عرض البلد فـط د معلوم و ايضاً مثلث ى ك ط زاوية ط منه قائمة فنسبة جيب تمام ط ى الى جيب تمام ى ك كنسبة الجيب الاعظم الى جيب تمام ك ط و ط ى معلوم و ى ك تمام ميل الشمس فتمام ط ك معلوم فـط ك معلوم و قد كان ط د معلوما فـك د الباقي معلوم و هو المطلوب



السمت اذا كان جنوبياً ادج نصف دايرة الافق الجنوبي و ه د نصف النهار و ا ك من معدل النهار و ل[43] قطبه و ه ز من دايرة الارتفاع فـا ز السمت المفروض و نجعل د ب مثل ا ز و دى مثل ه ك عرض البلد و نجيز ب ى ط و م جرم الشمس و نجيز ل م ن فـح ز حصة الارتفاع و م ح تعديل الارتفاع و ا ح حصة الداير و ن ح تعديل الدايرفعلى ما تبين في المقدمة ه ح مثل ط ز فاذا كان ه ح معلوماً و زد معلوماً كان ح ك معلوماً و تمامه ح ا معلوماً فدايرة معدل النهار و دايرة الارتفاع يتقاطعان على نقطة معلومة اما من مثلثى ه ح ك ه زد و اما من مثلثى اح ز اك د و مثلث م ن ح زاوية ن منه قائمة فنسبة جيب تمام م ن الى جيب تمام م ح كنسبة الجيب الاعظم الى جيب تمام ح ن فـح ن معلوم و المطلوب من

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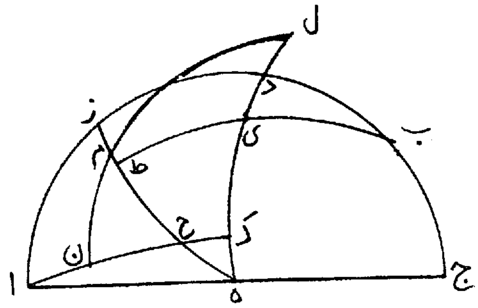
[41] V ج د instead of دج
[42] V مثلثى instead of مثلثا
[43] F ك instead of ل



هذه الصورة قوس م ز[٤٤] فمثلثا ه ط ى ه زد زاوية ه مشتركة و زاويتا ط ز قائمتان فنسبة جيب ه ى تمام عرض البلد الى جيب ى ط كنسبة ه د الجيب الاعظم الى جيب د ز تمام السمت فطى[٤٥] معلوم فتمامه ب ى معلوم و ايضاً مثلثا ب ى د[٤٦] ب ط ز زاوية ب مشتركة و زاويتا د ز قائمتان فنسبة جيب ب ى المعلوم الى جيب[٤٧] ى د عرض البلد كنسبة جيب ب ط الجيب الاعظم الى جيب ط ز المجهول[٤٨] فزط معلوم و قد تبين انه مساوٍ له ح فه ح معلوم فز ح معلوم و هو حصة الارتفاع و ايضاً ل قطب معدل النهار و م جزء الشمس فل م ن دايرة الميل و م ن الميل جنوبياً[٤٩] فمثلثا ح م ن ح ه ك زاويتا ح منهما[٥٠] متساويتان و زاويتا ك ن قائمتان و نسبة جيب ح م تعديل الارتفاع الى جيب م ن ميل الشمس كنسبة جيب ح ه تمام حصة الارتفاع الى جيب ه ك عرض البلد فح م معلوم و ز ح معلوم فم ز معلوم و هو الارتفاع المطلوب و ذلك ما اردنا ان نبين

