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GAṆITA BHĀRATĪ
BULL. IND. SOC. HIST. MATH.

KHAYYĀM'S SCIENTIFIC LEGACY

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BIOGRAPHY

1. His time

'Umar Khayyām, the famous Iranian poet, philosopher, mathematician and astronomer, lived in late 11th - early 12th centuries C.E. In Western countries, his fame rests mainly upon the free translation in verse of his quatrains (*ruba'iyyāt*) by E.Fitzgerald (1859).

Abu'l-Ḥasan Beyhaqī in the *Tatimma Ṣiwān al-ḥikma* (p.112) has given the horoscope of Khayyām's birth, without any reference to the date. In 1941, an Indian scholar, Swami Govinda Tirtha (pp. xxxii-xxxiv), by using this horoscope and collating it with some other uncertain data on the subject, determined the exact time of Khayyām's birth to have been at sunrise on Wednesday 18 May 1048. This date was later confirmed by the Institute for Theoretical Astronomy of the Soviet Academy of Science (Moṣaḥeb, p.130).

'Umar Khayyām

Tomb of 'Umar Khayyām

Niẓamī 'Arūzī Samarqandī in his *Chahār maqāla* mentions that he met Khayyām in Balkh in 506/1112-3,* and that Khyyām told him (p.100): "My tomb will be at a location where each spring the northern wind will pour flowers on me." He also mentions that in the winter of 508/1114-5, Khayyām was in Marv, and that the Sultan had asked him to determine a propitious day for hunting (p.101). Niẓāmī writes that in 530, four years after Khayyām's death, he visited his tomb (p.100). This indicates that Khyyām died in 526/1131-2. This date was confirmed by some recent researchers; however, other sources mention the year 517/1122-3

* In this usual notation, 506 represents th year in Hijri Era, and 1112-3 represents the corresponding year in Common Era, C.E. or A.D.

to be more plausible. The later date has been accepted by UNESCO.

2. His name

His complete name was Ghiyāth al Dīn Abu'l-Fatḥ 'Umar b. Ibrāhīm Khayyām Nīshābūrī; his later title was Ḥujjat al-Ḥaqq. Khayyām literally means "tent-maker", and this probably refers to his father's job. 'Abd al-Raḥmān Khāzinī, another famous scientist of Khayyām's time, mentions his *kunya* as Abū Ḥafṣ (instead of Abu'l - Fatḥ) in his *Mīzan al-ḥikma.* His name is stated as 'Umar b. Ibrāhīm Khayyāmī in most cases in his works. However, he is generally know as 'Umar Khayyām.

3. Biographical data

Khayyām was born in the city of Nīshābūr in northestern Iran of today, where he studied and flourished. He was greatly respected by the Seljukid Sultan Malikshāh, who invited him in 467/1074-5 to Isfahan to participate in the composition of the *Zīj-e Jalālī* and in the activities for improving the Iranian calender.

In 470/1077-8 he composed the treatise *Sharḥ mā ashkala min muṣādarāt kitāb Uqlīdis* (explanation of the difficulties in Euclid's postulates) (see below), probably in Isfahan.

In 485/1091-2 Sultan Malikshāh died, and Khayyām left Isfahan where he had lived for 18 years, and returned to Nīshābūr. He died in Nīshābūr where his tomb is now a favorite visiting-place for those who respect him either for his philosophical thoughts, poetically expressed in his quatrains, or for his outstanding achievements in mathematics.

Beyhaqī says he was ill-humored, and loath to teach his knowledge to others. He also refer to Khayyām's powerful memory. 'Arūzī says (p. 101) that Khayyām knew astrology although he did not believe in it. George Sarton, when attributing each half century to a brilliant scientist who lived in that period named the second half of 11th century after Khayyām as one of the greatest mathematicians of the Middle Ages, especially in algebra (vol. 1,pp.738-83). According to him Khayyām's treastise on algebra is probably the most outstanding algebraic work in the Middle Ages (vol. 2, part 1, p.8).

In the introduction to his treatise on algebra, Khayyaām complains about the uncongenial social and scientific environment in which he lived (Winter- 'Arafāt, pp. 29-30): "I was unable to devote myself to the learning of this *al-jabr* and the continued concentration upon it, because of obstacles in the vagaries of Time which hindered me; for we have been deprived of all the people of knowledge save for a group, small in number, with many troubles, whose concern in life is to snatch the opportunity, when Time is asleep, to devote themselves meanwhile to the investigation and perfection of a science; for the majority of people who imitate philosophers confuse the true with the false, and they do nothing but deceive and pretend knowledge, and they do not use what they know of the sciences except for base and material purposes; and if they see a certain person seeking for the right and preferring the truth, doing his best to refute the false and untrue and leaving aside hypocrisy and deceit, they make a fool of him and mock him."

