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Mathematical Problems of the Famous Iranian Poet Nāṣer-e Khosrow

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In this article I present three problems and their solutions from a newly discovered fragment of a lost mathematical work of Nāṣer-e Khosrow, a great Iranian poet and thinker who lived in the 11th century. The first and second problems lead to indeterminate linear equations and the third one leads to an indeterminate quadratic equation.

در این مقاله سه مسئله و حل آنها از کتاب ریاضیات نایافتهٔ ناصر خسرو شاعر و متفکر ایرانی قرن پنجم هجری بیان می شود . مسئلهٔ اول و دوم به معادلات سیالهٔ درجهٔ اول و مسئلهٔ سوم به معادلهٔ سیالهٔ درجهٔ دوم منجر می شود .

في هذه المقالة ثلاث مسائل وحلّها من كتاب الرياضيات المفقود للشاعر المفكّر الايراني ناصر خسرو في القرن الخامس الهجري . تنتهي المسألة الاولى والثانية الى المعادلات السيّالة من الدرجة الاولى والمسألة الثالثة تنتهي الى المعادلة السيّالة من الدرجة الثانية .

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KEY WORDS: Nāṣer-e Khosrow, indeterminate equations, Islamic mathematics, Iranian mathematics.

Abū Mo'īn Nāṣer ibn Ḥāreth Qobādiyānī, generally known as Nāṣer-e Khosrow, was born in A.D. 1003 at Qobādiyān in the vicinity of Balkh (Afghanistan) and died in A.D. 1088 at Badakhshān in modern Tajikistan.[1] He was a great Persian poet and prose writer as well as a noted Ismā'īlī thinker and propagandist. During his youth, he was engaged in official affairs at the courts of the Seljūk and the Ghaznavīd rulers. At the age of 43, he set out on a pilgrimage to Mecca. His journey lasted 7 years and included four pilgrimages to Mecca. He traveled through Ḥijāz, Asia Minor, Syria, Palestine, Egypt, and Sudan. In Egypt he embraced the Ismā'īlī doctrine, which was supported by the Fāṭimid Caliphs. Back again in his native land he met with opposition because of his propaganda for the Ismā'īlī beliefs. He found asylum in the Māzandarān province on the south shore of the Caspian Sea. This province was ruled by Shī'ite princes who tried to follow Iranian traditions and who were closer and more tolerant to the Ismā'īlī beliefs. After a while he moved to the town of Yomgān in the district of Badakhshān, where he spent the rest of his life, continued his propaganda, and produced the bulk of his literary output.

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[1] For information on Nāṣer-e Khosrow, see [4, 7:1006-7; 8, 2:305].

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Nāṣer-e Khosrow was a master of scholastic Islamic theology and philosophy; he knew the whole Koran by heart. He is known primarily for his *Safarnāma* (Book of Travels) [6] and his ethical and theological works, but he was also familiar with mathematics and astronomy, as is evident in his poetry and his philosophical writings.

Mathematics was one of his interests because it is the most abstract science and because of its close connections with logic. In one of his poems, he mentions his abilities in science and literature as the cause of opposition against him, and he refers to his skill in mathematics as well as poetry. In his philosophical work *Zād al-Mosāferīn* (Sustenance for Travelers), he discusses the application of the mathematical concepts of prime, deficient, and perfect numbers to general pedagogical problems [2, 121].

In the beginning of his *Safarnāma*, he narrates that he once attended a lecture by *Ostād* 'Alī Nesā'ī[2] in Semnān, east of Teheran. 'Alī Nesā'ī said: "I know nothing about *siyāq*[3] and I would like to learn some arithmetic." Nāṣer was astonished, and thought: "Knowing nothing, how can he teach?" [6, 3]. He also writes about his discussion with a leader of the Mu'tazilite [Islamic] sect in south Iran, saying: "He was an eloquent man with some claim to knowledge of geometry and arithmetic. We held discussions together on dialectic theology and arithmetic etc." [6, 97].

In his *Jāme' al-Ḥekmatayn* (Sum of Two Wisdoms), Nāṣer mentions that he has written a book on arithmetic in the following way:

> ... and I have composed a book on arithmetic, which I have called *The Curiosities of Arithmetic, and the Wonders of Arithmeticians* [*Gharāyib al-ḥisāb wa 'ajāyib al-ḥussāb*]. I have written it in the form of questions and answers and I have collected in it two hundred arithmetical problems; first comes the question, and then I have given the answer, the way to find it, and the proof of its validity. No science is so much based on proofs as arithmetic. Although nowadays in Khorāsān and the eastern territories no perfect arithmetician exists,[4] I have composed that book for the people of future times, because I have some ability in solving difficulties [i.e., difficult problems]. [7, 307, 308; see also 8, 2:306]

No complete manuscript of this Persian mathematical work has ever been reported, but a short fragment of it, consisting of three problems numbered 30, 31, and 36, is extant in manuscript no. 640/8 in the Malek Library in Tehran.[5] Problems 30 and 31 deal with a mixed rate of exchange between *Dinar* and *Dirham* (medieval units of money); problem 36 is about an indeterminate quadratic equation with rational roots. These types of problems were common in the Islamic mathematical tradition. They had been treated before Nāṣer-e Khosrow by Abū Kāmil (9th

[2] This is Abu'l-Ḥasan 'Alī ibn Aḥmad al-Nasawī, a well-known Iranian mathematician around 1040. See [3, 9:614-615].

[3] *Siyāq* or *siyāqat* was a special method of writing numbers in financial accounts. The method was used in Iran, Asia Minor, Egypt, and Arabian countries from the early 8th century until a few decades ago. It was based on stenographic symbols taken from Arabic calligraphy and on a pre-Islamic Iranian numeration system. See [5].