[٤٤] V م ن instead of م ز

[٤٥] V ى ط instead of ط ى

[٤٦] V ب د ى instead of ب ى د

[٤٧] F om. جيب

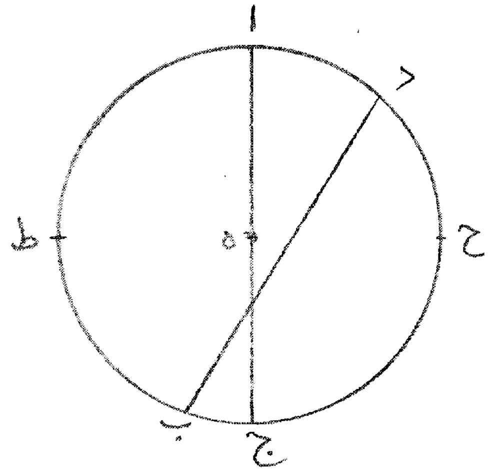
[٤٨] V المجهول instead of مجهول

[٤٩] V جنوب instead of جنوبياً

[٥٠] V om. منهما

الباب الخامس في البعد بين الكوكبين[51] لاحدهما عرض

اب ج د دايرة العرض على مركز **ه** و **اه ج** فلك البروج علـــى قطبـــى **ح ط** و مركـــز **ه** و نفرض الكوكب الذي له عرض نقطة **ب** و **دزب** دايرة تمر بالكوكب و تقطع فلك البروج على **ز** و **ز** هى الدرجة من فلك البروج او موضع الكوكب الذي لا عرض له و المطلوب قـــوس **ب ز** فـ**ج ب** عرض الكوكب و **ج ز** ما بين درجة الكوكب و الدرجة التي نريد بعد الكوكـــب



منها فمثلث **زج ب** زاوية **ج** منه قائمة فنسبة جيب تمام **ج ز** الى جيب تمام **زب** كنسبة الجيب الاعظم الى جيب تمام **ج ب** فـ**ب ز**معلوم و ذلك ما اردنا ان نبين

الباب السادس في البعد بين كوكبين ذوي عرض[52]

اب ج[53] دايرة العرض و **اه ب** فلك البروج و **ز** قطبه و نفرض الكـــوكبين اولاً نقطتـــى **ج ح** مختلفى الجهة و نخرج **ج ح ك ز ح ط ه ح د** و المطلوب قـــوس **ج ح** المـــار بـــالكوكبين فمثلث **ح ه ط**[54] زاوية **ط** منه قائمة فنسبة جيب تمام **ط ه** و هو تمام ما بـــين الكـــوكبين مـــن اجزاء فلك البروج الى جيب تمام **ه ح** كنسبة الجيب الاعظم الى جيب تمام **ح ط** عرض احـــد الكوكبين فـ**ه ح** معلوم[55] و مثلثا **ه ح ط ه د ا** زاوية **ه**[56] مشتركة و زاويتـــا **ط ا**[57] قائمتـــان

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[51] V كوكبين instead of الكوكبين

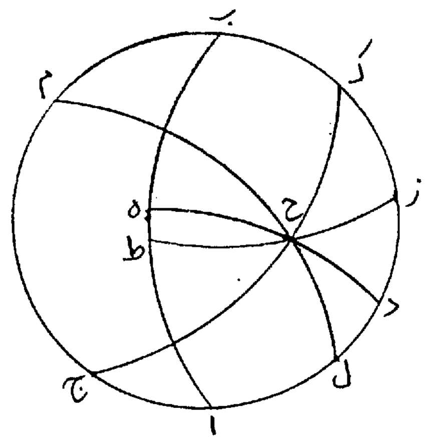
[52] V ذي عرضين instead of ذوي عرض

[53] F and V have اب ج د, but the correct order is ادب ج . Mss. A, Y, M and L have اب ج .

[54] V فمثلثا ه ح ط instead of فمثلث ح ه ط

[55] Marginal note in V: وجدت في الحاشية مكتوب فمثلثا ز ح د زط ا زاوية ط مشتركة و زاويتا ط ا قائمتان فنسبة ز ح الى ح د كنسبة ز ط الى ط ا ما بين الكوكبين في الطول فـح د معلوم فـه ح معلوم هذا انضم

فنسبة جيب **ه ح** المعلوم الى جيب **ح ط** عرض الكوكب الاول كنسبة جيب[58] **ه د** الجيب الاعظم الى جيب **د ا** فـ**د ا** معلوم و **ا ج** عرض الكوكب الآخر فجميع **ج د** معلوم و مثلث **ج ح د** زاوية **د** منه قائمة فنسبة جيب تمام **ج د** المعلوم[59] الى جيب تمام **ح ج** المطلوب[60] كنسبة الجيب الاعظم الى جيب تمام **ح د** المعلوم من قبل فـ**ح ج** معلوم