Niẓāmī 'Arūzī, a contemporary of Khayyām, considers himself a disciple and adherent of Khayyām, hails him as an astronomer, but makes no mention of his quatrains. Beyhaqī, who alledges to have attended Khayyām's teaching, does not refer to his quatrains, either. It seems that, because of the fanaticism and ignorance prevailing in his time, his quatrains were known only to a small circle of intimate friends. The earliest sources where we find a reference to his quatrains is the *Kharīdat al-qaṣr* by 'Imād al-Dīn Kātib Iṣfahānī, written in Arabic in 572/1176/7, i.e., over 50 years after Khayyām's death. In this work, Khayyām is mentioned among the poets of Khorāsān province. In many other sources , the authors have criticized Khayyām, accusing him to be an atheist (Hedāyat, p.11). The Russian orientalist V.A. Zhukovsky, has written the following about him (p.325); "He has ben regarded variously as a freethinker, a subverter of Faith, an atheist and materialist, a pantheist and a scoffer at mysticism, an orthodox Musulman, a true philosopher, a keen observer, a man of learning, a bon vivant, a profligate, a dissembler and a hypocrite, a blasphemer, nay, an incarnate negation of positive religion and of all moral beliefs; a gentle nature, more given to the contemplation of things divine than the worlly enjoyments; an epicurean skeptic; the Persian Abu'l -'Alā, Voltaire, and Heine. One asks oneself whether it is possible to conceive, not a philosopher, but merely an intelligent man (provided he be not a moral deformity) in whom were commingled and embodied such a diversity of convictions, paradoxical inclinations and tendencies, of high moral courage and ignoble passions, of torturing doubts and vacillations."

WORKS

All Khayyām's works are written in Arabic, except his quatrains. A Russian translation of his collected works except the treatise on the division of a quadrant and the one on music (see below) is prepared by B.A. Rosenfeld and A.P. Youschkevitch (Moscow, 1961).

1. Mathematics

Khayyām's major mathematical work is his *Risāla fi'l-jabr wa'l-muqābala* (Treatise on algebra and *muqābala*). It was Gerard Meerman who first discovered it in Leiden. Then J.E. Montucla brought it to the scholars' notice. Later, L.A. Sédillot found part of it in a manuscript in Bibliothèque Nationale (Paris), and used it to demonstrate that Muslim mathematicians had also attempted the solution of cubic equations. In 1851, F. Woepcke published in Paris the Arabic text and a French translation, which became the basis of later works on the subject. In 1981, Roshdi Rashed and Ahmad Djebbar published in Aleppo the Arabic text and his short treatise on the division of a quadrant. English translations were published in 1931 by D.S. Kasir, and in 1950 by 'Arafāt and Winter. A Persian translation together with the original Arabic text were published by Gh.-Ḥ. Moṣāḥeb in Tehran in 1317S. / 1938-9 (2nd ed. in 1339 S./1960-1).

Khayyām's algebra is mainly geometric. He first solves linear and quadratic equations by the geometrical methods propounded in Euclid's geometry, and then he shows that cubic equations can be solved by means of the intersection of conics sections. He was not the first to solve cubic equations by means of the intersection of conics and he himself refers to former works on the subject at the end of his algebraic treatise (van der Waerden, pp. 24, 29).

In the introduction to this work, Khayyām makes this statement: "One of the branches of knowledge needed in that division of philosophy known as mathematics, is the science of algebra, which aims at the determination of numerical and geometrical unknowns...." Then he begins the first chapter as follows (Berggren, pp.123-4):

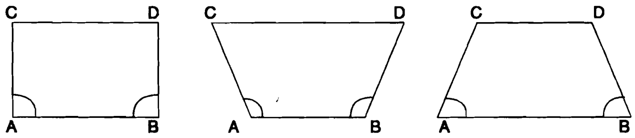
"By the help of God and with His precious assistance I say that algebra is a scientific art. The objects with which it deals are absolute numbers and geometrical magnitudes which, though themselves unknown, are related to things which are known, whereby the determination of the unknown quantities is possible.... What one searches for in algebraic art is the relation that lead from the known to the unknown...."

