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KŪSHYĀR IBN LABBĀN'S TREATISE ON HINDU ARITHMETIC

Mohammad Bagheri[1]

P.O.Box 13145–1785, Tehran, IRAN.

Abu'l-Hasan Kūshyār ibn Labbān Jīlī(=Gīlānī) was an eminent Iranian astronomer and mathematician who flourished about ten centuries ago. There is very little information about his life despite his remarkable scientific legacy. Kūshyār hailed from Gīlān province in Iran, situated on the southern coast of the Caspian Sea. For many years, he lived in the ancient city of Ray,near the present capital of Iran, where he studied, taught, and eventually met al-Bīrūnī. He then moved to Gorgān province in northern Iran, and worked as an astronomer at the court of Qābūs ibn Voshmgīr of the Ziārid dynasty. In March 1988, Kūshyār's millenium was celebrated at Gīlān University during the 19th Annual Iranian Mathematics Conference.

The province of Gīlān, Kūshyār's native land, is covered with forests and separated from central Iran by Alborz mountain range, and was therefore conquered by Arabs much later than the rest of the country. Hence, Iranian culture and civilization resisted Arab influence and lasted longer in the region. Even after the advent of Islam, most rulers of the region favored the Shi'ite sect thereof, also mostly accepted by Iranians. Some of these rulers encouraged science, arts, and Persian literature, and made their courts a place of refuge for thinkers and scholars. At the time of Kūshyār there were still many Zoroastrains in Gīlān. *Kūshyār* is a pure Persian name: its original

[1] E-mail: sut5@sina.sharif.edu

form, *Gōshyār*, is composed of the name *Gōsh* and the word *yār*, meaning 'a gift from, or aided by, *Gōsh*,' the name of the Zoroastrain angel of useful quadrupeds. Kūshyār had apparently accepted Islam as indicated by his Arabic *kunya* Abu'l-Hasan ("Hassan's father").

In classical Persian literature, there are references to Kūshyār as an eminent scientist. In the preface to a Persian medical encyclopedia *Dhakhīra-ye Khwārazmshāhī* ("The treasure of Khwārazmshāh"), composed in the early 12th century C.E. by Sayyed Esmā'īl Jorjānī, the author narrates that he met in the town of Qom (in central Iran) some of Kūshyār's descendants who possessed some of his works neatly calligraphed by Kūshyār himself. "Kūshyār," they explained, "wrote only when he was calm and relaxed. When he was told that this style of writing required too much time to complete a single book, his response was: 'Yes, it takes much time, but once I am gone, people won't be concerned with how long I took to write them, but rather with the quality and contents of the books'".

* * *

All Kūshyār's treatises are written in Arabic, the scientific language used in the world of Islam in that period. Kūshyār's most important astronomical work is *Zīj-e jāmi'* (lit., "The comprehensive astronomical tables"). Professor E.S.Kennedy has presented a summary of its contents in *A survey of the Islamic astronomical tables*. I am now preparing an edition of the Arabic text with English translation and commentary, using copies of several manuscripts kept in Istanbul, Leiden, Cairo, Alexandria, Berlin, and Moscow. About one century after the composition of this work, its first section, out of four sections, was translated into Persian a unique manuscript of which is extant in Leiden now. I have presented a paper on this Persian translation in Strasbourg in June 1995.

Kūshyār has also composed a similar work entitled *Zīj-ebāligh* (lit., "The expressive astronomical tables"). No complete copy of it has been found; however, a single chapter thereof entitled *Fī isti'māl adwār kawākib 'alā madhhab al-hind* ("On the application of the cycles of planets according to the Indians' method") is extant in the Mulla Firuz collection in the library of the Cama Oriental Institute in Mumbai. I have presented a paper on this chapter in Suwedia (Syria) in April 1993.

Kūshyār's treatise on astrology *Mujmal al-uṣūl fī aḥkām al-nujūm* ("A summary of the principles of astrology"), also named *Al-mudkhal fī ṣinā'at aḥkām al-nujūm* ("An introduction to the art of astrology") was widely known and used in the Islamic period. Prof. Michio Yano from Kyoto University has published an edition of the Arabic text

with English translation and commentary, and an old Chinese translation thereof in 1997. There are Persian and Turkish translations, commentaries and summaries of this well-known work.