و نفرض الكوكبين ايضاً نقطتى **ح ل** في جهة واحدة و نخرج **ل ح م** فعلى ما تقدم **ه ح** معلوم و **ح د** معلوم و **د ا** معلوم و **ا ل** عرض الكوكب الآخر فـ**ل د** معلوم و مثلث **ل ح د** زاوية **د** منه قائمة فنسبة جيب تمام **ل د** المعلوم الى جيب تمام **ل ح** المطلوب كنسبة الجيب الاعظم الى جيب تمام **د ح** المعلوم فـ**ل ح** معلوم و معلوم ان **د ا** ان كان[61] اقل من **ا ل** و ذلك اذا وقعت نقطة **د** فيما بين نقطتى **ا ل** نقص[62] من **ا ل** فيبقى **د ل** معلوماً و يكون مثلث **د ح ل** زاوية **د** منه قائمة فيكون **ل ح** معلوماً و ذلك ما اردناه[63]

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[56] F add. منه

[57] F and V and other mss. ا د instead of ا ط which is the correct form

[58] V om. جيب

[59] F ح ج المطلوب instead of ج د المعلوم

[60] F ج د المعلوم instead of ح ج المطلوب

[61] F om. ان كان

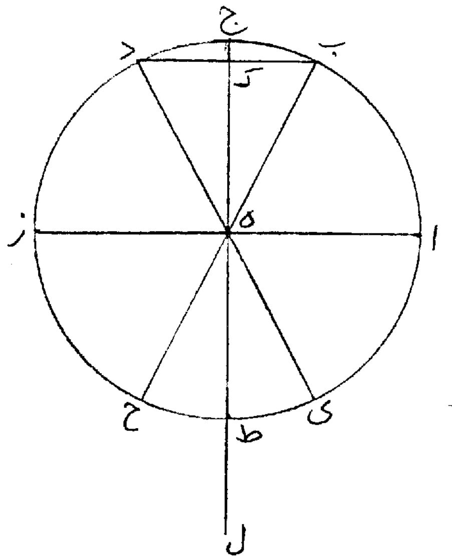
[62] F فنفرض instead of نقص

[63] V اردنا ان نبين instead of اردناه

الباب السابع في استخراج خط نصف النهار

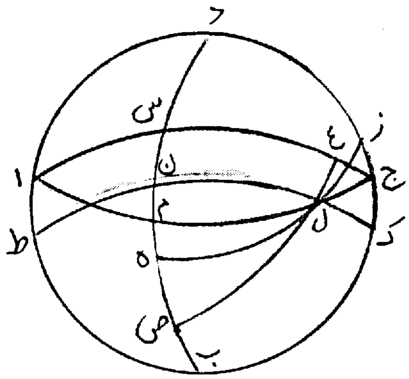
ليكن **ا ج ز** نصف دايرة الافق و **ا ط ز** نصف دايرة معدل النهار مطابقة لـدايرة الافـق او قائمة عليها و مركزاهما **ه** و نتوهم الشمس يدور عليها او على موازاتها بدور الكل و لـتكن نقطة **ط** وسط السماء و قوسا **ط ح ط ى** متساويتان و ليكن على مركز **ه** شخص قائم قيامـاً معتدلاً فاذا كانت الشمس عند نقطة **ح**[64] كان ظل الشخص[65] **ه ب**[66] و اذا كانت عند نقطة **ى**[67] كان ظلها **ه د**[68] و نصل **ب د** و نقسمه بنصفين[69] على **ك** و نخرج **ك ه** و ننفده الى **ل** فخـط **ك ل** هو[70] خط نصف النهار و ذلك ان ظل الشخص في مقابلة جرم الشمس ابداً فـ**ه د** علـى استقامة **ه ى** و **ه ب** على استقامة **ه ح** فقوس **ب د** مثل قوس **ح ى** فاذا قسمناها بنصفين على نقطة **ج** و وصلنا[71] **ج ه ل** كان الفصل المشترك بين سطح دايرة[72] نصف النهــار و سـطح دايرة[73] الافق و ذلك ما اردنا ان نبين