He succeeds in solving linear and quadratic equations to find the answers as absolute numbers. However, he fails to do so in the case of cubic equations. In his words (Berggren, p. 124): "As for administration of these types, if the object of the problem is an absolute number, neither we nor any of the algebraists have succeeded, except in the case of the first three degrees, namely number, thing and square; but may be those after us will. " Khayyām's expectation was realized in 1545 by G. Cardano, who found the solutions for cubic equations as absolute numbers in his *Ars magna*.

Khayyām takes into account only the positive coefficient and solutions of the equations. He discusses the six types of linear and quadratic equations, already mentioned by al-Khwārazamī. Then he speaks of 19 types of cubic equations, among which five types are reducible to linear or quadratic equations. Two types of the remaining 14 types were solved before Khayyām, and he solved the other 12 types by the use of conics. Although some cubic equations were solved by his predecessors, his work on the subject is coutstanding because it is both complete and systematic.

Another algebraic work by Khayyām is the *Risāla fī qismat rub 'al-dā' era* (Treatise on the division of a quadrant), composed before his algebra, the subject of which is a geometrical problem that leads to a cubic equation and then is solved by the intersection of conic sections. A manuscript of this work is in the Central Library of Tehran University. The Arabic text and a Persian translation thereof were first published by Moṣāḥeb in his book on Khyyām's algebra (1960). It has been translated into English by ‘A.-R. Amīr Mo‘ezz (1961), and into Russian by S.A. Krasnova and B.A. Rosenfeld (1963). A French translation by R. Rashed and A. Djebbar was published in their book on Khayyām's algebra (1981).

The second important mathematical work of Khayyam — especially from a historical point of view—is his *Sharḥ mā ashkala min muṣādarāt Kitāb Uqlīdis* (Explanation of the difficulties in Euclid's postulates). It consists of an introduction and three books. The first book is one Euclid's famous postulate on parallel lines; the second on ratios, proportions and their nature; the third on composite ratios. Manuscripts of this work are in Paris and Leiden. The original Arabic text was published in Tehran by T. Erāni in 1936. In 1961, A.I. Sabra published the text in



Alexandria. In 1967, the Arabic text with a Persian translation was published in Tehran by J. Humā'ī.

Half a century after Khayyām completed this work, Naṣir al-Dīn Ṭūsī quoted the first book thereof in his treatise on parallel lines. In this book, Khayyām refutes the proofs of Euclid's fifth postulate given by Hero, Nayrīzī and Ibn Haytham, and gives his own proof. He criticizes Ibn Haytham because the latter had based his demonstration on an axiom which implies movement. Ibn Haytham uses the axiom to prove that in any quadrangle with three right angles, the fourth angle would be also a right angle. Khayyām's proof is somehow similar to Ibn Haytham's. He uses an axiom, which he attributes to Aristotle, for proving that in any birectangular isosceles quadrangle, the two remaining equal angles are also right. He first assumes that these angles are acute and then, in another assumption, that they are obtuse. By *reductio ad absurdum,* he then shows that they can only be right angles. Khayyām's hypotheses of acute or obtuse angles thus correspond to the first theorems of the non-Euclidean geometries of Lobatchevsky and Riman, respectively.

Khayyām was not the first to try to prove the fifth postulate of the *Elements.*

At least 30 treatises on the subject were composed before thim. The study of the problem was continued by later mathematicians. Saccheri, Italian mathematicians of the 18th century, used the same birectangular isosceles quadrangle proposed by Khayyām in his discussion of the subject. Similarly Ibn Haytham's influence may be observed in the works of Levi ben Gerson who lived in France in the 14 th century, and of the Swiss mathematician Lambert (18th century). They based their discussions on the trirectangular quadrangle of Ibn Haytham. Like the latter, they tried to prove that in such a quadrangle the fourth angle would also be right.

In the second book, Khayyām states that Euclid's definition of proportion is correct, but that it is not a true definition of the concept ratio. The true meaning of a ratio is to be found by measuring one magnitude with another. His definition is based on continued mutual subtraction of two quantities. From a passage in the *Topica* of Aristotle we know that this definition of equality of ratios was used by Greeks before Euclid. Khayyām then proves through a series of theorems that this definition of the equality of ratios is equivalent to Euclid's.