Kūshyār's treatise on astrolabe is also extant in several manuscripts. It has been edited by Mr.Taro Mimura, a student of Prof. Michio Yano, in Japan. He plans to publish it with an English translation. I have prepared an edition of an old Persian translation of this work, due to be published in Iran.

* * *

Kūshyār's principal work on arithmetic is entitled *Uṣūl ḥisāb al-hind* ("Principles of Hindu reckoning") refers to the decimal numeral system and the arithmetical operations that can be made by using it. This system may be considered one of the most important contributions of Indians to the science of human kind. It was mainly introduced into Iran and the rest of Islamic territories in the 8th and 9th centuries, and thence it was transmitted to Europe. These numerals were called "Indian numerals" by the Islamic-period mathematicians, while Europeans who learned them from the Arabs called them "Arabic numerals".

Four manuscripts of Kūshyār's treatise on Hindu arithmetic have been located in Turkey, Iran, India and Egypt, up to now. The oldest known was in Aya Sofya library (MS.4857/7), but it is now kept in Sulaymaniya library, Istanbul. Paul Luckey first noticed it, and in his book on al-Kāshī, an Iranian mathematician and astronomer of the 15th century, published in 1951, he referred to it several times. This treatise is composed of two sections. The first one on decimal operations contains nine chapters as follows: 1. Introduction to the nine numerals; 2. Addition; 3. Subtraction; 4. Multiplication; 5. Order of a product; 6. Division; 7. Order of a quotient; 8. Extratracing square roots; 9. Checking the operations. The second section, on sexagesimal operations, has 16 chapters as follows: 1. On the sexagesimal table; 2. Writing numbers in the sexagesimal system; 3. Addition; 4. Subtraction; 5. Multiplication; 6. Order of a product; 7. Division; 8. Order of a quotient; 9. Extracting square roots; 10. Order of a square root; 11. Checking the operations; 12. Results of the previous chapters; 13. Sexagesimal table; 14. Table for the order of a product; 15. Table for the order of a quotient; 16. Extracting cube roots. Chapters 5,7, and 9 of this second section are found in MS. 4060/3 in Mar'ashīLibrary in Qom (Iran). In MS. 8/4 MMF in Cairo, which will be mentioned later, there is an excerpt of the second section of Aya Sofya manuscsript in six chapters with a lacuna at the beginning. These 6 chapters cover chapters 3–11 of the second section of the Aya Sofya text.

The chapters 13,14 and 15 of the second book are missing in the Aya Sofya manuscript. This manuscript, copied in 682 A.H./1283-4 C.E., has been the basis of the English translation by Martin Levey and Marvin Petruck published in 1956 with a facsimile of the manuscript. in 1967, the late Prof.A. S.Saidan published an edition of the Aya Sofya ms. with a useful introduction and explanations in *Majallat al-ma'had al-makhṭūṭāt* ("Journal of the Manuscripts Institute") in Cairo. In 1975, the French translation of the same text by the late Prof. Aly Mazahéri was published by Nice University, France.

There exists another version of Kūshyār's treatise on arithmetic in a manuscript (MS. 2092/4) in the Central Library of Tehran University. In 1971, the late Prof. Abolghassem Ghorbani, an eminent Iranian historian of mathematics who passed away about one year ago in Tehran, included a facsimile of this manuscript in his Persian bio-bibliographical work entitled "Iranian mathematicians". This version, entitled *'Uyūn al-uṣūl fi'l-ḥisāb* ("Sources of operations in arithmetic"), is written as a single section containing 12 chapters mainly on decimal operations with some remarks on sexagesimal fractions. These chapters cover most parts of the first section and some parts of the second section and some parts of the second section of the Aya Sofya text.