[74]**وجه آخر** و هو ان يكون سمت الشمس معلوماً فليكن **اج زط** دايرة الافق و **ا** مطلع الاعتدال و **ز** مغيبه فاذا كان سمت الشمس عند نقطة **ى** كان ظل الشخص **ه د** و ليكن **اى** و هو السمت معلوماً و هو مساوٍ لـ**زد** فـ**زد** معلوم فتمامه **دج** معلوم فاذا اخرجنا خط **ج ه ط** كان الفصـل المشترك بين سطح دايرة نصف النهار و سطح دايرة الافق و ذلك ما اردنا ان نعمل



[64] V ى instead of ح
[65] F الشمس instead of الشخص
[66] V ه د instead of ه ب
[67] V ح instead of ى
[68] V ظل الشخص ه ب instead of ظلها ه د
[69] F om. بنصفين
[70] F om. هو
[71] F وصلناها instead of وصلنا
[72] V om. دايرة
[73] V om. سطح دايرة
[74] F om. from here to the end of this chapter

اب ج د دايرة الافق و **ه** سمت الرأس و **ا ج** مطلع الاعتدال و مغيبه[75] و **ب ه د** نصف النهار و **ص** قطب معدل النهار و **اس ج** معدل النهار فقوس **ه س** عرض بلدنا و نفرض **س ع** ما بين طول بلدنا و طول مكة و نجيز **ص ع** فهو[76] ربع دايرة نصف نهار مكة و نجعل **ص** قطباً و ندير ببعد وتر تمام عرض مكة دايرة موازية لمعدل النهار و هي **ط ن ك** فهي تقطع قوس **ص ع** على **ل** فـ**ل** سمت رؤوس[77] اهل مكة و قوسا[78] **س ن ع ل** عرض مكة فـ**ه ن** ما بين العرضين و نجيز **ه ل ز** من دايرة تمر بسمت الرأس و هي دايرة البعد بين البلدين لان **ه ل** هو البعد بين بلدنا و بين مكة فقوس **زد** انحراف مكة فنرسم نصف دايرة تمر بمطلع الاعتدال و بنقطة **ل** و هي **ام ل ج**[79] فمثلثا **ص ل م ص ع س** زاوية **ص** منهما[80] مشتركة و



زاويتا **م س** قائمتان و نسبة جيب **ص ل** تمام عرض مكة الى جيب **م ل**[81] تعديل الطول كنسبة جيب **ص ع** الجيب الاعظم الى جيب **ع س** ما بين الطولين فـ**ل م** معلوم فتمامه **ل ج** معلوم و مثلثا **ج ل ع ج م س** زاوية **ج** مشتركة و زاويتا **ع س** قائمتان فنسبة جيب **ج ل** المعلوم الى جيب **ل ع** عرض مكة كنسبة جيب[82] **ج م** الجيب الاعظم الى جيب **م س** تعديل العرض فـ**م س** معلوم و **س ه** عرض بلدنا فـ**م ه** معلوم و هو عرض البلد المعدل[83] فـ**م د** معلوم و مثلثا **ج ل ز ج م د** زاوية **ج** مشتركة و زاويتا **زد** قائمتان فنسبة جيب **ج ل** المعلوم

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[75] F add. ل

[76] V فهي instead of فهو

[77] V روس instead of رؤوس

[78] V قوسي instead of قوسا

[79] V ال م ج instead of ام ل ج

[80] V om. منهما

[81] V ل م instead of م ل

[82] V om. جيب

[83] F om. المعدل

الى جيب **ل ز** تمام البعد بين البلدين كنسبة جيب **ج م** الجيب الاعظم الى جيب **م د** المعلوم فـ**ل ز** معلوم و هو تمام البعد بين البلدين فتمامه[84] **ل ه** معلوم و هو البعد و مثلثا **ه ل م** **ه ز د** زاوية **ه** مشتركة و زاويتا **م د** قائمتان فنسبة جيب **ه ل** البعد بين البلدين الى جيب **ل م** المعلوم كنسبة جيب **ه ز** ربع دايرة الى جيب **زد** انحراف مكة فانحراف مكة عن نصف النهار بلدنا معلوم و ذلك ما اردنا ان نبين و[85] على موجب هذه الصورة جوز[86] ان تكون العرضان متساويين[87] و مكة على غير خط المشرق و المغرب لان سمت رؤوس اهل مكة يقع الى ما يلي القطب[88] عن الدايرة المارة بمطلع الاعتدال و بسمت رؤوسنا و يتبين ذلك اذا خططنا الدايرة الموازية على قطب معدل النهار و ببعد وتر[89] تمام عرض بلدنا