[4] This statement is surprising because it is generally believed among modern historians that mathematics flourished in Iran in the 11th century A.D.

[5] The manuscript containing this fragment was written between A.D. 1258 and 1268 [1, 5:126]. The single page containing the fragment bears the number 309.

century A.D.) and others [9, 91, no. 3; 10]. Nevertheless, the fragment is of considerable interest because it is the work of a great Iranian poet and also because it is in Persian. The use of Persian instead of Arabic is rather uncommon in mathematical texts of the 11th century.

Below I present an edited Persian text of the fragment with an English translation. My own explanatory additions appear in square brackets. The problems and solutions are not presented in complete accordance with Nāṣer's own description of the work in his *Jāme' al-Ḥekmatayn*, which has been mentioned above.

TEXT

من عجائب الحساب وغرائب الحتاب لناصر بن خسرو

مسئله ل : سه نوع دینارست ، نوعی صرفش بپانزده درم و دوم صرفش بنوزده درم و سیم صرفش ببیست درم . بهژده درم دیناری خریدیم . از هر صرفی چند بود ؟ قسم دینار اول شیء نهیم ، در پانزده ، پانزده شیء بود وقسم دینار دوم دو شیء ، در نوزده ، سی و هشت بود ، و قسم دینار سیم دیناری الا سه شیء ، در بیست ، بیست عدد بود الا شست شیء . مبلغ بیست عدد بود الا هفت شیء ، و آن عدیل دیناری بود . پس شیء دو سبع دیناری بود .

مسئله لا : چهار نوع دینارست ، اول صرف پانزده درم ، دوم بشانزده ، سیم بهفده ، چهارم ببیست . بنوزده درم از هر چهار دیناری خریدیم . درین صورت قسم اول شیء و دوم هم شیء و سیم هم شیء و چهارم دینار الا سه شیء نهیم و مسئله تمام کنیم . بیرون آید شیء عدیل نیم دنگ .

مسئله لو : مالیست مجذور و با جذر بهم مجذور . جذر آن مال شیء نهیم . مجذورش مال بود ؛ با جذر بهم ، مال و جذرش بود و این هم مجذورست و جذر او زیادت از شیء بود . شیء و ثلث عدد نهیم . مربعش مال و دو ثلث شیء و تسع عدد بود ، عدیل مال و شیء . مقابله کنیم ، ثلث عدد شیء بود که مالش تسع عدد بود و ثلث و تسع بهم را جذر دو ثلث بود .

TRANSLATION

From *Curiosities of Arithmetic and Wonders of Arithmeticians*, by Nāṣer ibn Khosrow.

Problem 30: There are three kinds of *Dinar*s. One is exchanged for fifteen *Dirham*s, another for nineteen and the third for twenty *Dirham*s. We bought [a mixture of the three kinds, at the overall rate of] one *Dinar* for eighteen *Dirham*s. How much is there [in the mixture] from each kind?

[Solution:] Let the part of the first kind be the "thing."[6] Multiplication by fifteen yields fifteen things. Let the part of the second [kind of] *Dinar* be two things. Multiplied by nineteen, it becomes thirty-eight things. And let the part of the third [kind of] *Dinar* be one *Dinar* minus three things. Multiplied by twenty, [the result] is twenty units [i.e., *Dirham*s] minus sixty things. The sum will be twenty units minus seven things, and it is equal to one *Dinar* [exchanged by the overall rate]. Then one thing equals two-sevenths of a *Dinar*.[7]

[6] The algebraic vocabulary is the usual Arabic one: "Thing" (*shay'*) is used to designate the unknown, in modern notation x.

[7] In modern notation, the problem is equivalent to the equations $x + y + z = 1$, $15x + 19y + 20z = 18$. Assuming $y = 2x$, $z = 1 - 3x$, we obtain $20 - 7x = 18$, whence $x = \frac{2}{7}$.

Problem 31: There are four kinds of *Dinars*. The first is exchanged for fifteen *Dirhams*, the second for sixteen, the third for seventeen, the fourth for twenty. We bought a mixture of the four [kinds, at the rate of] nineteen *Dirhams* for one *Dinar*.

[Solution:] In this case, let the part of the first [kind] be one thing, and the second also one thing, and the third also one thing, and the fourth one *Dinar* minus three things; and solve the problem. The thing will be found equal to half of one-sixth [of a *Dinar*].[8]

Problem 36: There is a square quantity, and the sum of the quantity and its square root is also square. [Find the quantity.]

[Solution:] Let the square root of that quantity be the thing. Then, its square is equal to the quantity; [the latter] plus the square root is the quantity plus its square root, and this [sum] is also a square; and its square root is bigger than the thing. Let it be equal to one thing plus one-third of the unit. Its square will be the quantity plus two-thirds of the thing plus one-ninth of the unit, which is equal to the quantity plus the thing. By canceling out equals on both sides, we find the thing equal to one-third of the unit, its square is one-ninth of the unit, and the square root of one-third plus one-ninth is two-thirds.[9]

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[8] In modern notation, the problem is equivalent to the equations $x + y + z + v = 1$, $15x + 16y + 17z + 20v = 19$. Taking $x = y = z$ and $v = 1 - 3x$, we obtain $x = \frac{1}{12}$.

[9] This problem is equivalent to the indeterminate equation $x^2 + x = y^2$. Taking $y = x + a$ with $a = \frac{1}{3}$, we obtain $x^2 + x = x^2 + \frac{2}{3}x + \frac{1}{9}$, whence $x = \frac{1}{3}$. The additive term a has to be taken less than $\frac{1}{2}$ in order that the solution x be positive a restriction usual in ancient and medieval mathematics.