There is also a Hebrew translation and commentary of Kūshyār's treatise, entitled *Iyyūn hāiqārīm* by Shālôm ben Joseph 'Anābī, who lived in Constantinople in the 15th century. The manuscript of this work is extant in the Bodleian library, Oxford (MS. Oppenheim 211). Levey and Petruck have used this Hebrew manuscript in their English translation of the work, and have outlined the contents of the Hebrew manuscript in their introduction to the English translation. They conclude as follows: "The first eight chapters [of the Hebrew text] correspond to the first eight of the Arabic Book I.[i.e., the first section of the Aysa Sofya text]. The twelfth Hebrew chapter is a commentary on the last Arabic chapter of Book I. In chapter 9 the [Hebrew] commentary discusses the results [i.e., the order] of square root and in chapters 10 and 11 deals with the cube root, which is not taken up until Book II, chapter 16, of the Arabic. The commentary is, throughout, very detailed and in it not all of the Arabic Book II is missing. In this way the commentator attempted to cover the two books [sections] of [Kūshyār] ibn Labbān's work in a summary fashion."

A study of the chapter titles shows that the Hebrew translation was made from a version of the work similar to the Tehran manuscript. Mr. Claudio Cecotti from Udine (Italy) has studied the Hebrew text and has presented a paper on it at the University of Gīlān and Tehran University in June 1998; a Persian translation of his paper has

appeared in *Payandeh Memorial Volume* in Tehran (2001). Khurshid F.Abdullazada from Khujand State University of Tadjikstan has published a facsimile of the Tehran manuscript with a Russian translation in his Russian book entitled "Kūshyār Jili" (Dushanbe,1990).

A third manuscript of Kūshyār's treatise on arithmetic, kept in the said Mulla Firuz collection, Cama Oriental Inst. Mumbai(MS. 86[302]), and entitled *Uyūn al-uṣūl fi'l-hisāb al-hind,* is a combination of the other two titles. Its structure is also a mixture of the two previous texts. Like the Aya Sofya manuscript, it is composed of two sections, on decimal and sexagesimal operations, respectively. The text of the first section, in 12 chapters, is almost the same as that of Tehran University manuscript. The 2nd section consists of six chapters which summarize the 2nd section of the Aya Sofya manuscript. The most important advantage of this manuscript is the inclusion of sexagesimal multiplication tables and of tables for orders of products and quotients. In this respect, it complements the missing chapters of the 2nd section of the Aya Sofya text. The Mumbai manuscript, somewhat damaged and written in an inelegant hand, has not been critically studied yet.

Finally, the 4th manuscript of Kūshyār's treatise on arithmetic, kept in the Egyptian National Library (Cairo) (MS. 8/3 MMF), consists of 12 chapters and its contents are similar to those of Tehran University manuscript. However, it contains the tables for the order of products and of quotients missing in the Tehran manuscript.

* * *

Kūshyār's *Principles of Hindu reckoning* was thought to be the oldest extant Arabic text on Hindu reckoning unstil 1966 when Prof. A. S. Saidan made known *Al-fuṣūl fi ḥisāb al-hind* ("Chapters on Hindu Reckoning") by Abu'l-Ḥasan Aḥmad ibn Ibrāhīm Uqlīdisī (who worked in Damascus around 950 C.E.). Saidan published it in 1973 in Jordan. Nevertheless, Kūshyār's work has conserved its importance on account of its role in popularizing Hindu arithmetic and developing the relevant terminology.

In this work, Kūshyār has described, among other things, a complete sexagesimal system for both integral and fractional parts of numbers. Such a description of this system is not found in the works of his eminent predecessors and temporaries like al-Bīrūnī and al- Khwārazmī. However, al-Karajī in early 11th century and Jamshīd al-Kāshī in the 15th century were acquainted with the complete sexagesimal system.

Kūshyār has used a method for extracting roots that was later rediscovered and named "Ruffini-Horner's method." Kūshyār's method has some advantages in the op-

eration layout as compared with Uqlīdisī's, and the difference is such that makes it similar to the methods used by ancient Chinese mathematicians (from the 1st to the 10th centuries) for extracting roots. In this respect, Samaw'al ibn Yah-yā's and al-Nasawī's methods are similar to Kūshyār's, but al-Khwārazmī and al-Kāshī have used methods more similar to Uqlīdisī's. This rises the conjecture of Chinese influence on Kūshyār, and Kūshyār's influence on al-Nasawī(11th century) and Samaw'al ibn Yaḥyā (12th century). The scientific connection with China may have been made through the commercial road between China and France *(Firanja)* mentioned by Ibn Khurdādhbih in the 9th century. This road was used by Jewish merchants who spoke Arabic, Greek, Frankish, Slavonic, Spanish and Persian, and traveled both by land and sea. These mechants, called Radhanites *(al- Rādhāniya)*, have played a role in scientific and technological transmissions. According to ibn al-Nadīm, Chinese was understood in Khazar lands north of the Caspian Sea.