و بعد ان وفينا بما وعدنا في صدر المقالة من الابواب و البراهين فانا نختم الفصل بهذا الباب و المقالة بهذا الفصل و الكتاب بهذا المقالة و الحمد لله[90] وحده و كفى و صلوته على احسن خلقه محمد المصطفى

[و ختم النسخ في **يح** من رمضان سنة **ثمه** للهجرة على يدى محمود بن احمد بن الحسين المعلمي]

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[84] F ف instead of فتمامه

[85] V om. و

[86] V يجوز instead of جوز

[87] V متساويان instead of متساويين

[88] V add. القطب

[89] F om. وتر

[90] From here to the end of the treatise, in V: ولي الحمد و اهله و السلم على نبيه محمد و آله الصلحين [و فرغ من نسخه في اواخر العشر الاول من شوال من سنة سبع و عشرين و اربعمائة للهجرة]

Samenvatting

Het proefschrift dat u nu leest bevat een Arabische editie met Engelse vertaling van de Boeken I en IV van de *Jāmiᶜ Zīj* ("Omvattend Sterrenkundig Handboek met Tabellen", spreek uit: Dzjaami Ziedzj). Dit werk is omstreeks 1025 na Christus gechreven door de Iraanse sterrenkundige Kūshyār (spreek uit: Koesjaar) ibn Labbān in de stad Jurjān in Iran. Het woord *Zīj* komt uit het Perzisch en betekent oorspronkelijk draad of koord. Hiervan zijn andere betekenissen afgeleid: netwerk van draden in een weefsel; netwerk van horizontale en verticale lijnen in een tabel met getallen; de tabel zelf; en uiteindelijk een heel handboek, inclusief tabellen en alle verdere uitleg, waarmee een sterrenkundige de posities van hemellichamen kon berekenen.

Tussen 800 en 1500 zijn in het Islamitisch cultuurgebied meer dan 200 verschillende handboeken van dit type samengesteld. Hiervan zijn er meer dan 100 teruggevonden, veelal in verscheidene handschriften in bibliotheken over de hele wereld. Uit dit rijke materiaal zijn maar enkele handboeken in de moderne tijd uitgegeven. De belangrijkste hiervan zijn de instructies bij de *Zīj* van Ulugh Beg (ca. 1420, Perzische editie met Franse vertaling verschenen ca. 1850), de *Ṣābī Zīj* van Al-Battānī (ca. 900, Arabische tekst met Latijnse vertaling verschenen ca. 1900), en *al-Qānūn al-Masᶜūdī* van al-Bīrūnī (ca. 1035, Arabische text en Russische vertaling allebei verschenen ca. 1955).

Dit proefschrift is het eerste deel van mijn project om de *Jāmiᶜ Zīj* van Kūshyār ibn Labbān in zijn geheel te publiceren. De *Jāmiᶜ Zīj* bestaat uit vier "Boeken". De eerste twee Boeken bevatten de standaardinhoud van elke *Zīj*: uitleg van berekeningsmethoden in Boek I, en bijbehorende tabellen in Boek II. De Boeken III en IV geven de *Jāmiᶜ Zīj* een bijzondere opbouw. In Boek III wordt de structuur van het heelal behandeld en worden allerlei sterrenkundige begrippen uitgelegd. Boek IV bevat "bewijzen" van de berekeningsmethoden in Boek I. De Boeken I en IV hebben een parallelle structuur. In de oudste versie van de *Jāmiᶜ Zīj*, die in dit proefschrift uit de handschriften wordt gereconstrueerd, bestaan de Boeken I en IV uit acht "afdelingen", die elk in een aantal hoofdstukken onderverdeeld zijn. Bij de meeste hoofdstukken in een afdeling van Boek I hoort een bijpassend hoofstuk in dezelfde afdeling van Boek IV. In de acht afdelingen behandelt Kūshyār de volgende onderwerpen: 1: de Syrische en Islamitische kalender, en de Perzische zonnekalender die hij zelf voor zijn berekeningen gebruikt; 2: de sinus en koorde, en de berekening van sinus- en koordentabellen; 3: de tangens en cotangens; 4: de berekening van de posities van de zon, maan en planeten; 5: berekeningen in verband met tijdsbepaling, ascendant (punt van de ecliptica dat aan de Oostelijke horizon opgaat), en de astrologische "huizen"; 6: berekening van zons- en maansverduisteringen; 7: berekening van astrologische aspecten en progressies en van het "wereldjaar" uit de Perzische astrologie; en 8: diverse berekeningen op de hemelbol die niet zo vaak nodig zijn, de bepaling van de richting van Mekka, en verder een lijst met heldere vaste sterren en maanhuizen.