In the third book, Khayyām defines the multiplication of ratios, thus filling a logical gap in Euclid's *Elements*. This subject was applied in arithmetic, geometry, theoretical music (see below), and trigonometry. He also raises the question whether ratios can be regarded as a kind of number in a broad sense. He leaves this phiosophical question unanswered. But later, mathematicians such as Naṣīr al-Dīn Ṭūsī considered all ratios as number (van der Waerden, pp.30-1).

Khayyām had a treatise entitled *Mushkilāt al-ḥisāb* (Problems of arithmetic), which is mentioned in the list of contents at the beginning of a collection of manuscripts in the library of the University of Leiden (Cod. or. 199). Unfortunately this treatise is not actually included in the collection. Khayyām refers to this work in his algebra (Winter-'Arafat, pp.34, 71) as follows: "The Hindus have their own methods for extracting the sides of squares and cubes based on the investigation of a small number of cases, which is [through] the knowledge of the squares of the nine integers, that is, the squares of 1, 2, 3, and so on, and of their products into each other, that is the product of 2 with 3, and so on. I have written a book to prove the validity of those methods and to show that they lead to the required solutions, and I have supplemented it in kind, that is, finding the sides of the square of the square, and the quadrato.cube, and the cubo-cube, however great they may be; and these proofs are only algebraical proofs based on the algebraic parts of the book of *Elements*."

The algorithmic method for extracting square and cube roots found in early Islamic-period works on arithmetic—such as Uqlīdisī, Kūshyār's and Nasawī's treatises—seems to have been influenced by earlier Indian and Chinese work on the subject (Chemla, p. 233). This method was rediscovered by Ruffini and Horner at the beginning of the 19th century. The earliest Arabic account of the general method for extraction of roots with positive integer exponents from natural numbers is found in Naṣir al-Dīn Ṭūsī's *Jāmi 'al-ḥisāb. bi'l-takht wa'l-turāb* (Collection on arithmetic by means of board and dust). Since Ṭūsī made no claim to priority of discovery, and since he was well acquainted with the work of Khayyām, it seems likely that the method he presented is Khayyām's own (Youschkevitch and Rosenfeld, p. 326). In the catalog of Persian and Arabic manuscripts in the Oriental Library of Bankipore (Calcutta, 1908) is mentioned a treatise by Khayyām entitled *Risāla dar ṣiḥḥat-e ṭuruq-e hindī barāye istikhrāj-e jadhr o ka'b* (Treatise on the correctness of the Indian methods for extracting square and cube roots; Moṣāḥeb p.132). A manuscript of this work was reported to be in Munich (*ibid.*) in Minorsky's article in *Encyclopaedia of Islam* (vol. 6, p.985). A. Qurbānī (1989, p. 62) writes that he personally tried to find this manusacript in Munich but in vain. Matvievskaya and Rosenfeld (p. 317) have mentioned the anonymous treatise *Mushkilāt fī ilm al-ḥisāb* (Problems in arithmetic) in Baku (ms. no. B5545/14) as a probable copy of this work. I received a photo of this manuscript in summer of 1996 and found that it has no material on the subject of root extraction.

Khayyām wrote a treatise on music entitled *Sharḥ al-mushkil min kitāab al-mūsīqā* (Explanation of the difficulties in the book on music) which he mentions in his treatise on parallel lines (Erānī, p. 40). This may be a commentary on Euclid's book on music, *Kitāb al-mūsīqā or Kitāb al-nigham,* mentioned by Ibn al-Nadīm (p. 326). No complete manuscript thereof exists; however, a short chapter of it, entitled *al-Qaul 'alā ajnās allati bi'l-arba'a* (Discussion on the genera in a tetrachord), is extant as MS 1705/8 (general) in Manisa, fols. 97-9. This chapter was published by J. Humā'ī (pp. 341-4). Here, Khayyām takes up the problem - already tackled by the Greeks, and particularly by Euclid in the *Sectio canonis* - of dividing a tetrachord (fourth) into three intervals, thus producing four musical notes. Since the tetrachord is an interval with the ratio 4:3, the three intervals into which it may be divided are defined by ratios the product of which is equal to 4:3. Khayyām lists twenty one examples of the tetrachord, two of which were his own findings. In these two additions, he divideds the interval 4:3 into combinations of the intervals 5:4-36:35-28:27 and 5:4-40:39-26:25 (Humā'ī, pp. 343-4). He says that these two types were not mentioned in the works of former authors, although they are pleasant to hear The other types were drawn from Ptolemy's *Theory of harmony*, al-Fārābi's *Kitāb al-mūsīqā al-kabīr* (Great book of music), and from Ibn Sina, either from his *Kitāb al-shifā'* (The book of healing) or from the *Dānish-nāma* (The book of knowledge). Each example is further evaluated in terms of aesthetics (Youschkevitch and Rosenfeld, p. 326). A Persian free translation of Khayyām's treatise on music by T. Bīnesh is published in Iran in 1994 (see bibliography). I have prepared a critical edition of this work with an introduction, Persian translation and commentary which will appear in the Iranian journal *Rahpūye-ye honar.*