* * *

As Prof. Saidan has mentioned, the transmission process of Hindu reckoning to the Islamic-period mathematics is not clear enough. He believes that it possibily took place indirectly or orally, for the following reasons: 1) The sectioning of the Islamic-period treatises on Hindu mathematics is different from that of the original Indian sources on the subject; 2) Thesre is no trace of Sanskrit terminology in the Islamic-period treatises on Hindu arithmetic; 3) The operations of duplication, bisection and the checking of operations by casting out nines are usually mentioned in the relevant Islamic-period treatises, but we do not find them in the Indian sources; 4) The Indian approach to proportions which was descsribed in al-Bīrūnī's treatise *Rāshīkāt al-hind*("The Indian *rashīks*") are not found in the relevant Islamic-period treatises which rather follow Euclid's approach in his *Elements*.

I hope that with the newly found manuscripts of Kūshyār's treatise on Hindu arithmetic, further researches will be undertaken using the existing data on the intercultural connections, which in turn will enrich the present data. This deserves such an interest because of the enduring importance of the decimal numeral system devised by Indians, which continues to be universally used, thus superseding the numerous numeral theories and methods propounded over past centuries.

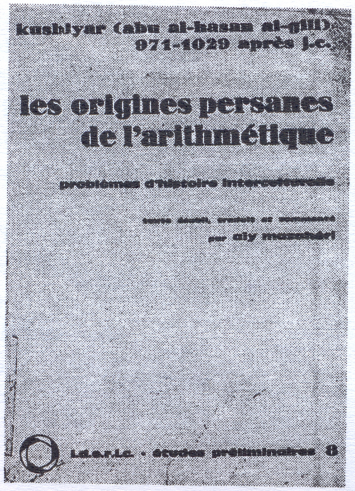
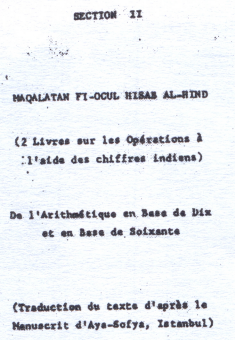


Figure 3. Titlepage of the book containing the French translation of Kūshyār's arithmetic treatise



SECTION II

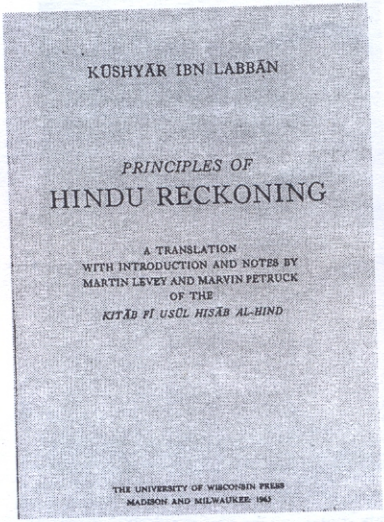
MAQALATAN FI-OCUL HISAB AL-HIND

(2 Livres sur les Opérations à l'aide des chiffres indiens)

De l'Arithmétique en Base de Dix et en Base de Soixante

(Traduction du texte d'après le Manuscrit d'Aya-Sofya, Istanbul)