Er zijn diverse redenen waarom uit de meer dan 100 ongepubliceerde sterrenkundige handboeken de *Jāmiᶜ Zīj* voor dit project uitgekozen is. De auteur Kūshyār ibn Labbān komt uit hetzelfde gebied als de auteur van dit proefschrift, namelijk de landstreek Gīlān aan de Kaspische Zee. De *Jāmiᶜ Zīj* is geschreven in een tamelijk vroege periode in de Islamitische sterrenkundige traditie. Enkele tabellen en tekstgedeelten uit de *Jāmiᶜ Zīj* hebben sinds het midden van de negentiende eeuw de aandacht getrokken van historici zoals L. Ideler, E. Wiedemann, J.L. Berggren, G. van Brummelen en B. van Dalen. Hierdoor ontstond de behoefte aan een complete utgave van de *Jāmiᶜ Zīj*. Door de aanwezigheid van Boek IV over "bewijzen" geeft de *Jāmiᶜ Zīj* meer inzicht in de denkwijze van Islamitische sterrenkundigen dan de meeste andere handboeken, waarin een soortgelijk deel met bewijzen ontbreekt. Tenslotte was de *Jāmiᶜ Zīj* belangrijk in de Islamitische sterrenkunde. Dit blijkt uit het feit dat omstreeks 1050 een Arabisch commentaar met uitgewerkte getallenvoorbeelden verscheen, en uit het feit dat Boek I rond 1090 in het Perzisch vertaald is. Enkele handschriften in Hebreeuwse lettertekens laten zien dat de *Jāmiᶜ Zīj* ook door Joodse sterrenkundigen werd bestudeerd, Voorzover bekend is het werk in de middeleeuwen niet in het Latijn vertaald.

De *Jāmiᶜ Zīj* behoort tot de traditie van de *Almagest* van Ptolemaeus (ca. 150 na Chr.), die in de negende eeuw in het Arabisch vertaald was. Ptolemaeus en zijn Islamitische opvolgers gingen uit van een bolvormige aarde in het midden van een bolvormig heelal. Ptolemaeus beschreef de bewegingen van de zon, maan en planeten met behulp van combinaties van cirkelbewegingen. Hij probeerde de parameterwaarden in deze meetkundige modellen uit waarnemingen te bepalen, en de modellen vervolgens ook voor positieberekeningen te gebruiken. De verschillen tussen deze berekeningen en nieuwe waarnemingen konden in de tijd van Ptolemaeus met het blote oog niet of nauwelijks worden opgemerkt. In de negende eeuw na Christus werden grotere afwijkingen gevonden tussen de berekeningen op grond van de *Almagest* en de nieuwe sterrenkundige waarnemingen waarmee men in het Oosten van de Islamitische wereld was begonnen. Door het aanbrengen van kleine correcties in parameterwaarden in de modellen van Ptolemaeus, probeerde men deze modellen ook voor de nieuwe tijd geschikt te maken. Dit proces ging een aantal eeuwen door, en de correcties kregen langzamerhand een fundamenteler karakter. In de *Jāmi*ᶜ *Zīj* gebruikt Kushyar, voor zover bekend als eerste in de Islamitische wereld, een waarde voor de eccentriciteit van Mars die afwijkt van de Ptolemaeische. Helaas geeft hij niet precies aan hoe hij deze nieuwe waarde uit waarnemingen heeft afgeleid.