2. Astronomy

After the fall of the Sasanian dynasty in Iran, the method for calculation of leap years was abandoned, and the lunar calendar used by Arabs replaced the Iranian solar calender. But the lunar calendar was not suitable for agricultural activities and caused certain problems in the collection of taxes. In 467/1074-5, Malikshāh ordered a group of astronomers - including Khayyām-to improve the Iranian calendar. This task was performed in the observatory built by Malikshāh's order in Isfahan. The new calendrar was called *Jalālī* or *Malikī* after the name of Sultan Jalāl al-Dīn Malikshāh who proclaimed it as the formal calendar of Iran. If is still the basis of the current official calendar in Iran. It is the most accurate calendar in the world, because it takes into account the actual year transfer (i.e., the exact moment when the Sun enters Aries) accurately determined by astronomical calculations for each year, whereas other calendars have conventional rules for this purpose.

Khayyām participated also in composing the *Zīj-e Malikshāhī* in Isfahan observatory. Quṭb al-Dīn Shīrāzī, in his treatise *Nihāyat al-idrāk fī -dirāyat al-aflāk* written in 675/1276-7, has unjustly criticized Khayyām's method for determining the leap years in that *zīj* (Taqīzādeh, p. 174). No copy of this *zīj* has survived. However, its catalog of 100 fixed stars, giving their ecliptic coordinates and magnitudes, exists in the anonymous ms. Ar. 5968 in the Bibliothèque Nationale, Paris.

3. Physics

Khayyām has two short treatises on weighing, both preserved in the *Mīzān al-ḥikma by* 'Abd al-Raḥmān Khaāzinī (pp.87-92, 151-3). The first, entitled *Mīzān al-mā',* (The water balance) or *Risāla fī iḥtiyāl ma'rifat miqdāray al-dhahab wa'l-fiẓẓa fī jism murakkab minhumā* (Treatise on the art of determining the quantities of gold and silver in a body composed of both), was published in A'ẓamgarh (1932) and Hyderabad (1940). Its German, Russian and Persian translations have also been published (Youschkevitch and Rosenfeld, p. 332; Qurbānī, p. 334).

The second treaise, *Fi'l-qusṭās al-mustaqīm* (On the right balance), describes as balance with a mobile weight and variable scales.

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BIOGRAPHY

1. His time

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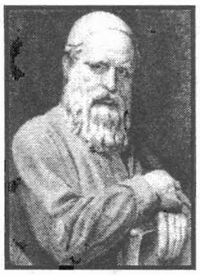
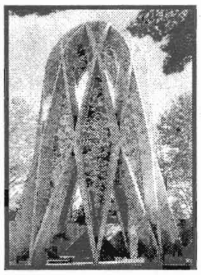
Abu'l-Ḥasan Beyhaqī in the *Tatimma Ṣiwān al-ḥikma* (p.112) has given the horoscope of Khayyām's birth, without any reference to the date. In 1941, an Indian scholar, Swami Govinda Tirtha (pp. xxxii-xxxiv), by using this horoscope and collating it with some other uncertain data on the subject, determined the exact time of Khayyām's birth to have been at sunrise on Wednesday 18 May 1048. This date was later confirmed by the Institute for Theoretical Astronomy of the Soviet Academy of Science (Moṣaḥeb, p.130).



'Umar Khayyām



Tomb of 'Umar Khayyām

Niẓamī 'Arūzī Samarqandī in his *Chahār maqāla* mentions that he met Khayyām in Balkh in 506/1112-3,* and that Khyyām told him (p.100): "My tomb will be at a location where each spring the northern wind will pour flowers on me." He also mentions that in the winter of 508/1114-5, Khayyām was in Marv, and that the Sultan had asked him to determine a propitious day for hunting (p.101). Niẓāmī writes that in 530, four years after Khayyām's death, he visited his tomb (p.100). This indicates that Khyyām died in 526/1131-2. This date was confirmed by some recent researchers; however, other sources mention the year 517/1122-3

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