Figure 4. Beginning of the section of the book containing the French translation


KŪSHYĀR IBN LABBĀN

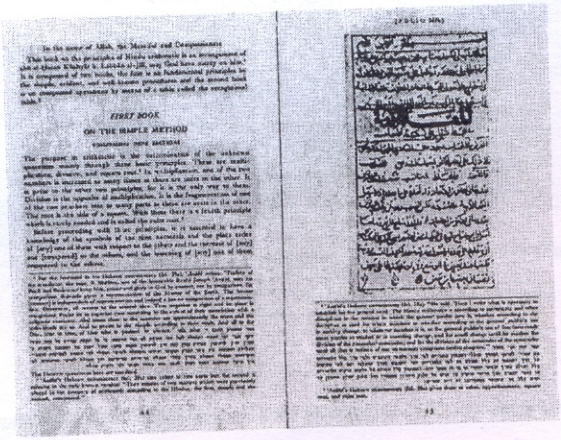
PRINCIPLES OF

HINDU RECKONING

A TRANSLATION
WITH INTRODUCTION AND NOTES BY
MARTIN LEVEY AND MARVIN PETRUCK
OF THE
KITĀB FĪ USŪL HISĀB AL-HIND

THE UNIVERSITY OF WISCONSIN PRESS
MADISON AND MILWAUKEE 1965

Figure 1. Titlepage of the English translation of Kūshyār's treatise on Hindu arithmetic


FIRST BOOK

ON THE SIMPLE METHOD

Figure 2. A page from the English translation with the facsimile of the corresponding folio of AYa Safya ms.

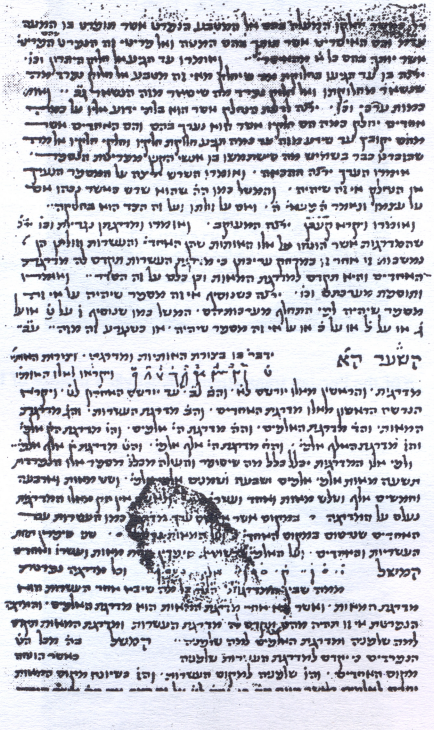
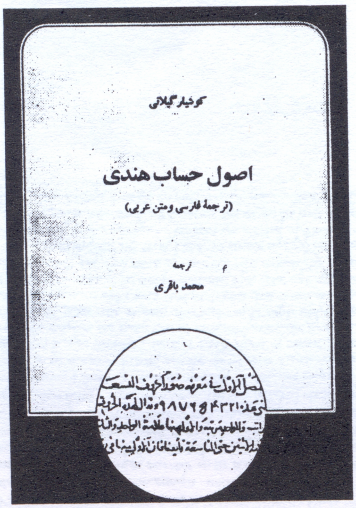


Figure 5. A folio of the ms. containing the Hebrew translation of Kūshyār's arithmetic treatise


کوشیار گیلانی

اصول حساب هندی

(ترجمهٔ فارسی و متن عربی)

ترجمه

محمد باقری

Figure 6. Titlepage of the Persian translation of Kūshyār's treatise of Hindu arithmetic

Figure 7. Beginning of Kūshyār's treatise on Hindu arithmetic in Mumbai ms.

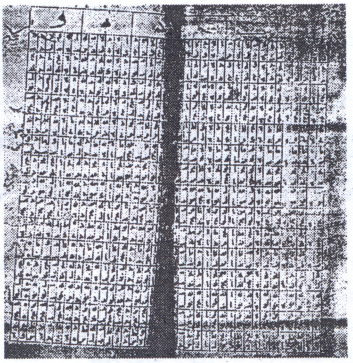


Figure 8. Beginning of the sexagesimal multiplication tables in Mumbai ms.

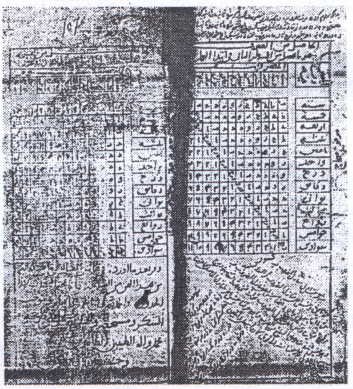


Figure 9. Tables for the order of a product and the order of a quotient in Mumbai ms.