De belangrijkste verschillen tussen de *Jāmi*ᶜ *Zīj* en soortgelijke eerdere werken hebben te maken met wiskunde. Een voorbeeld: In de theorie van zonsverduisteringen geeft Ptolemaeus benaderende berekeningsmethoden, die in de praktijk voldoende nauwkeurig zijn. Hierin wordt aangenomen dat een klein stukje van een bol als plat vlak beschouwd kan worden. Kūshyār geeft exacte methoden met bewijzen op basis van boldriehoeksmeetkunde. Omgekeerd vereenvoudigt Kūshyār sommige interpolatiemethoden van Ptolemaeus voor het berekenen van de eclipticale lengte van de maan en de planeten. Voor het samenstellen van tabellen hoeft Kūshyār minder rekenwerk te doen maar het resultaat wordt ook minder nauwkeurig.

Van de twaalf Arabische handschriften van de *Jāmi*ᶜ *Zīj* die tegenwoordig bekend zijn, waren er uiteindelijk acht voor deze editie beschikbaar (te Alexandrië, Berlijn, Cairo, Istanbul (3), Leiden, Moskou). Ook is de middeleeuwse Perzische vertaling gebruikt, die in een handschrift in Leiden bewaard is. Vier Arabische handschriften in Cairo waren niet beschikbaar. In dit proefschrift wordt geargumenteerd dat drie van de beschikbare Arabische handschriften de oudste versie van de *Jāmi*ᶜ *Zīj* representeren, en uit deze drie handschriften is de oudste versie gereconstrueerd. Alle varianten uit deze drie handschriften, en de belangrijkste varianten uit de andere handschriften, zijn aangegeven in het kritisch apparaat onderaan de pagina's van de Arabische tekst. In de letterlijke Engelse vertaling zijn in puntige en ronde haken woorden respectievelijk opmerkingen toegevoegd om de gedachtengang begrijpelijk te maken. Enig beknopt commentaar is te vinden aan het eind van elk van de acht afdelingen van Boek I en Boek IV. De lezer vindt in de General Introduction informatie over Kūshyār ibn Labbān, zijn andere werken, en de handschriften van de *Jāmi*ᶜ *Zīj*, en ook worden een aantal termen en begrippen uit de Griekse en Islamitische sterrenkunde uitgelegd die nuttig kunnen zijn om de tekst en vertaling te begrijpen.

Curriculum Vitae

Mohammad Bagheri was born on January 5, 1951 in Rasht, Iran, and he currently lives in Tehran, Iran. His nationality is Iranian. He was trained as an electrical engineer in Sharif University of Technology in Tehran (ex Arya-Mehr) and received his B.Sc. in 1975. He has been specializing in history of science since 1990. From 1993 to 2004 he was the director of the History of Science Department in the Encyclopaedia Islamica Foundation in Tehran, and now he is associate professor in this institution. He also taught history of mathematics at Sharif University of Technology in Tehran from 1993 until 2002. Between 1997 and 2005, he was a member of the Executive Committee of the History of Science Division of the International Union of History and Philosophy of Science.

In 1994 Mr. Bagheri discovered in an Iranian library a hitherto unknown letter by the famous fifteenth-century Islamic mathematician Jamshīd al-Kāshī. The letter was published in Persian in his book, *From Samarkand to Kashan: al-Kāshi's Letters to his Father*, Tehran 1996, and in English translation with commentary in *Historia Mathematica* **24** (1997), pp. 241-256. He also published a dictionary of mathematical terms: *A Vocabulary of Mathematics (English-Persian, Persian-English)*, Tehran 1984, which was reprinted several times.

Since 1995 Mr. Bagheri has been working on the *Jāmiʿ Zīj* of Kūshyār ibn Labbān. This dissertation contains the edition, translation and commentary of Books I and IV of the *Jāmiʿ Zīj*. Between 1995 and 2006, he visited the Department of Mathematics of the University of Utrecht six times in connection with his research. From September 2006 to January 2007 he is working as a research fellow at the Netherlands Institute of Advanced Studies in Wassenaar, on the edition and translation of the remaining Books II and III of Kūshyār's *Jāmiʿ Zīj*.